GENERAL PHYSICS

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PREFACE TO THE SECOND EDITION

This book provides a course in General Physics which will enable pupils to build up, on an experimental basis, a knowledge of physical principles and an appreciation of their importance in daily life. For the second edition, the sections on electrolysis have been rewritten in accordance with modern ideas, new chapters have been added on alternating currents and on discharge tubes, X-ray tubes, and valves, and various alterations and additions have been made throughout, to meet present-day requirements in Physics for the General Certificate of Education at the Ordinary Level of the various examining bodies.

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GENERAL PHYSICS

SECTION I—MECHANICS

CHAPTER I

INTRODUCTION. MEASURING INSTRUMENTS

Introduction

The study of physics is important because so much of life to-day consists of applying physical principles to our needs and pleasures. The electrical and hot-water arrangements in our houses; cycles, motor cars, trams, trains, and aeroplanes; cameras and the cinema; gramophones and radio sets; and the machines which do so much of the work formerly done by hand, all require a knowledge of physics for their understanding. A complete understanding of many of these things requires a much deeper knowledge than can be obtained from a school course or from this book, but a knowledge of what is included here should enable you to recognise the physical principles underlying much that you see around you and will form a basis for further study if you wish to pursue it. A number of applications of physics are mentioned in the following chapters but, whenever you have learned one of the principles discussed, you should try to find other instances in which it is applied. Thus you will find that the laws, formulae, and calculations of physics are not merely things to remember for examinations, but knowledge which will greatly increase your interest in matters of everyday life.

Measurement

Careful measurement is often required to obtain exact knowledge of physical matters. It is assumed that you already know the Metric system of units and tables, but memorising of these and the notes on page 15 will be useful. It is also assumed that you have practised the use of metre scales, measuring jars, pipettes, and the physical balance. The metre scale enables lengths to be measured to the nearest tenth of a centimetre or inch, but both the physicist and the engineer often need to make more exact measurements, and some instruments for doing this should be known.

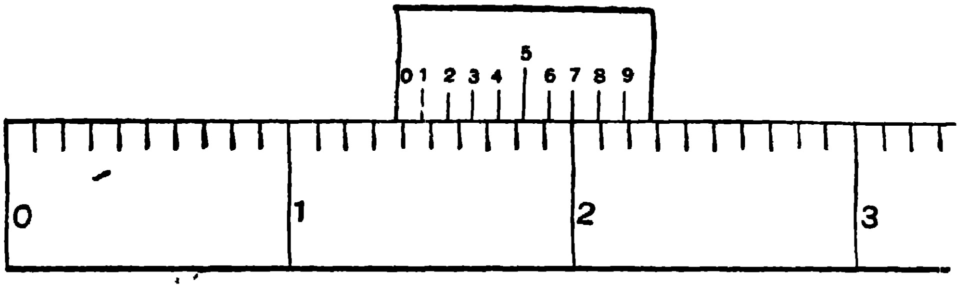


FIG. 1. VERNIER SCALE.

The **Vernier Scale** will be best understood if you make one. Along the edge of a piece of cardboard mark off $\frac{9}{10}$ of an inch. Divide this into ten equal parts and number the marks as shown in Fig. 1. Cut out the piece of cardboard so marked. Place it beside the edge of a scale which is divided into inches and tenths of an inch. Each small division on the cardboard is one-tenth of ·9 in., *i.e.* ·09 in., and is therefore ·01 inch shorter than the small divisions on the scale.

Place the zero mark on the cardboard opposite to one of the marks on the main scale. To bring mark number 1 on the cardboard opposite a mark on the main scale, it must be moved ·01 in. to the right. Similarly to bring mark number 2 opposite a mark on the main scale it must be moved ·02 in., and so on. Now slide the cardboard to the position shown in the diagram. The zero on the cardboard scale is a little more than 1·3 in. from the zero of the main scale. Division 7 on the cardboard is opposite to a mark on the main scale so, to bring

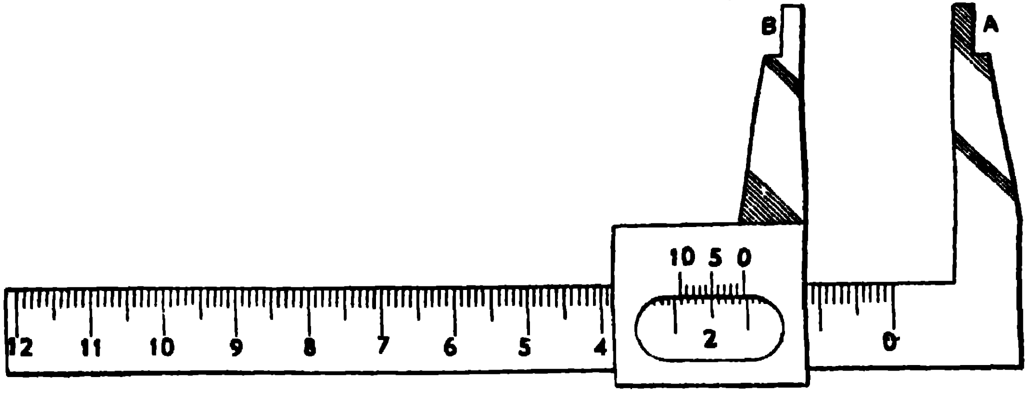


FIG. 2. SLIDE CALIPERS.

the cardboard zero to the 1·3 in. mark, it would have to be moved ·07 in. to the left. Thus the distance between the two zeros is 1·37 in., *i.e.* whole inches and tenths of inches are read from the main scale and the little bit over is read in hundredths of an inch from the mark on the cardboard which is opposite to a mark on the main scale.

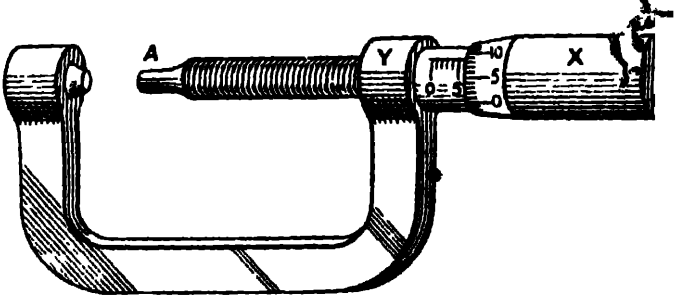


Fig. 3. Screw Gauge.

The **Slide Calipers** are illustrated in Fig. 2. The stem to which the jaw A is fixed is graduated in inches or centimetres and tenths of those units. The jaw B slides along the stem and carries a vernier scale. The zero of the vernier should be just opposite to that of the fixed scale when the jaws are tightly closed. If this is not the case the "zero error" should be noted and correction made for it after taking a reading. When the jaws are opened their distance apart will be given to the nearest hundredth of an inch or centimetre by the reading of the scales.

This instrument is useful for measuring the dimensions of small objects such as the diameters of rods and tubes.

The **Screw Gauge** (Fig. 3) is used to measure very small distances, such as the diameter of a wire, to the nearest hundredth of a millimetre. The collar X is attached to a screw passing through the stem Y. The screw thread is accurately cut so that one complete turn of the collar moves the end A one millimetre lengthwise. A scale of millimetres is engraved along Y and the bevelled edge of X is divided into 100 equal parts. If X is turned sufficiently for one of these divisions to pass the line along Y, the point A moves ·01 mm. When the jaws are closed the edge of X should be at the zero of the scale on Y and the zero of the circular scale should be opposite the line on Y. When the jaws are opened, the number of complete turns—millimetres—is read from the main scale and the number of additional hundredths of a millimetre is read from the mark on X opposite to the line on Y.

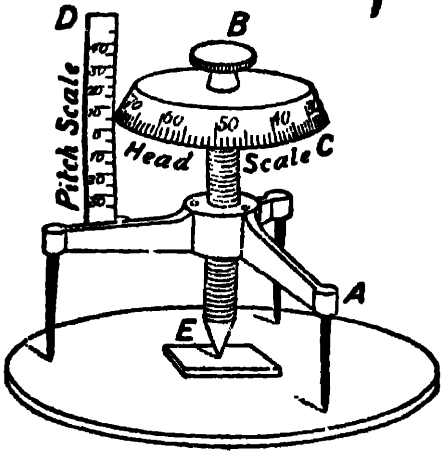


Fig. 4. Spherometer.

The **Spherometer** (Fig. 4) is based on the same principle as the screw gauge. A rigid frame is supported on three fixed feet. An accurate screw carrying a disc on its head passes through the centre of the frame. A vertical scale of millimetres is fixed on the frame nearly touching the edge of the disc which is divided into 100 equal parts. The reading of the circular scale is taken opposite to the fixed scale. Zero readings on both scales should be obtained when the instrument is standing on a flat surface which is just touched by the movable foot. The fixed scale is graduated upwards and downwards from a zero in the middle so that the distance of the movable foot above or below the plane of the fixed feet may be measured.

In using all these instruments preliminary examination for zero error is necessary, and note should be made as to whether the correction should be added to or taken from the readings. Further, the above descriptions give the principles of the instruments but differences in details occur, *e.g.* it may require two complete turns of a spherometer screw to move the point one millimetre. In that case, if the circular scale has 100 divisions, each of them represents ·005 mm. Such details should be noted before the instrument is used.

CHAPTER II

INERTIA, FORCE, MASS, WEIGHT

Most people have a general idea of what is meant by the word "force," but for scientific purposes a definite meaning must be attached to it. If you consider a few cases in connexion with which you would use the word, *e.g.* throwing a cricket ball, kicking a football, pulling in a tug-of-war, the action of a magnet on a nearby pin, you will realise that it is usually associated with actions producing motion or altering the motion of bodies.

Inertia

Bodies at rest resist attempts to set them in motion. This resistance will be felt if you pull a horizontal thread attached to a brick suspended from a string, and may be so great that a sharp tug breaks the thread without causing much movement of the brick. Common experience also shows that a moving body tends to maintain its motion. When a train or motor car is suddenly stopped the passengers are shot forward because their bodies tend to maintain their forward motion. When loaded trucks are being pushed along level rails, considerable effort is often needed to start them, but, once started, little effort is needed to keep them moving, and a big effort is needed to bring them to a sudden stop. Numerous similar examples might be given.

(A moving body tends to maintain not only its kind of motion but also the speed with which it is moving and its direction of motion. Owing to the effects of friction (Chapter VIII.) and gravitation (page 9), these facts are only approximately shown by actual experiences, but the tendencies can be seen in cases where friction and gravitation have very small effects. A top with a sharp point spinning on a hard smooth surface loses its speed of rotation very slowly. This is also true of the wheel of a toy gyroscope which has smooth, well lubricated bearings (Fig 5). A smooth piece of ice sent skimming along the surface of a smooth sheet of ice will travel a long distance with little loss of speed, and it travels in a straight line.

The tendency to maintain direction of motion is shown when a stone is projected from a sling. While it is in the sling the stone has a circular motion. As soon as it is released it shoots off along a straight

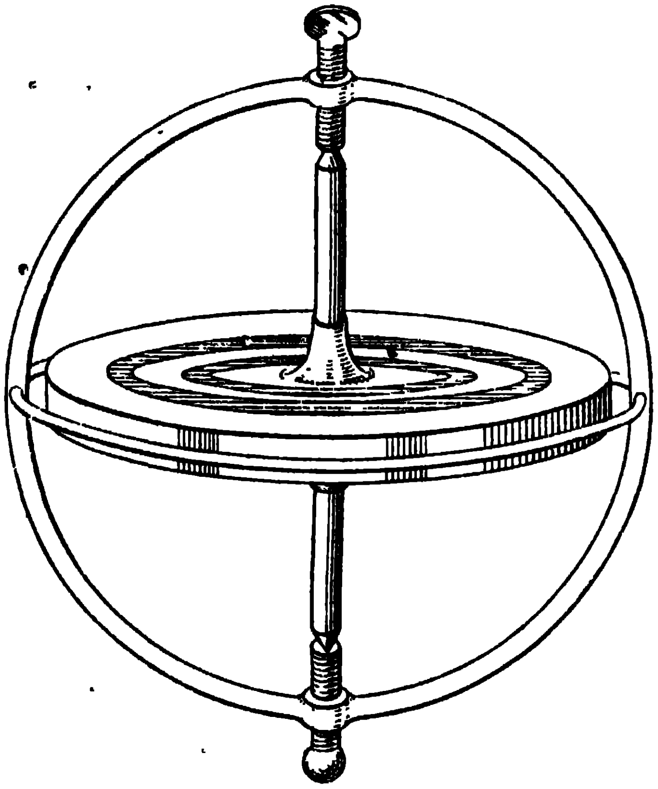


Fig. 5. Gyroscope.

line (Fig. 6) continuing its direction at the moment of release. The same tendency is shown by drops of water flung off from the rim of a revolving wheel.

Owing to this *tendency of bodies to remain at rest, or to continue any motion they are performing*, they are said to possess the property of inertia. It is because of this property that heavy fly-wheels are introduced in many machines. The impulses given to the machine are intermittent and would produce a jerky action. Once the fly-wheel has started revolving, its tendency to continue at the same speed gives a steadiness to the motion of other parts geared to it. The same principle is applied in the *gyroscopic compass* (Fig. 7). The magnetic compass does not point to the true north, and it may be made unreliable by masses of iron near it. The gyroscopic compass contains a heavy wheel mounted on a horizontal axis. This is mounted in a frame free to move about a vertical axis. The wheel is kept revolving by electrical means and, if it is started with its plane in an east to west direction, its inertia will tend to maintain it in that direction however the ship may turn.

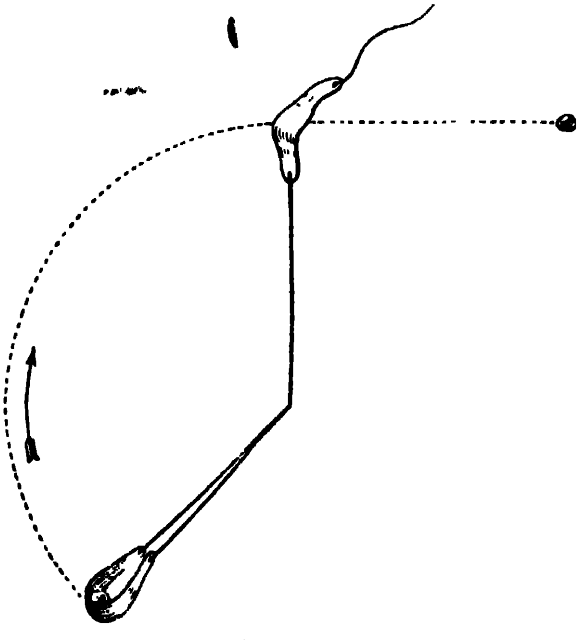


Fig. 6.

The property of inertia is also utilised in the picking up of water by a travelling locomotive. When the scoop A of the travelling locomotive (Fig. 8) moves along the water trough, the resistance of the water in front to being given a forward motion forces the water in the mouth of the scoop up the pipe and into the tank.

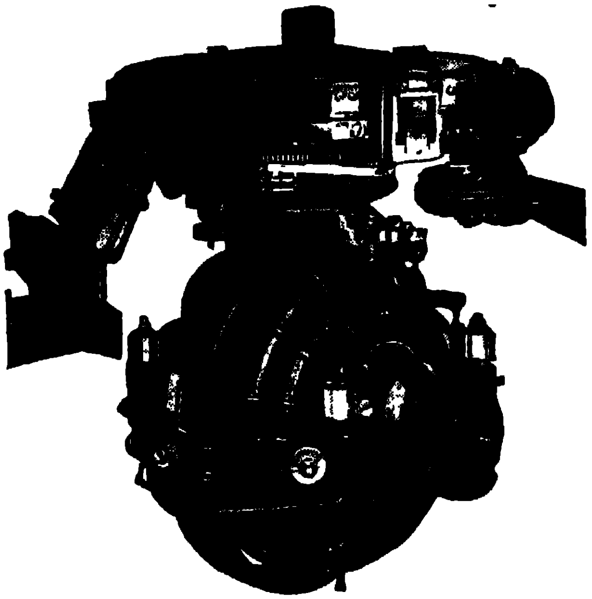


FIG. 7. GYROSCOPIC COMPASS.

Newton's First Law. Force

Newton, who made a study of the motion of bodies during the latter part of the seventeenth century, came to the conclusion that **every object remains at rest or moves with uniform speed along a straight line unless compelled to do otherwise by forces acting on it.** From this law the term "force" is often defined as *anything that will cause a change in the state of rest or motion of a body.* This does not really say what a force is, but reminds us that whenever we observe a body start or stop moving, or change its speed or direction of motion, forces are operating to produce these changes.

Mass. Units of Mass

Every body is said to have **a mass** which is usually defined as *the amount of matter in the body.* Consider a number of ounce weights, some of iron and some of brass. Each of the iron ones will have the same volume and, since they consist of the same kind of matter, each must contain the same amount of matter. Similarly every brass weight will contain the same amount of matter as each of the others. A brass weight will be slightly smaller than an iron weight, but, in spite of this difference in size, any one

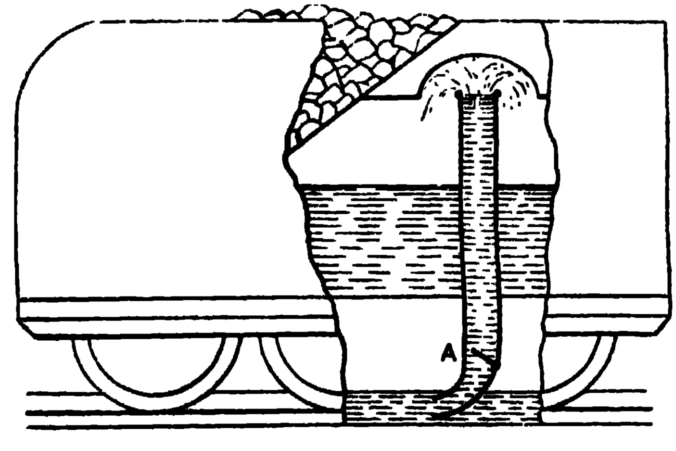


FIG. 8.

of the brass weights will balance any one of the iron weights on a true balance, and each brass weight is said to contain as much matter as each of the iron weights. It is assumed that the matter is more closely packed in the brass than in the iron. A piece of cork which would balance one of the weights would have a much larger volume but would only contain the same amount of matter, in this case less closely packed.

It will be seen that the practical test of equality of masses is that they will balance on a true balance. The purpose of the common balance is to measure masses by counterpoising them with standard masses which are called "weights."

7 Units of Mass.—The standard of mass in the British system is the Imperial Standard Pound, which is a piece of platinum kept at the Board of Trade Offices. A mass which will balance the standard pound is kept at each Weights and Measures office for testing tradesmen's "weights."

In the metric system the unit of mass is the gramme, which is the mass of 1 c.cm. of pure water at 4° C. As standard, a piece of platinum, the Standard Kilogramme, which has a mass equal to that of a litre of water at 4° C. and therefore equal to 1000 grammes, is kept at Sévres, near Paris.

Weight

The terms "mass" and "weight" are often used as though they have the same meaning, and the measurement of mass by the common balance is called "weighing." Actually the two words refer to different things. Bodies raised from the earth tend to fall when support is withdrawn from them. Newton appears to have been the first to think out clearly the meaning of this. According to tradition, the blow he received from an apple falling in his orchard started the train of thought which led him to his *Theory of Gravitation*, according to which each body that exists attracts all other bodies. By means of it Newton was able to explain such things as the motion of the planets and the tides.

Applying the theory to a body near the earth, such as an apple or a brick, the body and the earth attract one another. The body also attracts and is attracted by all other bodies such as the sun, stars, and planets. These bodies are so far away that their effects are very small and, for our purpose, may be neglected. Also the attraction of the body has very little effect on the huge mass of the earth, and that, too, may be neglected so that we may consider that the body experiences a force directed towards the centre of the earth which causes it to fall if it is not supported. *This force, tending to pull a body towards the centre of the earth, is the " weight " of the body.*

The Spring Balance

If we fix one end of a spiral spring and pull the other end, a resistance to the pull is experienced and, when we let go, the spring flies back to its original length. The stretching sets up forces in the spring tending to restore it to its original length and, the more it is stretched, the greater these forces become. This property of a spring is utilised in a spring balance to measure weights of bodies. When a body is hung on a spring, the force due to the earth will pull it downwards until an equal opposing force is set up in the spring which stops further motion.

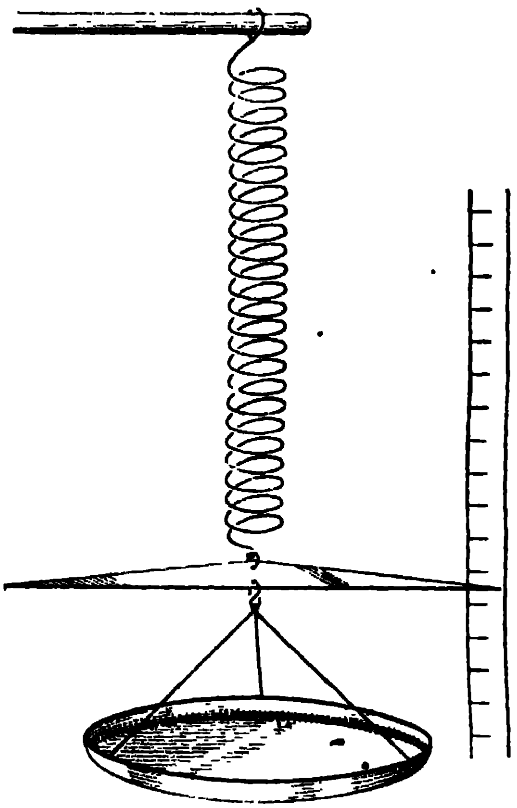


Fig. 9.

To the lower end of a spiral spring hung vertically fix a pointer and a pan just heavy enough to stretch the spring a little (Fig. 9). Stand an upright scale behind the pointer and note the reading. Obtain a number of equal masses, such as ounce weights, and put each in turn in the pan, noting that the same extension of the spring is produced by each. This indicates that equal masses *at the same place* have equal weights. It is for this reason that the terms mass and weight are often used as though they meant the same thing. Also for this reason the weight of a unit of mass is taken as a unit of weight. The weight of a pound or the weight of a gramme, *i.e.* the force with which a mass of 1 lb. or 1 grm. is pulled towards the centre of the earth, are such units. They are usually written 1 lb. wt. or 1 grm. wt.

Now determine the extensions produced by 1, 2, 3, etc., of the equal masses taken together, and draw up a table as follows:—

WEIGHT	READING	EXTENSION	$\frac{\text{EXTENSION}}{\text{WEIGHT}}$
0	67·7 cm.	0	—
1 oz. wt.	70·0 cm.	2·3 cm.	$\frac{2\cdot3}{1} = 2\cdot3$
2 oz. wt.	72·3 cm.	4·6 cm.	$\frac{4\cdot6}{2} = 2\cdot3$
3 oz. wt.	74·5 cm.	6·8 cm.	$\frac{6\cdot8}{3} = 2\cdot2(7)$
4 oz. wt.	76·9 cm.	9·2 cm.	$\frac{9\cdot2}{4} = 2\cdot3$
5 oz. wt.	79·3 cm.	11·6 cm.	$\frac{11\cdot6}{5} = 2\cdot3(2)$

The constant value of the quotient in the last column shows that the extension increases proportionally to the weight causing it.

Plot weight against extension into a graph from your results. Determine the extension caused by a piece of metal placed in the pan, and from the graph find the weight of the piece of metal. Alternatively the weight may be calculated as follows. Suppose the piece of metal produces an extension of 10·8 cm. From the table above, each oz. wt. produces an extension of 2·3 cm.;

$$\therefore \text{ Weight to produce 10·8 cm. extension} = \frac{10\cdot8}{2\cdot3} = 4\cdot7 \text{ oz. wt.}$$

Mass and Weight Compared

From the foregoing, *the common balance measures mass and the spring balance measures weight.* Actually it is the weight of the body and the weight of the "weights" which are balanced by the common balance, but, as both pans are in practically the same place, equality of weights indicates equality of masses. It follows that the spring balance will compare masses so long as it is used in one place only.

An important difference between mass and weight is seen when we consider bodies at different places. Moving a body from one place to another will not alter the amount of matter in it, so that we can say that the mass of a body is constant. The weight of a body, however, does vary with its position. The gravitational attraction between two bodies becomes less as the distance between them increases, so the weight of a body decreases as it gets further from the centre of the earth, and a very sensitive spring balance would show a difference between the weight of a body at the bottom of a mountain and the weight of the same body at the top of the mountain. This difference would not be shown by the common balance since, while the weight of the body would decrease as it was carried up the mountain, the weight of the "weights" which balance it would decrease to the same amount, so that they would still give balance at the top.

Measurement of Forces

Since the weight of a body is a force, units of weight may be used as units of force. By a force of 8 lb. wt. we mean a force equal to the gravitational pull of the earth on a mass of 8 lb. It will be realised that such units are not constant and that a force of 8 lb. wt. at one place on the earth's surface may not be equal to a force of 8 lb. wt. at another place. Such variations from place to place on the earth's surface are small, and engineers, on whose calculations they would have little effect, find it convenient to use gravitational units of force for practical purposes.

FIG. 10.

When a body is raised, the force to be overcome is its weight, so a force equal to its weight must be used. For example, to raise a mass of 7 lb. will require at least a force of 7 lb. wt. When other kinds of movement take place, the force to be overcome is not the weight of the body. For example, when an engine draws a train such forces as the friction between the wheels and the axles and the resistance of the air are being overcome. The force necessary to do this may be measured by coupling the train to the engine by a very strong spring which has been graduated by hanging known masses from it. If, when the engine moves the train, the spring is stretched as much as when a mass of 15 tons is hung from it, the force exerted on the train is 15 tons weight. A spring used in this way for measuring forces is often referred to as a **dynamometer.**

On following page are suggestions for measuring forces in certain cases in the laboratory.

(1) FORCE NEEDED TO CAUSE A BODY TO SLIDE.—(*a*) Attach a spring balance to the body and pull it horizontally till the body just slides (Fig. 10). The reading of the spring balance indicates the force used.

(*b*) Attach a light scale-pan to the body by means of a string passing over a pulley (Fig. 11). Put weights in the scale-pan till the body just slides. Their weight indicates the force used.

(2) FORCE NEEDED TO OVERCOME FRICTION IN THE CHAIN AND REAR BEARINGS OF A BICYCLE.—Support the bicycle off the ground. Turn the cranks until they are horizontal. Hang weights from one pedal until the wheel just begins to turn (Fig. 12). Their weight indicates the force used.

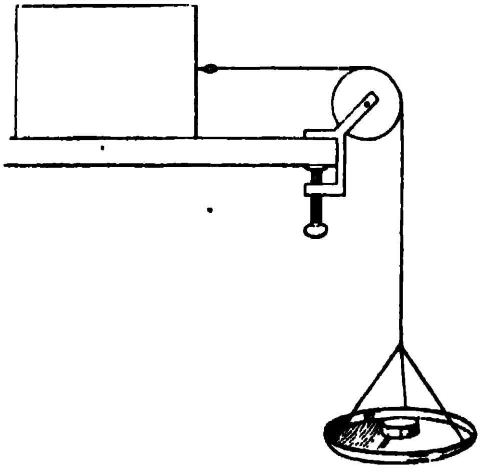


FIG. 11.

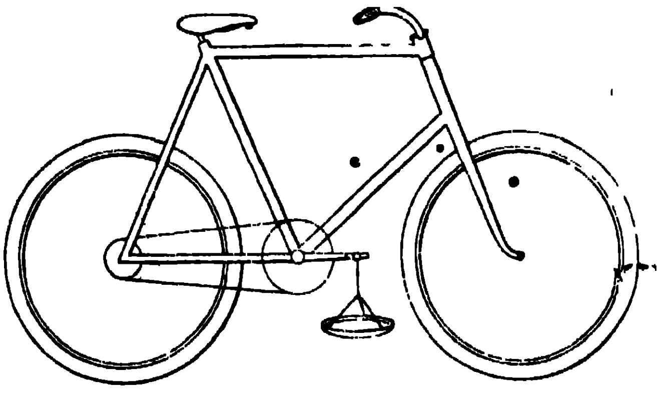


FIG. 12.

QUESTIONS ON CHAPTER II

1. Explain carefully what you mean by (*a*) the mass, and (*b*) the weight of a piece of iron, and how you would measure each.

2. Why does the weight of a body vary at different places on the earth? Would you expect a body to weigh more or less on the moon than on the earth? Give reasons for your answer.

3. In which of the following places would you expect the weight of a brick to be (*a*) more, and (*b*) less than when it is at ground level in London:—The Equator, the North Pole, the top of Snowdon, the bottom of a coal mine in Kent? Give reasons.

Would any difference be found if the brick were weighed on a common balance at each of those places? Explain.

4. Explain what you understand by a force. In what units may forces be measured? How could the force exerted by a tug-of-war team pulling on a rope be measured?

5. State Newton's First Law of Motion, and give three examples from ordinary experience to illustrate it.

6. What do you understand by the inertia of a body? Explain three cases in which use is made of the inertia of bodies.

7. Distinguish between the *mass* of a body and its *weight*.
Do you think that it is possible to (*a*) change the mass of a body without changing its weight, (*b*) change the weight of a body without changing its mass?
If so, describe how you would produce the change. [L.U.

8. Describe the spring balance. What exactly is determined when an article is "weighed" by a spring balance? A quantity of tea is bought in the latitude of the equator, a spring balance being used in the "weighing." What would be the difference if the same tea were weighed by the same spring balance (1) high up in an aeroplane in the latitude of the equator, (2) at the north of Scotland?

9. When you buy a pound of tea are you buying a weight or a mass? Would a penny weigh more or less at sea level than at the top of a mountain? Could the difference be detected by perfectly accurate scales?

10. Hold the ring of a spring balance in your right hand, and the hook in your left, and pull until the reading is 6 lb. wt. Which hand is exerting this force? A boy weighing 100 lb. is standing still. What forces are acting on him?

CHAPTER III

VOLUME, DENSITY, AND RELATIVE DENSITY

Density

When designing a large building or bridge an architect or engineer will need to know the masses of iron girders he intends to use so that he may plan supports of sufficient strength for them. From his plans he can calculate their volumes and, if he knows the mass of a cubic foot of iron, simple multiplication sums will give their masses. Similarly, in the case of a tank lorry to carry petrol, the mass of the load should not be excessive for the strength of the axles, etc., and a knowledge of the mass of one cubic foot of petrol would enable the size of tank to carry the greatest permissible load to be calculated. These examples illustrate the usefulness of a knowledge of the mass per unit volume of a substance. That quantity is called the density of the substance. British engineers frequently express densities in pounds per cubic foot, but in scientific work they are more frequently expressed in grammes per cubic centimetre.

It is evident that it will often be inconvenient to try to obtain exactly 1 cub. ft. or 1 c.cm. of a substance in order to find its density, but, if both the mass and the volume of any piece of a substance are found, its density can be found by dividing the mass by the number of units of volume.

$$\text{Density} = \frac{\text{Mass}}{\text{Volume}}$$

From the definition of a gramme (page 8) it follows that the density of water is 1 grm. per c.cm. In British units the density of water is about 62·5 lb. per cub. ft.

Volume

A volume measurement is usually required when a density is to be found. Rules for finding the volumes of a number of regular solids have probably been learned already but, for reference, several of them are noted below.

Volume of rectangular block = length × breadth × height.
Volume of cylinder = $\pi \times (\text{radius})^2 \times$ height. $\pi r^2 h$
Volume of any solid of uniform cross-section = area of cross-section × length.
Volume of sphere = $\frac{4}{3}\pi \times (\text{radius})^3$.
Volume of cone = $\frac{1}{3}\pi \times (\text{radius of base})^2 \times$ height.

The following relations for a circle should also be known:—

Circumference = $\pi \times$ diameter = $2\pi \times$ radius.
Area of circle = $\pi \times (\text{radius})^2$.

In numerical work the value $\frac{22}{7}$ should be used for π unless other directions are given. It is more correct than the contraction 3·14 which is sometimes used.

The volume of a piece of solid of irregular shape can be found by measuring the volume of liquid it will displace. The story of the discovery of this method by Archimedes when he stepped into a brimming bath is well known. Because of his joyful shouts of "Eureka" ("I have found it") in the excitement of his discovery, pieces of apparatus for finding volumes by this method are called "Eureka cans." Small ones for dealing with rather small solids do not give very accurate results, but bodies from about the size of a cricket ball upwards may be measured with sufficient accuracy as follows.

A hole is punched through the bottom of a tin can large enough to hold the body and a piece of metal tube is soldered into this hole so that it comes about three-quarters of the way up inside the can (Fig. 13). Water is poured into the can until it overflows through the tube. When the flow ceases the can will be filled to the top of the tube. A measuring jar is then placed below the tube and the solid is gradually lowered into the water until it is completely covered. The volume of water which flows out into the jar is equal to the volume of the solid.

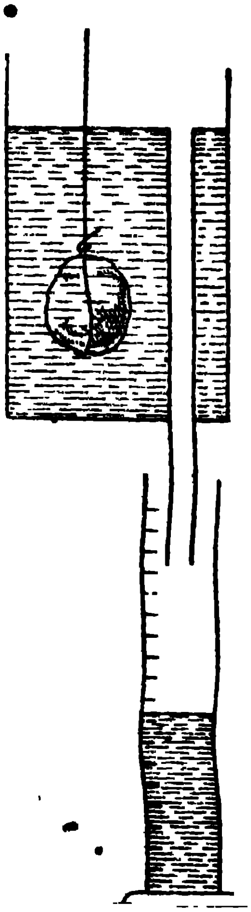


FIG. 13.

Smaller bodies may be more accurately measured as follows. A beaker is partly filled with water and the body, tied to a thread, is lowered into it. A piece of wire bent as shown in Fig. 14 has one end wound round a stand so that it is firmly supported but may be slipped

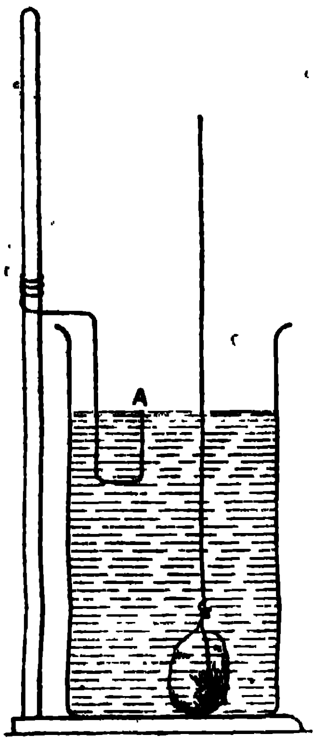


FIG. 14.

up or down the stand. It is carefully adjusted until the point A just touches the surface of the water. If you look upwards from just below its level, the water surface will act as a mirror. When the image of the point just touches the point itself, without overlapping, the point is just at the surface of the water. Withdraw the solid, shaking any drops of water clinging to it back into the beaker. Find what volume of water must be run into the beaker from a burette to bring the surface back to the wire point. This will be equal to the volume of the body.

The above methods only apply to bodies which will sink in water. They may be adapted to bodies which would float by tying below the body a heavy sinker which will drag it under water. In the case of the can, the sinker, but not the body, is lowered into the water before the jar is put in position (Fig. 15), and then allowed to pull the body under. In the other method the sinker is left in the water when the body is removed.

Determination of Densities

(A) SOLIDS.—(1) Solids of regular shape are weighed and the necessary measurements for calculating their volumes are made. In each case $\frac{\text{Mass}}{\text{Volume}}$ gives the density. When dimensions exceed 5 cm., measurement with a metre scale, estimating to the nearest half-millimetre, will give sufficient accuracy. Dimensions of 1 to 5 cm. should be measured with slide calipers with vernier scales. Dimensions of less than 1 cm. should be measured with a screw gauge.

The following experiments are suggested: Find the density of wood, using a rectangular block about 10 cm. × 6 cm. × 4 cm. Find densities of various metals using cylinders about 4 cm. high and 2 cm. diameter. Find the density

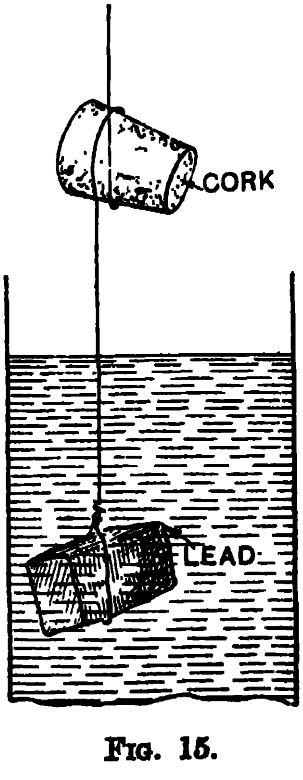


FIG. 15.

of copper using a length of about 10 cm. of stout copper wire. In the last case the wire is a long thin cylinder. Measure its length with a metre scale and its diameter with a screw gauge. The diameter should be measured in several places along the wire and the measurements averaged in case there are slight variations.

(2) Solids of irregular shape may be weighed and their volumes measured by the methods described on pages 15, 16. Glass stoppers, iron bolts, large corks, pieces of coal and stone might be used.

(B) LIQUIDS.—(1) Weigh a small beaker. Run into it a measured volume of the liquid from a pipette or burette. Weigh again and find the mass of the liquid by difference. Calculate the density in the usual way. Densities of milk, paraffin, turpentine, and various solutions may be found in this way. It is not a suitable method if the liquid vaporises easily or if it gives off fumes.

(2) A **density bottle** may be used. This is a small bottle with a close fitting glass stopper through which a narrow hole is bored from top to bottom. If the bottle is filled to the brim and the stopper dropped in, liquid displaced by the stopper will be forced out through the hole and the bottle will be filled to the top of the hole. Thus, exactly the same volume of liquid is contained by the bottle every time it is filled in this way (Fig. 16).

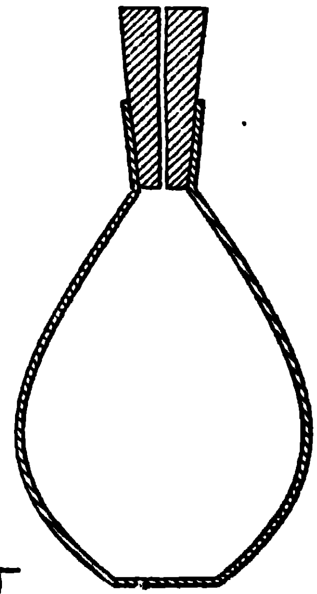


FIG. 16.

Weigh the clean, dry, empty bottle. Fill as above with water, wipe dry on the outside and weigh again. Drain out the water as completely as possible, fill with the liquid whose density is to be found, and weigh once more. Calculate as follows:—

Mass of empty bottle ... ... = 22·44 grm. ......(1)
Mass of bottle filled with water ... ... = 74·15 grm. ......(2)
Mass of bottle filled with liquid ... ... = 80·66 grm. ......(3)
From (1) and (2),
Mass of water which fills bottle ... ... = 51·71 grm.
But 1 c.cm. of water weighs 1 grm.;

$\therefore$ Capacity of bottle ... ... = 51·71 c.cm.

From (1) and (3),

Mass of liquid which fills bottle ... ... = 58·22 grm.;

$\therefore$ 51·71 c.cm. of liquid weighs 58·22 grm.;

$$\therefore \quad 1 \text{ c.cm. of liquid weighs } \frac{58{\cdot}22}{51{\cdot}71} = 1{\cdot}13 \text{ grm.,}$$

i.e. the density of the liquid is 1·13 grm. per c.cm.

SOME COMMON DENSITIES

IN GRAMMES PER CUBIC CENTIMETRE

SUBSTANCE	DENSITY (GRM. PER C.CM.)	SUBSTANCE	DENSITY (GRM. PER C.CM.)
Aluminium	2·6	Lead	11·4
Brass	8·5	Marble	2·6
Brick	1·4–2·2	Silver	10·5
Copper	8·94	Zinc	7·1
Glass	2·6	Alcohol	0·8
Gold	19·3	Glycerine	1·3
Iron (cast)	7–7·7	Mercury	13·6
Iron (wrought)	7·8	Petrol	0·9

The above densities may be expressed in pounds per cubic foot by multiplying each by 62·5.

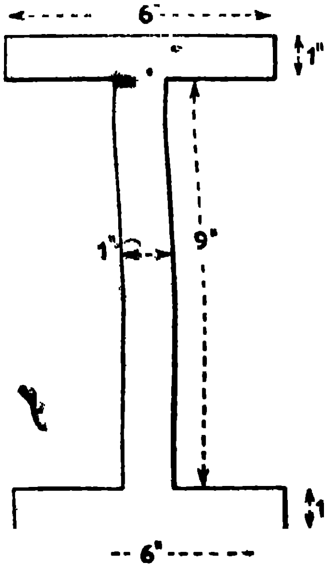


FIG. 17.

EXAMPLES.—(1) *An iron girder has a cross-section of shape and dimensions as shown in Fig.* 17. *It is* 30 *ft. long. Find its mass, taking the density of iron as* 7·5 *grm. per c.cm.*

Area of cross-section =

$$2\,(6 \times 1) + (9 \times 1) = 21 \text{ sq. in.}$$

$$\text{Volume} = \tfrac{21}{144} \times 30 \text{ cub. ft.}$$

$$\text{Mass} = \frac{21}{144} \times 30 \times 7{\cdot}5 \times 62{\cdot}5 = \frac{16406{\cdot}25}{8} \text{ lb.}$$

$$= 2051 \text{ lb. (to nearest lb.)} = 18 \text{ cwt. } 35 \text{ lb.}$$

(2) *Bottles each to hold* 50 *grm. of glycerine are to be designed. What must be the capacity of each?*

Note. **Since density is mass of unit volume, the number of times it is contained in a given mass will be the number of units of volume in that mass. Volume $= \frac{\text{Mass}}{\text{Density}}$.**

$$\text{Density of glycerine} = 1{\cdot}3 \text{ grm. per c.cm.};$$

$$\therefore \text{ Volume of 50 grm. of glycerine} = \frac{50}{1{\cdot}3} = 38{\cdot}5 \text{ c.cm.}$$

(3) *A yellow metal ornament weighs* 2125 *grm. and will displace* 250 *c.cm. of water. Is it made of gold?*

$$\text{Density of metal} = \frac{\overset{425\ 85}{\cancel{2125}}}{\underset{50}{\cancel{250}}} = 8{\cdot}5 \text{ grm. per c.cm.}$$

$$\text{But density of gold is } 19{\cdot}3 \text{ grm. per c.cm.};$$

$$\therefore \text{ The metal is not gold.}$$

Relative Density

In many cases, instead of dealing with the density of a substance, it is preferable to consider the number of times the substance is as dense as water. This is called the Relative Density or Specific Gravity of the substance.

$$\text{Relative Density} = \frac{\text{Density of substance}}{\text{Density of water}}$$

√. I.

Since the density of water is 1 grm. per c.cm., the relative density of a substance will be represented by the same number as its density in grm. per c.cm., *e.g.* the density of lead is 11·4 grm. per c.cm.;

$$\therefore \text{ Relative density of lead} = \frac{11{\cdot}4 \text{ grm. per c.cm.}}{1 \text{ grm. per c.cm.}} = 11{\cdot}4.$$

Note that the relative density is just a number since it only says *how many times* the substance is as dense as water.

If masses of equal volumes of a substance and of water are known we may write:—

$$\text{Relative density of substance} = \frac{\text{Density of substance}}{\text{Density of water}}$$

$$= \frac{\dfrac{\text{Mass of substance}}{\text{Volume}}}{\dfrac{\text{Mass of water}}{\text{Volume}}} = \frac{\text{Mass of substance}}{\text{Mass of water}}$$

$$\text{i.e. } \left.\begin{array}{c}\text{Relative density of a}\\ \text{substance}\end{array}\right\} = \frac{\text{Mass of any volume of the substance}}{\text{Mass of an equal volume of water}}$$

This makes the finding of the relative density of a substance much simpler than the finding of its density, since no actual measurement of a volume is required, but only the finding of the mass of a quantity of water which has the same volume as the substance which is weighed. Thus from the results on page 17 the relative density of the liquid might be calculated as follows:—

$$\text{Mass of liquid} = 58{\cdot}22 \text{ grm.}$$

$$\text{Mass of an equal volume of water} = 51{\cdot}71 \text{ grm.};$$

$$\therefore \text{ Relative density of liquid} = \frac{58{\cdot}22}{51{\cdot}71} = 1{\cdot}13.$$

The density bottle may be used for finding the relative density of a solid in small pieces, *e.g.* lead shot, in the following way.

Fill the bottle with water and weigh it. Place some of the shot on the pan with the bottle and weigh again. Now place the shot in the water in the bottle and replace the stopper. A volume of water equal to the volume of the shot will have been displaced from the bottle. Weigh again and record your results as below.

$$\text{Mass of bottle filled with water} = 75{\cdot}46 \text{ grm.} \quad \ldots(1)$$

$$\text{Mass of bottle filled with water} + \text{shot outside} = 128{\cdot}23 \text{ grm.} \ldots(2)$$

$$\text{Mass of bottle with shot inside} = 123{\cdot}59 \text{ grm.} \ldots(3)$$

$$\text{From (1) and (2):—Mass of shot} = 52{\cdot}77 \text{ grm.}$$

$$\text{From (2) and (3):—Mass of water displaced} = 4{\cdot}64 \text{ grm.}$$

Since the shot and the water displaced had the same volume:—

$$\text{Relative density of lead} = \frac{52{\cdot}77}{4{\cdot}64} = 11{\cdot}4.$$

Other methods of finding relative densities are given in Chapters X. and XII. From the relation:—

$$\text{Relative density} = \frac{\text{Mass of substance}}{\text{Mass of an equal volume of water}}$$

it follows that the result will not depend on the units of mass used, so long as the same unit is used for both masses, *e.g.* if the masses recorded above had been taken in ounces, the result would still be 11·4.

The density of a substance in pounds per cubic foot may be found from its relative density by multiplying by 62·5, *e.g.* if a substance has a relative density of 7, it is 7 times as dense as water. But the density of water is 62·5 lb. per cub. ft.; ∴ the density of the substance is 62·5 × 7 lb. per cub. ft.

Questions on Chapter III

QUESTIONS ON CHAPTER III

Use the table of densities on page 18 when necessary.

1. Calculate densities in the following cases:—
   - 150 c.cm. of iron weigh 1100 grm.
   - 60 cub. in. of wood weigh 1½ lb.
   - 75 c.cm. of salt solution weigh 82·5 grm.
   - A brick 9 in. × 4½ in. × 3 in. weighs 6 lb.

2. Calculate masses of:—
   - 6 cub. ft. of marble.
   - 96 c.cm. of mercury.
   - 1 gallon of petrol.
   - A lead cylinder 10 cm. long and 3 cm. in diameter.

3. Calculate volumes of:—
   - 1 kilog. of mercury.
   - 1 cwt. of lead.
   - 700 grm. of alcohol.

4. A good quality milk has a density between 1·029 and 1·033 grm. per c.cm. How would you expect its density to change if (*a*) cream was removed from it, (*b*) water was added to it?

200 c.cm. of a sample of milk were found to weigh 209 grm. What had probably been done to it?

5. A piece of white metal weighs 109·34 grm. and displaces 15·4 c.cm. of water. Of what metal does it probably consist?

6. Describe fully how you would find the density of (*a*) cork, (*b*) common salt.

7. A coil of copper wire of diameter 1·2 mm. weighs 150 grm. What length of wire is in the coil?

8. From the following data calculate the density of turpentine.

Mass of empty beaker ... ... = 15·2 grm.
Burette readings:—
(*a*) before running turpentine into beaker ... = 1·3 c.cm.
(*b*) after running turpentine into beaker ... = 24·6 c.cm.
Mass of beaker and turpentine ... ... = 35·5 grm.

9. 10 grm. of paraffin-wax of density 0·89 grm. per c.cm. were melted with 15 grm. of beeswax of density 0·96 grm. per c.cm. Calculate the density of the mixture when it had solidified.

10. A trench 4 ft. wide, 9 ft. deep, and 50 yd. long is to be dug. The average density of the soil is 150 lb. per cub. ft. Calculate to the nearest ton the mass of material to be removed.

11. An empty density bottle weighs 20·24 grm. When filled with water it weighs 72·75 grm., and when filled with sulphuric acid it weighs 116·86 grm.

Find: (*a*) The capacity of the bottle in c.cm.
(*b*) The relative density of sulphuric acid.
(*c*) The mass of 10 gallons of sulphuric acid.

12. A cylindrical oil tank is 20 ft. deep and 10 ft. in diameter. It is filled with oil of specific gravity 0·89. What mass of oil does it contain? Answer to nearest ton.

13. A density bottle filled with water weighs 77·59 grm. 10·25 grm. of glass beads are put into it and the stopper replaced. The total mass is then 83·74 grm. Find the relative density of the glass.

14. A box made of elm wood of specific gravity 0·55 weighs 100 lb. What would be the mass of a similar box made of oak of specific gravity 0·85?

15. " The *relative density* of a sample of methylated spirit is 0·82."

Explain the meaning of this statement and describe an experiment by which you could verify it, being supplied with a physical balance and any other apparatus you may require. [L.U.

16. Define *specific gravity.*

How would you find the specific gravity of glass if you were provided with a short length of glass tubing and any other necessary apparatus? Explain your method.

If the specific gravity of window glass is 2·6, find the weight of a pane which is half a metre square and 3 mm. thick. [L.U.

17. Distinguish between the *density* and the *specific gravity* of a substance.

A specific gravity bottle weighed 24·20 grm. when empty, 67·81 grm. when filled with turpentine, and 74·20 when filled with distilled water. What was the specific gravity of the turpentine?

The same bottle, cleaned and dried, had a little salt added to it. It then weighed 27·70 grm. Turpentine was poured in carefully so as just to fill the bottle. The total weight was then 69·91 grm. Calculate the specific gravity of the salt. [J.M.B.

CHAPTER IV

MOTION

Speed

In Chapter II. the word "speed" was used a number of times without exact definition. By the speed of a body we mean the rate at which it travels through space. It is measured in such units as miles per hour, feet per second, and centimetres per second, *i.e.* by the distance travelled in a unit of time. If the speed of a body is uniform, it is determined by finding the distance travelled in any given time and applying the relation

$$\text{Speed} = \frac{\text{Distance travelled}}{\text{Time taken}}.$$

When the speed varies from moment to moment, the speed at any particular moment is the distance that would be travelled in a unit of time if the speed at the moment were maintained for that unit of time. Later paragraphs will show how such momentary speeds can be determined in certain cases.

Velocity

"Velocity" has a meaning similar but not identical to "speed." If six boys each started to walk from school at the same time and walked at the same speed along different roads for an hour, they would all travel the same distance, but their positions at the end of the hour would probably be very different. To determine the position at which a moving body arrives we need to know the direction of its motion as well as its speed. The term velocity indicates speed in a given direction. Thus we may speak of a velocity of 6 mi. per hr. towards the north, or a velocity of 50 cm. per sec. along a given straight line. A change of either speed or direction of motion is a change of velocity. A car travelling round a circular track at a uniform speed would have a constantly changing velocity because, at each instant, its direction of motion would be altering.

Acceleration

In connexion with motor cars, trains, etc., the word "acceleration" is used to denote increase in speed, and slowing down is often referred to as retardation. In mechanics any change of velocity is called an acceleration, retardation being regarded as a negative acceleration. Change of direction also involves acceleration, even though there is no change of speed. The car mentioned at the end of the last paragraph would be undergoing continuous acceleration because its velocity is changing continuously.

Acceleration is measured by *the rate at which velocity changes.* Suppose a tram on a long flat stretch of rails starts from rest and gradually increases its velocity so that, at the end of each second after starting it has the velocities shown in the table below.

Time	0	1	2	3	4	5	sec.
Velocity	0	2	4	6	8	10	ml. per hr.

In each second its velocity has increased by 2 ml. per hr., so it is said to have undergone an acceleration of 2 ml. per hr. per sec. As 2 ml. per hr. is approximately 2·9 ft. per sec., this acceleration might be written 2·9 ft. per sec., per sec., which is often abbreviated to 2·9 ft. (per sec.)^2.

Uniform Velocity and Uniformly Accelerated Motion

(1) UNIFORM VELOCITY.—Consider a body moving with a uniform velocity of 2 ft. per sec. For the first ten seconds of its motion the following figures could be obtained:—

Time	0	1	2	3	4	5	6	7	8	9	10	sec.
Space described	0	2	4	6	8	10	12	14	16	18	20	ft.

These could be plotted into a "space-time" graph as shown in Fig. 18. It will be observed that the graph is a straight line. The distances represented by PQ and YZ are travelled in times represented by OQ and XZ respectively. Hence $\frac{PQ}{OQ} = \frac{YZ}{XZ}$, since each fraction is equal to the velocity which is constant. Conversely, had the space-time graph been given, the velocity could have been found by taking the ratio of any pair of corresponding lines such as YZ and XZ.

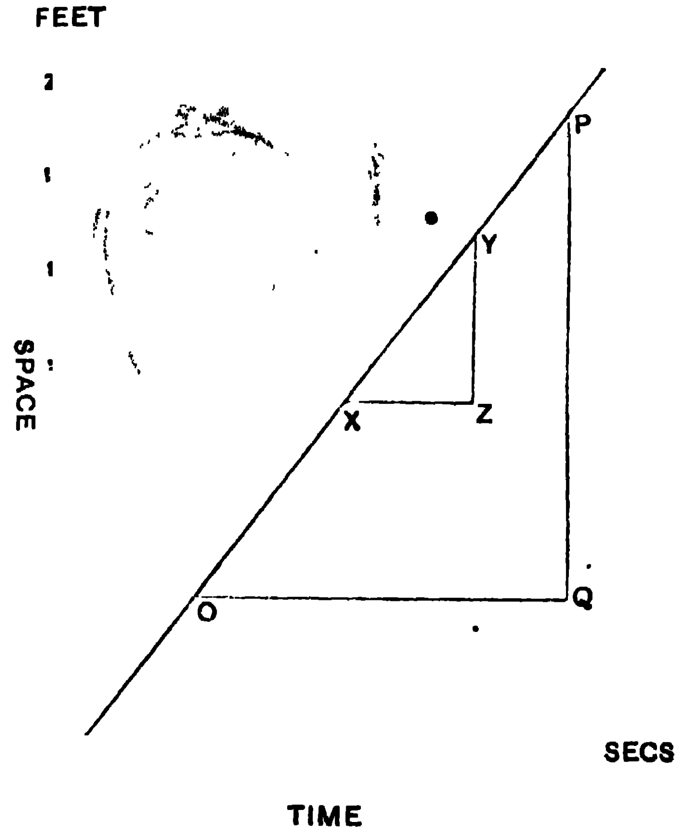


FIG. 18.

(2) UNIFORMLY ACCELERATED MOTION.—Allow a marble to roll down a groove in a slightly sloping board which is 8 or 9 ft. long. It will gain speed, *i.e.* undergo acceleration as it rolls. Set up a metronome to beat half seconds. Release the ball at the top of the groove as one beat is heard and mark the point it has reached as the next beat is heard. Measure the distance it rolled in the first half second. Repeat several times and take the average of the distances recorded. In a similar way find the distance travelled in 1, 1½, 2, 2½, 3, etc., seconds. Such an experiment gave the table below.

Time	0	½	1	1½	2	2½	3	3½	4	sec.
Distance	0	1·35	5·40	12·15	21·60	33·75	48·60	66·15	86·40	in.

These figures plotted into a graph give the curve of Fig. 19.

We cannot in this case take the ratio $\frac{PQ}{OQ}$ as meaning the velocity, since it was different at different moments between the times represented at O and Q. Between the two points X and Y the curve is almost straight so we may say that between the times 2½ and 3 seconds the velocity was almost constant and equal to $\frac{YZ}{XZ}$. At the beginning of this interval the velocity would be slightly less than this, and at the end slightly more, so at some instant in between, which may be taken as half-way between, the velocity would have this value. Thus we may say that at the time 2¾ sec. the momentary velocity was $\frac{YZ}{XZ} = 29\cdot7$ in. per sec. Similarly velocities at other instants may be obtained and the following table constructed:—

Time	¼	¾	1¼	1¾	2¼	2¾	3¼	3¾
Velocity	2·7	8·1	13·5	18·9	24·3	29·7	35·1	40·5

in. per sec:

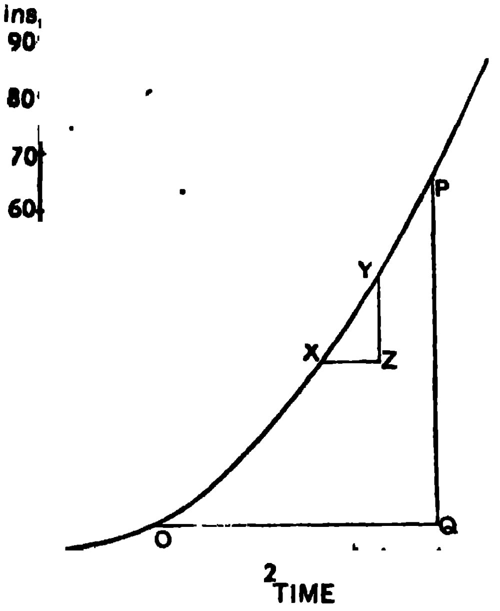


Fig. 19.

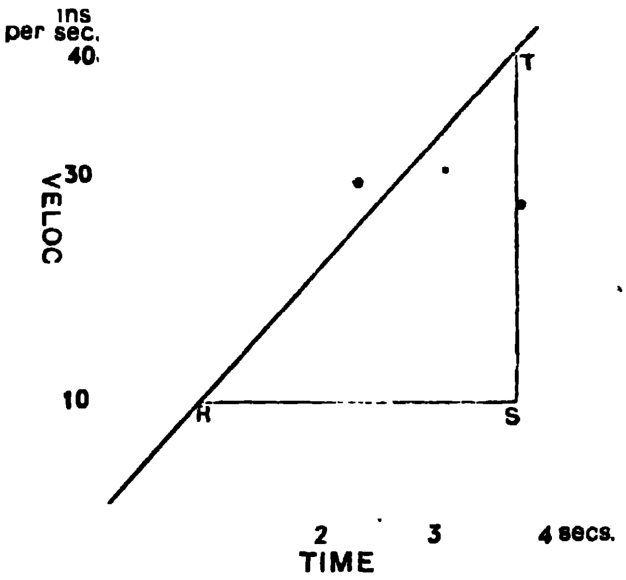


Fig. 20.

These numbers plotted into a velocity-time graph give Fig. 20. This is a straight line showing that in equal intervals of time equal increases of velocity occur, *i.e.* the acceleration was uniform.

In the time represented by RS the velocity increased by an amount represented by ST. Therefore the rate at which the velocity increased is equal to $\frac{ST}{RS} = 10\cdot8$ in. (per sec.)^2. The graph also shows that, during the 4 sec. considered the velocity increased uniformly from zero to 43·2 in. per sec. Therefore the average velocity during the interval $\frac{0 + 43\cdot2}{2}$ in. per sec. = 21·6 in. per sec. In 4 sec. a body moving with a uniform velocity of 21·6 in. per sec. would travel 21·6 × 4 = 86·4 in., which Fig. 19 shows to have been the distance actually travelled by the marble. Thus we arrive at two important conclusions regarding a body moving with uniform acceleration.

(*a*) The average velocity during any interval of time

$$= \frac{\text{Velocity at beginning} + \text{Velocity at end of interval}}{2}.$$

(*b*) Distance travelled in any interval = average velocity × time.

Equations of Motion

Equations (*a*) and (*b*) enable many simple problems on uniformly accelerated motion to be solved as in the following examples:—

(1) *A body, starting from rest, travels for* 10 *sec. with an acceleration of* 5 *ft.* (*per sec.*)^2. *Find its final velocity and the distance it covers in* 10 *sec.*

The velocity increases by 5 ft. per sec. in 1 sec.;

$\therefore$ in 10 sec. it increases by 5 × 10 = 50 ft. per sec.

Final velocity is 50 ft. per sec.

Since initial velocity was zero, average velocity was $\frac{50}{2}$ ft. per sec.;

$\therefore$ Distance covered in 10 sec. $= \frac{50}{2} \times 10 = 250$ ft.

(2) *A motor car travelling at* 30 *ml. per hr. is brought to rest with a uniform loss of speed in* 5 *sec. by the application of the brakes. Find the acceleration produced by applying the brakes and the distance the car travels after their application.*

30 ml. per hr. = 44 ft. per sec.

A velocity of 44 ft. per sec. is uniformly reduced to zero in 5 sec.

$\therefore$ the acceleration $= -\frac{44}{5} = -8{\cdot}8$ ft. (per sec.)^2.

Average speed after application of brakes $= \frac{0 + 44}{2} = 22$ ft. per sec.;

$\therefore$ Distance covered in 5 sec. $= 22 \times 5 = 110$ ft.

Problems can often be solved more neatly by using certain equations which can be proved by using the above principles.

Let a body undergo a uniform acceleration of f units. In t seconds its velocity will increase by ft units. Therefore, if it started from rest its velocity v after t seconds will be given by

$$v = ft \qquad \text{.................................(1)}$$

and, starting from a time when its velocity was u units, t seconds later its velocity will be given by

$$v = u + ft \qquad \text{.................................(2)}$$

A body to which Equation (1) applies will have had an average velocity of $ft/2$ units during the t seconds considered. Therefore the distance s it travels during those t seconds is given by

$$s = \frac{ft}{2} \times t$$

or

$$s = \tfrac{1}{2}ft^2 \qquad \text{.................................(3)}$$

If Equation (2) applies, the average velocity for the t seconds is

$$\frac{u + (u + ft)}{2} = \frac{2u + ft}{2} = u + \tfrac{1}{2}ft$$

and so

$$s = (u + \tfrac{1}{2}ft) \times t,$$

i.e.

$$s = ut + \tfrac{1}{2}ft^2 \qquad \text{.................................(4)}$$

The average velocity may also be written $\frac{v + u}{2}$;

$$\therefore s = \frac{v + u}{2} \times t.$$

From Equation (2)

$$t = \frac{v - u}{f};$$

$$\frac{v + u}{2} \times \frac{v - u}{f} = \frac{v^2 - u^2}{2f};$$

$$\therefore v^2 - u^2 = 2fs \qquad \text{.................................(5)}$$

Examples (1) and (2) above may be worked as follows by using these equations:—

(1) Using equation $v = ft$,

$$v = 5 \times 10 = 50.$$

Using equation $s = \frac{1}{2}ft^2$,

$$s = \frac{1}{2} \times 5 \times 10^2 = 250;$$

$\therefore$ Final velocity is 50 ft. per sec. and distance travelled 250 ft.

(2) Initial velocity = 30 ml. per hr. = 44 ft. per sec. Final velocity = 0.

Using equation $v = u + ft$,

$$0 = 44 + 5f; \quad \therefore\ 5f = -44; \quad \therefore\ f = -8{\cdot}8.$$

Using equation $s = ut + \frac{1}{2}ft^2$,

$$s = (44 \times 5) + \frac{1}{2}(-8{\cdot}8 \times 5^2) = 220 - 110 = 110;$$

$\therefore$ The acceleration was $-8{\cdot}8$ ft (per sec)^2 and distance travelled during braking was 110 ft.

The following example illustrates the use of Equation (5):—

A body undergoing uniform acceleration has its velocity increased from 20 *cm. per sec. to* 50 *cm. per sec., while it travels* 100 *cm. What is its acceleration and how long does it take to travel the* 100 *cm.?*

Using equation $v^2 - u^2 = 2fs$,

$$50^2 - 20^2 = 2f \times 100; \quad \therefore\ 200f = 2500 - 400 = 2100;$$

$$\therefore\ f = 10{\cdot}5.$$

Using equation $v = u + ft$,

$$50 = 20 + 10{\cdot}5t; \quad \therefore\ 10{\cdot}5t = 30; \quad \therefore\ t = 2{\cdot}38;$$

$\therefore$ The acceleration is 10·5 cm. (per sec.)^2 and the time 2·38 sec.

Motion Due to Gravity

It is well known that different bodies released from the same height take different times to fall to the ground. A little investigation will show that air resistance is largely responsible for this. A sheet of paper held horizontally and released falls much more slowly than if it is rolled into a tight ball and then dropped. Also, if a small piece of tissue paper and a penny are held side by side and dropped, the penny falls more quickly than the paper. If, however, the paper is placed on the penny and the latter dropped so that it remains horizontal while falling, the two will fall together. In this case the penny experiences air resistance but shields the paper from it.

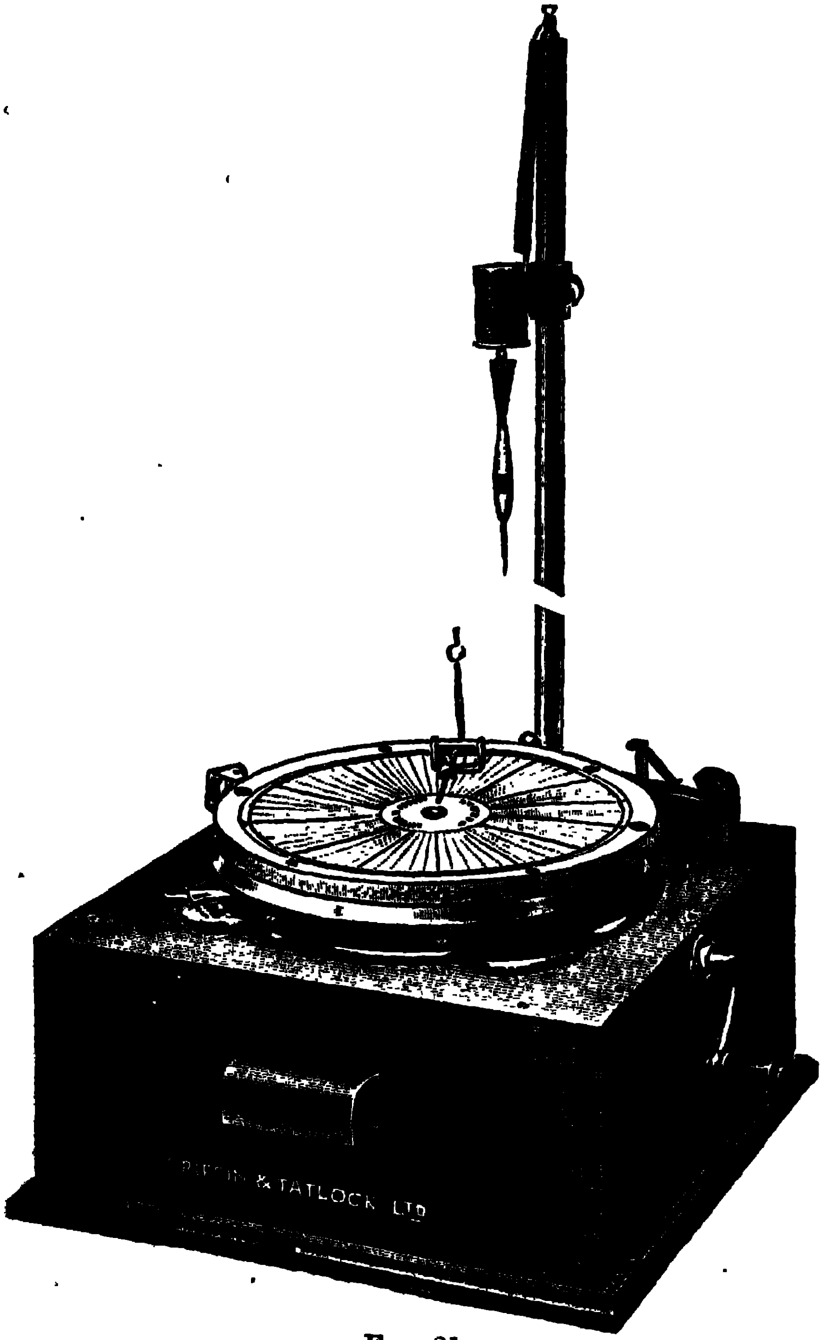


Fig. 21.

The last experiment suggests that all bodies might fall at the same rate in vacuum. That bodies of different weights could fall at the same rate was shown by Galileo (1564-1642) who dropped metal balls, one weighing ten times as much as the other, at the same moment from the top of the Leaning Tower of Pisa. The balls struck the ground together. Later, Newton (1642-1727) showed that a feather and a golden coin would fall at the same rate in a vacuum by arranging so that they could be released at the same moment at the top of a long tube from which the air had been pumped.

Air resistance has little effect on the falling from moderate heights of bodies of compact shape made of dense materials, and so long as we confine our attention to such cases it may be neglected.

It is obvious that a body such as a stone is accelerated as it falls. The magnitude of this acceleration may be found by the apparatus illustrated in Fig. 21. In this an electromagnet is supported on a tall rod fixed vertically on the side of a box containing a gramophone motor. On the turntable is a disc of cork carrying a circular card divided into sectors. When the motor is running freely one sector will pass a point near the edge of the disc each hundredth of a second.

There is a gap in the electrical circuit controlling the magnet which can be closed by resting a metal ball on two metal plates. With the ball in position a dart with an iron cap is suspended from the magnet as shown. The motor is started and, when it has attained its steady speed, a pivoted arm attached to the disc is flung over and knocks the ball away. This breaks the circuit and releases the dart just as the zero line on the scale is passing below its point. When the dart has stuck in the cork mat the motor is stopped. The number of sectors between the zero line and the point where the dart struck the card gives the number of hundredths of seconds taken by the dart in falling. As it is possible to estimate tenths of sectors, the time can be measured to the nearest $\frac{1}{1000}$ of a second. The distance the dart fell can be found by measuring the height from the turntable to the point of the dart while it is hanging on the magnet. The experiment should be repeated a number of times and the average of the times recorded should be taken.

The height of the electromagnet on the vertical rod may be adjusted so that a series of measurements of times taken to fall varying distances may be made. Assuming that the acceleration is uniform we can use the equation $s = \frac{1}{2}ft^2$ to calculate a value of f from each set of measurements made. An approximately constant value for f will be found showing that the assumption of uniform acceleration was correct. Accurate experiments show that the acceleration of a falling body near sea-level at about the latitude of London is approximately 32 ft. $(\text{per sec.})^2$ or 980 cm. $(\text{per sec.})^2$. This is referred to as the *acceleration due to gravity,* and is usually represented in formulae by the symbol "g." Thus the equations of motion proved in the previous section may be applied to falling bodies, g being substituted for f.

Examples.—(1) *A stone dropped from the top of a cliff takes* 6 *sec. to reach the ground below. What is the height in feet of the cliff?*

Using equation $s = \frac{1}{2}gt^2$,

$$s = \frac{1}{2} \times 32 \times 6^2 = \frac{1}{2} \times 32 \times 36 = 576;$$

$\therefore$ Height of cliff is 576 ft.

(2) *A bullet is fired vertically upwards, leaving the rifle with a velocity of* 800 *ft. per sec. Find* (*a*) *the height to which it rises, and* (*b*) *the time taken for its ascent.*

In this case there is a negative acceleration as the bullet rises and its highest point will be reached when its upward velocity is reduced to zero.

Using equation $v = u + (-g)t$,

$$0 = 800 - 32t; \quad \therefore \ 32t = 800; \quad \therefore \ t = 25.$$

Using equation $s = ut + \frac{1}{2}(-g)t^2$,

$$s = (800 \times 25) - (\tfrac{1}{2} \times 32 \times 25^2) = 20000 - 10000 = 10{,}000 \text{ ft.};$$

$\therefore$ Time of ascent is 25 sec. and bullet ascends 10,000 ft.

(3) *How far will a stone fall during the fifth second of falling from rest?*

Using equation $s = \frac{1}{2}gt^2$.

For 4 sec. $s = \frac{1}{2} \times 32 \times 4^2 = 256.$

For 5 sec. $s = \frac{1}{2} \times 32 \times 5^2 = 400.$

$\therefore$ During fifth second stone falls $400 - 256 = 144$ ft.

Force and Acceleration

The action of a force is necessary to produce any change in the motion of a body and so any acceleration must be associated with a force producing it.

The relation between force used and acceleration produced may be found by use of the trolley illustrated in Figs. 22 and 23. The trolley is about 8 in. long and is fitted with two springs V_1 and V_2, each of which makes a

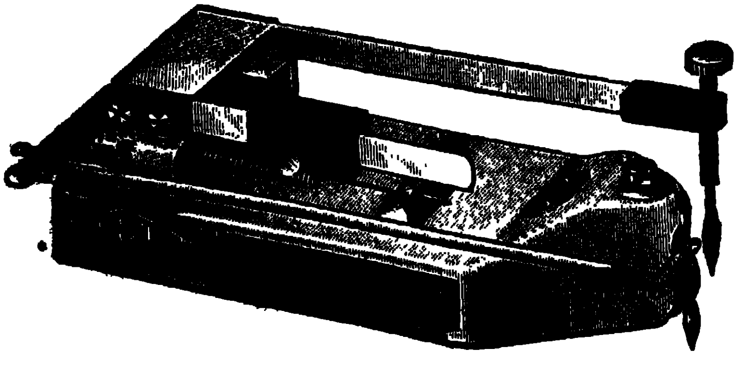


Fig. 22.

complete vibration in $\frac{1}{10}$ sec. V_2 carries an inked brush which just touches the surface over which the trolley runs. V_1 acts as a balancer to prevent irregular movements of the trolley being caused by the vibrations of V_2. AB is a draw bar. The attachments to AB may be used to measure the pull on the trolley, but for present purposes may be neglected. The mass of the trolley may be varied by inserting lead blocks of known weight into a slot in its side.

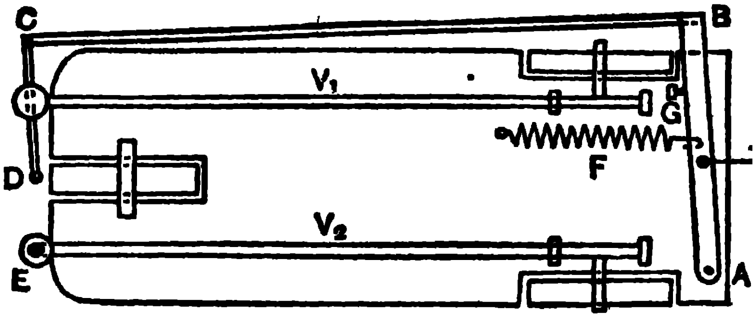


FIG. 23.

The trolley is placed on a sheet of paper and attached to a light weight pan by means of a cord passing over a pulley as in Fig. 11. The two vibrators are pinched together and suddenly let go. The weight pulls the trolley forward, and the vibrating brush traces out a wavy line as in Fig. 24. If a straight line is drawn down the middle of the wave trace, spaces marked off on it by each complete wave show the distance the trolley travelled in each $\frac{1}{10}$ sec. The lengthening out of the waves as the trolley proceeds shows that the trolley undergoes acceleration.

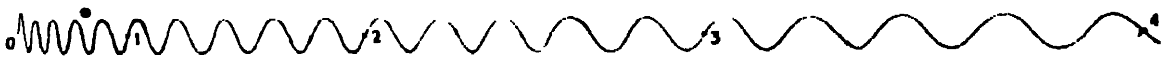


FIG. 24.

If the end of every fifth wave is marked off—points 1, 2, 3, etc.—the distances between these marks were travelled in successive half-seconds. Thus in 0·5 sec. the trolley travelled the distance O1; in 1 sec. the distance O2, and so on. Thus the distance O4 was travelled in 2 sec. Suppose this distance to be 32·5 cm.

Using equation $s = \frac{1}{2}ft^2$,

$$32{\cdot}5 = \tfrac{1}{2}f \times 2^2; \quad \therefore\ 2f = 32{\cdot}5; \quad \therefore\ f = 16{\cdot}25;$$

$\therefore$ The acceleration was 16·25 cm. (per sec.)^2.

As the actual starting point of the trace is difficult to determine it is better to measure two distances from the same point, such as 1-3 and 1-4. Let these be 16·3 cm. and 30·4 cm. respectively. These distances were travelled in 1 sec. and 1·5 sec. respectively, but there was an unknown velocity u at the beginning of each of these periods.

Using equation $s = ut + \frac{1}{2}ft^2$.

For first distance $16{\cdot}3 = (u \times 1) + (\frac{1}{2}f \times 1^2)$,

i.e. $$16{\cdot}3 = u + \tfrac{1}{2}f \quad \text{.............................} \quad (1)$$

For second distance $30{\cdot}4 = (u \times 1{\cdot}5) + (\frac{1}{2}f \times 1{\cdot}5^2)$,

i.e. $$30{\cdot}4 = 1{\cdot}5u + 1{\cdot}125f \quad \text{..........................} \quad (2)$$

Multiplying (1) by 1·5, $24{\cdot}45 = 1{\cdot}5u + 0{\cdot}75f$ .... (3)

Subtracting (3) from (2), $5{\cdot}95 = 0{\cdot}375f$;

$$\therefore f = \frac{5{\cdot}95}{0{\cdot}375} = 15{\cdot}9;$$

∴ The acceleration was 15·9 cm. (per sec.)^2.

In this way determine the accelerations produced by various weights in the pan, keeping the mass of the trolley constant. It will be found that $\frac{\text{Acceleration}}{\text{Weight in pan}}$, i.e. $\frac{\text{Acceleration}}{\text{Force exerted}}$ is approximately constant. From this it follows that the acceleration produced in a body is proportional to the force exerted on the body.

It should be noted that the weights in the pan do not quite accurately measure the force exerted on the trolley since part of their weight is used in causing their own motion. This error is not large if the mass of the weights is small compared with the mass of the trolley.

A further set of experiments may be carried out, varying the mass of the trolley but keeping the weight in the pan constant. It will be found in this case that acceleration × mass is a constant. That means that, if the mass is multiplied by any number, the acceleration is divided by the same number. A short statement for this is that the acceleration is inversely proportional to the mass.

Newton's Second Law

The two results of the preceding section may be combined to give the equation:—

$$\frac{\textbf{Mass} \times \textbf{Acceleration}}{\textbf{Force applied}} = \textbf{a constant.}$$

This may be verified by further trolley experiments in which both the mass of the trolley and the weights used to set it in motion are varied. The equation shows that **the force necessary to produce a given acceleration in a body is proportional to the product of the mass of the body and the acceleration produced.**

The above statement expresses the second of the laws of motion which were enunciated by Newton.

Absolute Units of Force

In the equation $\frac{\text{Mass} \times \text{Acceleration}}{\text{Force applied}} = \text{a constant}$, the value of the constant will depend on the units used in measuring the quantities on the left-hand side. If we decide to consider that one unit of force has been used when one unit of mass has been given one unit of acceleration, the value of the constant becomes 1 and we can write

$$\text{Mass} \times \text{Acceleration} = \text{Force}.$$

This enables units of force to be defined which, unlike the gravitational units previously used, are constant since they are based on mass and not on weight. Such units are called absolute units. The absolute unit of force in the Metric System is the **dyne**, which is the force necessary to impart an acceleration of 1 cm. (per sec.)^2 to a mass of 1 grm. In the British System the absolute unit is the **poundal**, which is the force necessary to impart an acceleration of 1 ft. (per sec.)^2 to a mass of 1 lb. Thus, to give a car weighing 10 cwt. an acceleration of 5 ft. (per sec.)^2 requires a force of $10 \times 112 \times 5$ pndl.

The relation between the gravitational and absolute units of force may be obtained by considering a falling body. Thus a mass of 1 lb. falls with an acceleration of g ft. (per sec.)^2. The force producing this acceleration is its own weight, *i.e.* 1 lb.-wt. But, from the definition above, the force required is $1 \times g$ pndl. Hence 1 lb.-wt. $= g$ pndl. Similarly, by considering a mass of 1 grm., we find that 1 grm.-wt. $= g$ dynes.

Thus taking the appropriate values of g for sea-level at the latitude of London, it may be said that approximately 1 lb.-wt. = 32 pndl. and 1 grm.-wt. = 980 dynes.

EXAMPLE.—*A car weighing* 1 *ton starts from rest on a level road and is uniformly accelerated so that its speed is* 30 *ml. per hr. after* 10 *sec. Find the force exerted on it. Answer in both absolute and gravitational units.*

$$30 \text{ ml. per hr.} = 44 \text{ ft. per sec.};$$
$$\therefore \text{ Acceleration} = \tfrac{44}{10} \text{ ft. (per sec.)}^2.$$
$$\text{Force} = \text{Mass} \times \text{Acceleration} = 2240 \times 4{\cdot}4 = 9856 \text{ poundals.}$$
$$9856 \text{ poundals} = \tfrac{9856}{32} = 308 \text{ lb.-wt.}$$

Newton's Third Law of Motion

This law states that to every action there is an equal and opposite reaction. This means that when one body exerts a force on another, the second exerts an equal and opposite force on the first. Thus, when you draw a heavy trolley by means of a string, you can feel a backward pull of the string on your hand. If two spring balances, pointing in opposite directions, were linked and attached to the string, when the pull was given through them both would give the same reading, showing that the two opposite pulls are equal. Again, if you hold a stone in your hand, it is exerting a downward force equal to its weight on your hand. Your hand must be exerting an equal upward force on it to prevent it from falling. Similar reasoning will indicate that, if the stone is lying on a table, the table must be exerting an upward force equal to the weight of the stone on the latter. It should be particularly noted that, whenever a body is resting on a support, the latter exerts on the body a reaction equal and opposite to the force exerted on the support by it.

Momentum

It has been shown that, in absolute units, force = mass × acceleration. Now acceleration is rate of change of velocity so we may write force = rate of change of (mass × velocity). For this reason, in the case of a moving body, the product:—

Mass of body × Its velocity

is often said to measure its quantity of motion, and is called the momentum of the body. No special names are given to units of momentum. If British units have been used in measuring mass and velocity, momentum is measured in foot-pound units. When Metric units are used, they give momentum in gramme-centimetre units.

Consideration of momentum is important in connexion with problems on blows and action and reaction between moving bodies. At the seaside, or in connexion with bridge building, you may have seen huge beams of wood, called piles, being driven into the ground by a pile driver. A heavy weight is raised by means of a rope passing over a pulley and then released so that it falls on the head of the pile. In falling it acquires a considerable momentum. When it strikes the pile it meets with a resistance which quickly brings it to rest. Thus its momentum is lost at a very great rate and consequently a very great force is exerted on the pile.

Example.—*A pile driver weighing* 300 *lb. falls through* 16 *ft. and is brought to rest in* $\frac{1}{5}$ *sec. after striking the pile. Find the force on the pile.*

Using equation $s = \frac{1}{2}gt^2$,

$$16 = \frac{1}{2} \times 32 \times t^2; \quad \therefore t^2 = 1; \quad \therefore t = 1.$$

Using equation $v = ft$,

$$v = 32 \times 1 = 32;$$

$\therefore$ Velocity on striking pile is 32 ft. per sec.

and Momentum $= 300 \times 32$ ft.-lb. units;

$$\text{Rate at which momentum is lost} = \frac{300 \times 32}{\frac{1}{5}} \text{ units per sec.}$$

$\therefore$ Force exerted $= 300 \times 32 \times 5 = 48{,}000$ pndl. or 1500 lb.-wt.

This is the force of the blow, but the total force will also include the weight of the driver, so that

Total force exerted $= 1800$ lb.-wt.

It should be clear from this example that the rapidity with which momentum is destroyed is a big factor in producing an effective blow. You may have experienced this if you have tried to drive a nail into a springy board. The "give" of the board causes the momentum of the hammer to be destroyed much less quickly than in the case of a firm board, so that each stroke has much less effect on the nail.

When you are sliding on ice, if you overtake someone sliding more slowly and clasp him so that you go on together, your velocity will be less than it was before, but greater than that of the one you have overtaken, *i.e.* you have lost momentum and he has gained momentum. If your weights and the various velocities were measured it would be found that your combined momentum after "coalescing" was equal to the sum of your separate momenta before the collision. This is an example of the *Law of Conservation of Momentum*, which states that the total momentum in a given direction of a set of bodies remains constant if no external force acts on them.

In applying the law of Conservation, momenta in opposite directions must be taken to have opposite signs. Thus, if two sliders going in opposite directions with equal momenta collide, presuming they keep their feet, they will both come to a dead stop, for if the momentum of one is x units, that of the other is $-x$ units, and the sum is zero; so that the two, joined into one "body," will have no momentum. Again, if two of you stand back to back on a slide and push off from one another, you will move off from one another with equal momenta, *i.e.* the lighter will move off with the greater velocity, so that mass × velocity is the same for both. In this case the pair of you had no momentum before pushing off, so when you begin to move you must have equal and opposite momenta for your total momentum still to be zero.

Examples.—(1) *In shunting, a detached truck weighing* 20 *tons and moving with a velocity of* 20 *ml. per hr. overtakes a truck weighing* 35 *tons and moving in the same direction at* 10 *ml. per hr. With what velocity will the two go on together? (Neglect friction, etc.)*

$$\text{Momentum of 1st truck} = 20 \times 20 \text{ ton-ml. units}$$

$$\text{Momentum of 2nd truck} = 35 \times 10 \text{ ton-ml. units;}$$

$$\therefore \text{ Total momentum} = 750 \text{ ton-ml. units}$$

$$\text{Total mass} = 55 \text{ tons;}$$

$$\therefore \text{ Velocity after collision} = \tfrac{750}{55} = 13{\cdot}6 \text{ ml. per hr.}$$

(2) *If the trucks in the above example were moving in opposite directions, what would be the velocity after collision and in what direction would the final motion be?*

Taking the direction of the first truck as positive.

$$\text{Momentum of 1st truck} = 400 \text{ ton-ml. units.}$$

$$\text{Momentum of 2nd truck} = -\,350 \text{ ton-ml. units;}$$

$$\therefore \text{ Total momentum} = 50 \text{ ton-ml. units.}$$

Since this is positive, the direction of motion would still be that of the first truck.

$$\text{The velocity would be } \tfrac{50}{55} = {\cdot}91 \text{ ml. per hr.}$$

(3) *A projectile of mass* 150 *lb. leaves a cannon of mass* 6 *tons with a velocity of* 1200 *ft. per sec. With what velocity does the cannon recoil?*

In this case, by Newton's Third Law, the force exerted on the cannon will be equal to that exerted on the projectile so that they will move apart. At the moment of firing projectile and cannon together have no momentum so they must move apart with equal momenta in order that the sum of the momenta will remain zero. Owing to the much greater mass of the gun its velocity will be much less than that of the projectile for an equal momentum.

$$\text{Momentum of projectile} = 150 \times 1200 \text{ ft.-lb. units.};$$

$$\therefore \text{ Momentum of cannon} = -\,150 \times 1200 \text{ ft.-lb. units};$$

$$\therefore \text{ Velocity of cannon} = \frac{-\,150 \times 1200}{6 \times 2240} = -\,13{\cdot}4 \text{ ft. per sec.}$$

The minus sign indicates that the velocity is in the opposite direction to that of the projectile.

QUESTIONS ON CHAPTER IV

Take g as 32 ft. (per sec.)^2 or 980 cm. (per sec.)^2 where necessary.

1. Distinguish between *speed*, *velocity*, and *acceleration*. Give examples to show what is meant by (*a*) a uniform velocity, (*b*) a uniform acceleration.

2. The following table refers to cases of uniform velocity. Fill in the empty spaces.

VELOCITY	DISTANCE	TIME
6 ft. per sec.	yd.	1 hr.
30 ml. per hr.	264 ft.	sec.
cm. per sec.	1 Km.	1 min.
ml. per hr.	704 ft.	16 sec.
30 Km. per hr.	cm.	5 sec.

3. The following table refers to cases of uniform acceleration. Fill in the missing figures.

INITIAL VELOCITY	FINAL VELOCITY	ACCELERATION	TIME	DISTANCE
0	50 ft. per sec.		5 min.	
0		10 cm. (per sec.)^2	15 sec.	
0			10 sec.	200 cm.
16 ft. per sec.		2 ft. (per sec.)^2	5 sec.	
20 ml. per hr.	45 ml. per hr.	2·2 ft. (per sec)^2		
	100 cm. per sec.		½ min.	21 m.

4. The following table refers to falling bodies starting from rest. Fill in.

DISTANCE FALLEN	TIME	FINAL VELOCITY
500 ft.	sec.	ft. per sec.
m.	6 sec.	cm. per sec.
1 Km.	sec.	m. per sec.
ft.	sec.	384 ft. per sec.
ft.	1 min.	ft. per sec.

5. Construct space-time and velocity-time graphs from the following table:—

Time ...	0	1	2	3	4	5	6	7	8	sec.
Distance	0	1	4	9	16	25	36	49	64	ft.

From your graphs determine if the acceleration is uniform and, if so, what is its value ?

6. A car starts from rest, accelerates at the rate of 2 ft. (per sec.)^2 for $\frac{1}{2}$ min., then travels with uniform velocity for a quarter of an hour, after which it is braked and uniformly brought to rest in 25 sec. Find (*a*) its maximum velocity, (*b*) its acceleration after the brake was applied, (*c*) the total distance it travelled.

7. Describe experiments (*a*) to illustrate that all bodies fall equal distances in equal times in vacuum, (*b*) to find the value of the acceleration due to gravity.

8. The following figures were obtained with the apparatus described on page 28.

Distance dart fell	100	90	80	70	cm.
Time of falling	0·452	0·429	0·404	0·378	sec.

Find the average value they give for *g*.

9. At what velocity must a bullet leave the muzzle of a rifle in order to rise 1000 ft. when the rifle is fired vertically ? How long will it take to rise to its highest position, and how long to fall again to the level of the rifle muzzle ? What will its velocity be on reaching that level again ?

10. An object is seen to fall from an aeroplane and observed to take 15 sec. in reaching the ground. Assuming that air resistance can be neglected, calculate (*a*) the height in feet of the aeroplane, (*b*) the velocity with which the object strikes the ground.

11. Describe how you would carry out an experiment to show that, when a force is applied to a body,

$$\frac{\text{Mass of body} \times \text{Acceleration}}{\text{Force applied}} = \text{a constant.}$$

12. Define *dyne* and *poundal*. Why are they called absolute units of force? Explain how they are related to the gravitational units of force.

13. Fill in the spaces in the following table:—

MASS	ACCELERATION PRODUCED	FORCE ACTING
600 grm.	20 cm. (per sec.)^2	
1 st.	8 ft. (per sec.)^2	
	15 ft. (per sec.)^2	30 pndl.
	50 cm. (per sec.)^2	250 grm.-wt.
90 lb.		18 lb.-wt.
100 grm.		10,000 dynes

14. Find the average force in tons-weight required to stop a train weighing 200 tons and travelling at 60 ml. per hr. in half a minute from the application of the brakes. What distance will the train travel in that half-minute?

15. A sack of coal weighing 1 cwt. is raised by means of a rope passing over a pulley. It undergoes uniform acceleration and rises 24 ft. in 5 sec. Find (*a*) its acceleration, (*b*) the tension (pull) in lb.-wt. of the rope.

16. Define *momentum*, and explain why the momentum of a body is often referred to as the amount of motion in it. Why does a flying bullet give a much more serious blow to a rigid board than to a bag of sand?

17. Find the force exerted by a pile driver which weighs 500 lb. when it is allowed to fall 12 ft. and is brought to rest in $\frac{1}{8}$ sec.

18. A boy weighing 8 st. and walking at a speed of 4 ml. per hr. steps on to a stationary trolley which is free to move in the direction

in which he was walking. The trolley weighs 80 lb. With what speed will it move off as he steps on it?

19. A gun weighing 1 ton fires a shot weighing 2 cwt. which leaves the gun with a velocity of 1200 ft. per sec. What will be the initial velocity of recoil of the gun and what force must be applied to it to bring it to rest in a distance of 1 yd.?

20. Distinguish between *mass*, *weight*, *momentum*, and *inertia*.

Explain how force may be defined and measured from Newton's Laws of Motion. [L.U.

21. Describe an experiment to show (*a*) that the weight of a body is proportional to its mass, (*b*) that a body falling freely under the action of gravity has an acceleration of 32 ft. per sec. per sec. approximately. [L.U.

22. Explain the meaning of the statement that the acceleration due to gravity is uniform, its value being 32 ft. per sec. per sec.

A weight is dropped down the shaft of a mine. Draw a graph showing the relation between distance travelled and time taken, after calculating the distance at the end of each complete second up to 6 sec. From this graph obtain the time taken to reach the bottom if the depth is 300 ft. [J.M.B.

CHAPTER V

COMBINATIONS OF FORCES

Scalar and Vector Quantities

Quantities such as forces, velocities, and accelerations differ from quantities such as volumes and masses in having an idea of direction as well as magnitude connected with them. Quantities of the former kind are called **vector quantities**; those of the latter kind are **scalar quantities.** Scalar quantities of the same kind may be added to one another by the ordinary arithmetical rules. Thus 6 pt. and 4 pt. will always make 10 pt. This is not the case with vector quantities, since, in considering their combined effect, the direction as well as the magnitude of each must be considered. Thus, if a body at O (Fig. 25) is acted on by forces of 6 lb.-wt. and 4 lb.-wt. acting in the directions OX and OY, it will tend to move in some such direction as OP. A single force, acting in the direction OP, could be substituted for the two forces along OX and OY without altering the effect on the body, but this force would be less than 10 lb-wt. **The single force which, acting alone, would produce the same effect as a set of forces acting together is said to be the resultant of those forces.**

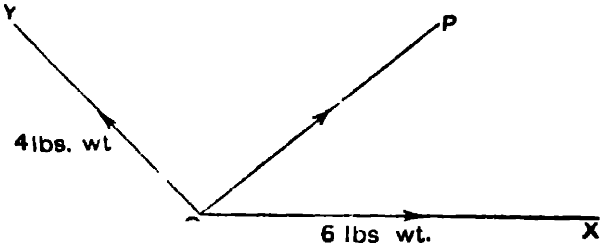


FIG. 25.

The principle of the last paragraph is illustrated if the hooks of three spring balances are tied together and the three balances are pulled in different directions. It will be found that no one balance gives a reading equal to the sum of the readings of the other two.

If two forces act along the same straight line and in the same direction, their resultant will be equal to their sum. If they act in opposite directions it will be equal to their difference and will act in the direction of the larger one.

Parallelogram of Forces

The way in which the resultant of two forces acting at an angle to one another can be found is shown by the following experiment.

Connect masses of 3, 4, and 5 lb. by light cords, two of which pass over light, easy running pulleys, as shown in Fig. 26. On releasing them, the weights will come to rest in a position similar to that shown. Forces of 4 and 3 lb.-wt. act along the strings OP and OQ and just balance the pull of 5 lb.-wt. along OR. Hence their resultant must be a force of 5 lb.-wt. acting in the direction of RO produced.

Place a sheet of paper behind the strings and mark their directions

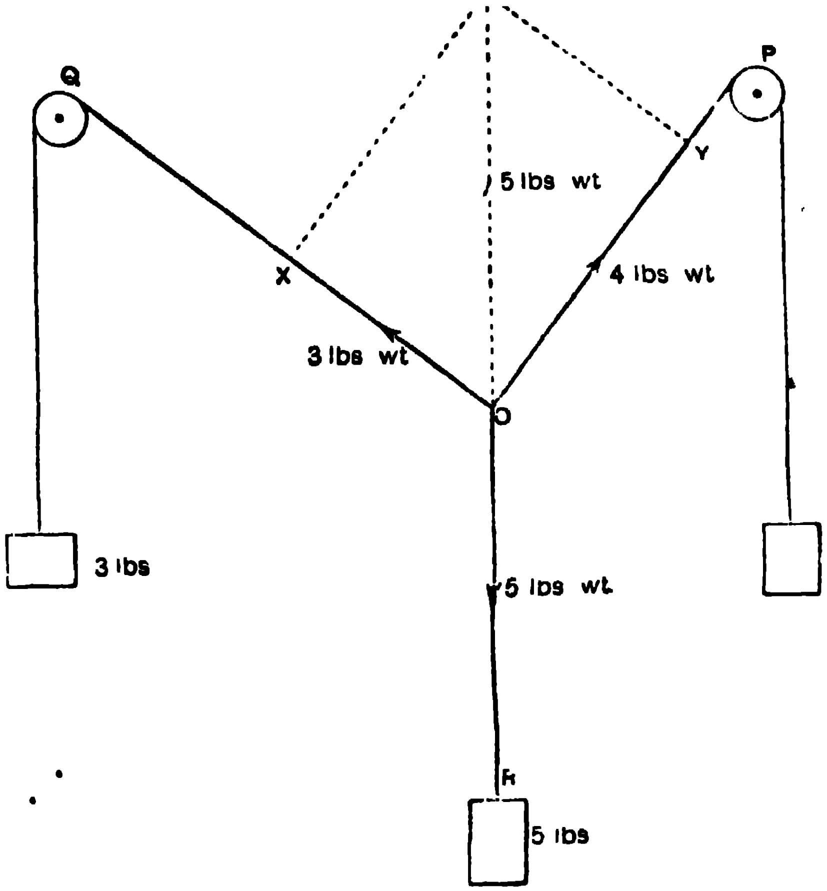


Fig. 26.

on it. Choosing some such scale as 1 in. to 1 lb.-wt., mark off lengths OX and OY along OQ and OP to represent the forces of 3 and 4 lb.-wt. which act along them. Complete the parallelogram OXZY and draw its diagonal OZ. It will be found that OZ is along RO produced, and that on the same scale as OX and OY, it represents a force of 5 lb.-wt. This illustrates the theorem of the **parallelogram of forces**, which states that, if two forces acting at a point are represented in magnitude and dire their point of intersection represents the magnitude and direction of their resultant. This should be verified for a number of cases.

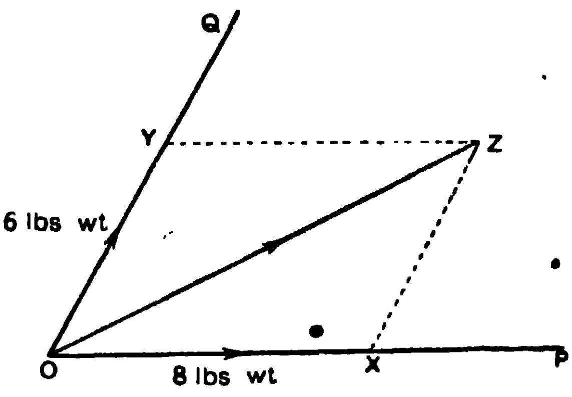


FIG. 27.

EXAMPLES.—(1) *Forces of* 6 *and* 8 *lb.-wt. respectively act along lines inclined to one another at an angle of* 60°. *Find the magnitude and direction of their resultant.*

From O (Fig. 27) draw OP and OQ, making an angle of 60°. Mark off on OP a length OX 8 cm long and on OQ a length OY 6 cm. long. Complete the parallelogram OXZY and draw the diagonal OZ. Then OZ represents the resultant of the forces represented by OX and OY. Measurement shows that OZ is 12·1 cm. long and makes an angle of 25° with OP. Therefore the required resultant is 12·1 lb.-wt. and its direction makes an angle of 25° with that of the force of 8 lb.-wt.

All such problems may be approximately solved by a graphical method as above. The following example shows how, when the two forces are at right angles, the problem may be solved by calculation.

(2) *Two forces of* 7 *and* 11 *lb.-wt. act at right angles to one another. Find the magnitude and direction of their resultant.*

If OP and OQ (Fig. 28) represent the two forces, OR represents their resultant.

$$\text{Now } OR^2 = OP^2 + PR^2 = OP^2 + OQ^2$$
$$= 11^2 + 7^2 = 121 + 49 = 170;$$
$$\therefore\ OR = \sqrt{170} = 13{\cdot}04.$$

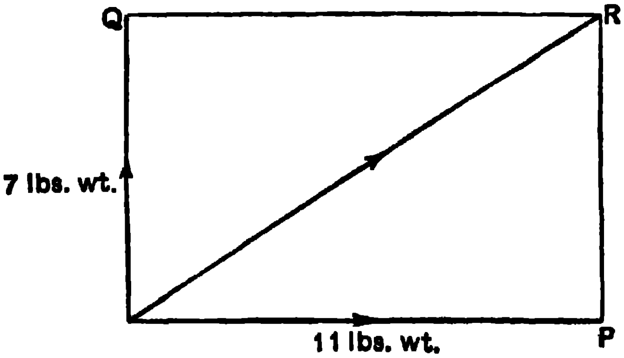


FIG. 28.

Also $\tan \angle ROP = \frac{RP}{OP} = \frac{7}{11} = 0{\cdot}6364$. Reference to tables shows that the angle whose tangent $= 0{\cdot}6364$ is approximately $32\frac{1}{3}°$.

Therefore the resultant is a force of 13·04 lb.-wt. and its direction makes an angle of $32\frac{1}{3}°$ with that of the larger force.

Resolving Forces

When forces are "added" by means of the parallelogram theorem they are said to be compounded. The reverse process, that of finding two forces which, acting in different directions, may be substituted for a single force, is called *resolving* a force into component forces. The following example will illustrate the graphical method of doing this.

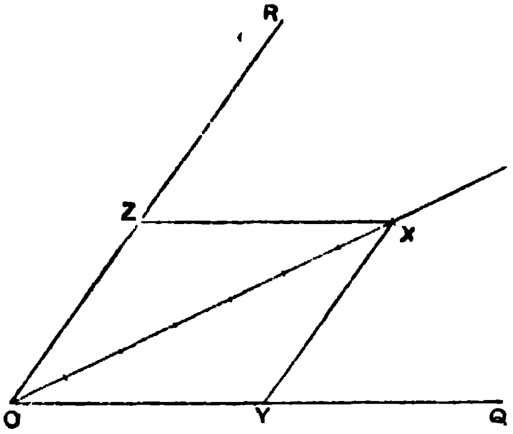


FIG. 29.

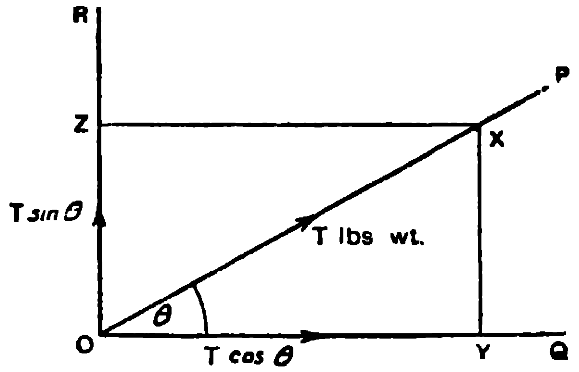


FIG. 30.

Let a force of 7 lb.-wt. act in the direction OP (Fig. 29) on a small body at O. It is required to find the components of this force in the directions OQ and OR. Along OP cut off OX, 7 units long. From X draw XY parallel to OR to meet OQ at Y and XZ parallel to OQ to meet OR at Z. OY and OZ measure 3·4 and 2·7 units respectively. Hence, by the parallelogram theorem, a force of 3·4 lb-wt. along OQ together with a force of 2·7 lb.-wt. along OR would have the same effect on the body as the force of 7 lb.-wt. along OP.

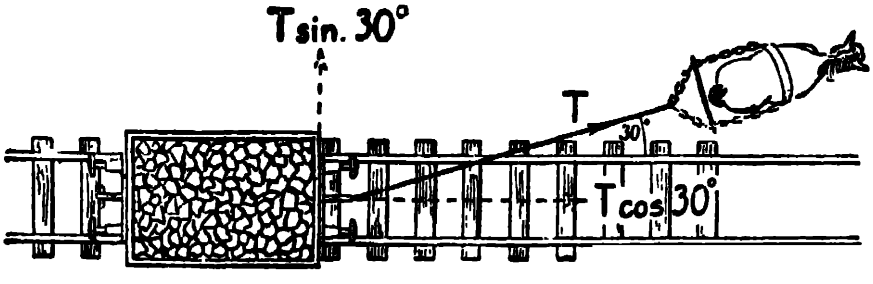


FIG. 31.

The finding of components of a force which are at right angles to one another is frequently required. The following shows how such components may be calculated. Let a force of T lb.-wt. be acting at O along OP (Fig. 30), and OQ and OR be two lines at right angles to one another, OQ making an angle θ with OP. If $OX = T$ units and the rectangle OYXZ is completed,

$$\frac{OY}{OX} = \cos\theta, \text{ i.e. } \frac{OY}{T} = \cos\theta; \quad \therefore\ OY = T\cos\theta,$$

$$\frac{OZ}{OX} = \frac{YX}{OX} = \sin\theta, \text{ i.e. } \frac{OZ}{T} = \sin\theta; \quad \therefore\ OZ = T\sin\theta,$$

so the components along OQ and OR are $T\cos\theta$ lb.-wt. and $T\sin\theta$ lb.-wt. respectively, *e.g.* if T is 200 and θ is 30°,

$$\text{Component along OQ} = 200 \times \frac{\sqrt{3}}{2} \text{ lb.-wt.}$$

$$\text{Component along OR is } 200 \times \tfrac{1}{2} \text{ lb.-wt.}$$

The importance of the last case can be seen from Fig. 31. The horse is giving a pull of T lb.-wt. on the rope which is at an angle of 30° to the rails. This force may be resolved into components of $T\cos 30°$ lb.-wt. in the direction of the rails and $T\sin 30°$ lb.-wt. at right angles to them. The latter component evidently has no tendency to move the truck forwards. Its only effect is to make the flanges of the near wheels press on the rail. Thus, the effective force pulling the truck forward is $T\cos 30°$ lb.-wt. and, since $\cos 30° = \frac{\sqrt{3}}{2}$, nearly one-fifth of the pull given by the horse is wasted. Perhaps you will understand now why you are urged to "keep the rope straight" when you are pulling in a tug-of-war.

Equilibrium. Triangle of Forces

If two forces are acting on a small body, a third one which is equal to their resultant but in the opposite direction will neutralise their effects and prevent any motion of the body, in other words, the resultant of the three forces will be zero. Such a set of forces is said to be in equilibrium.

In Fig. 32 (*a*) the line OC represents the resultant R of the forces P and Q represented by OA and OB. If the shaded triangle only had been drawn as in (*b*), OC would still represent the resultant of the forces P and Q. This indicates how the resultant of two forces acting at a point may be found by drawing a triangle instead of a parallelogram.

If now a force equal and opposite to that represented by OC is considered as in (*c*), it would be in equilibrium with two forces P and

Q acting at O in directions parallel to OA and AC respectively. This gives the theorem known as the **triangle of forces**, namely:—**if three forces acting at a point can be represented in magnitude and direction by the three sides, in order, of a triangle, those forces are in equilibrium.**

Examples.—(1) *Forces of* 6 *and* 4 *lb.-wt. act on a small body, the first towards the east and the second in a direction* 30° *east of north. Find the magnitude and direction of the force required to keep the body in equilibrium.*

In Fig. 33 (*a*) the arrangement of the two forces is shown. Fig. 33 (*b*) is constructed as follows. O′P′ is drawn parallel to OP and 6 units long. P′Q′ is drawn parallel to OQ and 4 units long. O′Q′ is joined and represents the required force. Q′O′ is 10·6 units long and angle RQ′P′ is 24°; ∴ the required force is 10·6 lb.-wt. in a direction 24° south of west.

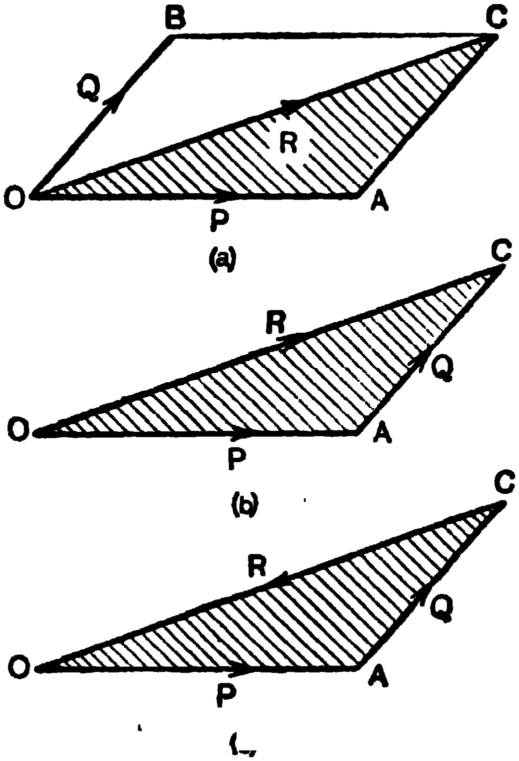


Fig. 32.

(2) *A vertical spring balance reads* 200 *grm.-wt. when a piece of metal is hung on it. The metal is then pulled to one side by means of a horizontal thread until the balance makes an angle of* 30° *with the vertical. Calculate the new reading of the balance and the tension in the thread.*

The body will be in equilibrium under the action of the three forces, its weight (200 grm.-wt.) acting vertically downwards, the pull (x grm.-wt.) of the spring acting at an angle of 30° to the vertical, and the tension (y grm.-wt.) in the thread acting horizontally.

Draw Fig. 34 to show the direction of these forces. Now draw the triangle of forces, Fig. 35, as follows:—

Draw AB vertically to represent the weight of the body on some suitable scale. (Note, begin with the force whose size is known.) From B draw BC horizontally and from A draw AC, making an angle of 30° with AB. (Note, the sides of the triangle must be so arranged so that all the arrows indicating the directions of the forces proceed either clockwise or anti-clockwise round the triangle.)

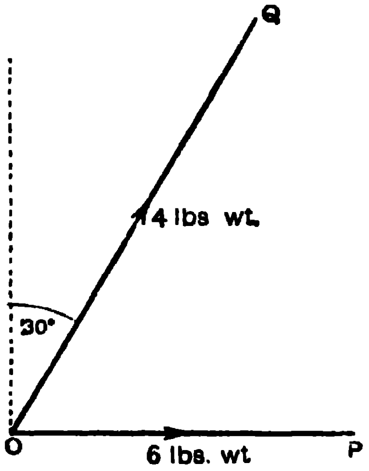


FIG. 33 (*a*).

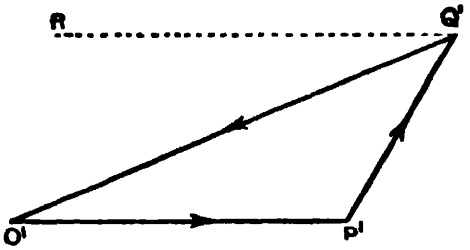


FIG. 33 (*b*).

The values of x and y may then be determined by measurement of CA and BC or they may be calculated as follows:—

$$\cos 30^\circ = \frac{AB}{AC}; \quad \therefore AC = AB \times \frac{1}{\cos 30^\circ} = 200 \times \frac{1}{0{\cdot}8660} = 230{\cdot}9,$$

$$\tan 30^\circ = \frac{BC}{AB}; \quad \therefore BC = AB \times \tan 30^\circ = 200 \times 0{\cdot}5774 = 115{\cdot}5;$$

$\therefore$ Reading of spring balance = 230·9 grm.-wt.

and Tension in thread = 115·5 grm.-wt.

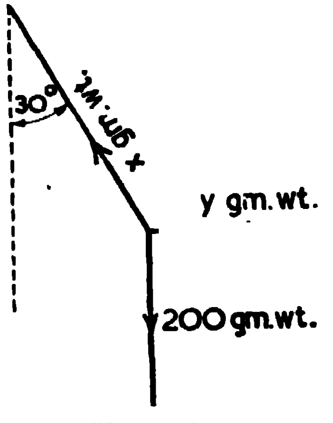


FIG. 34.

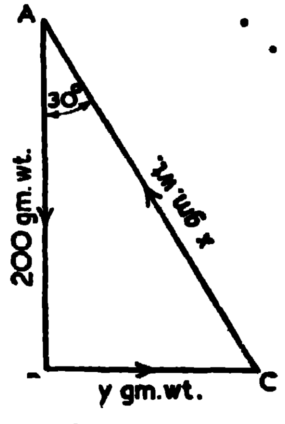


FIG. 35.

Interesting applications of the resolution of forces are found in connexion with the flight of aeroplanes and the sailing of yachts.

Fig. 36 (*a*) illustrates an aeroplane in level flight. The forces acting on it are the forward pull, P, of its propeller, its weight, W, acting vertically downwards, and the air resistance, R, on its wings which acts approximately at right angles to the under surface of the wings. R may be resolved into the components L and B which are respectively vertically upwards and horizontally backwards. If the wings are at the proper angle, L and W will be equal so that the vertical forces have a zero resultant and the plane neither rises nor falls. If P is greater than B there will be a resultant forward force equal to P − B which will give a forward acceleration to the aeroplane. If P

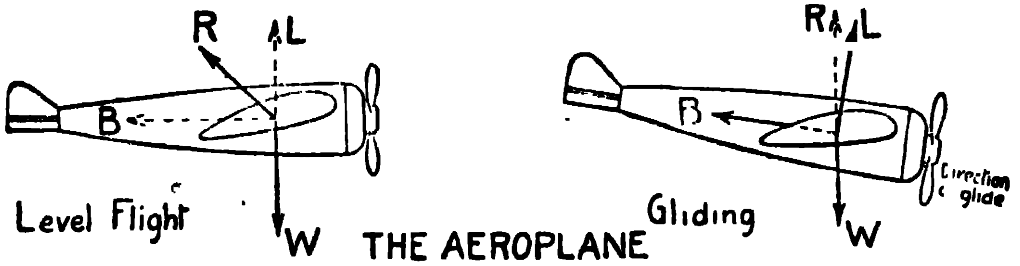


Fig. 36 (*a*). Fig. 36 (*b*).

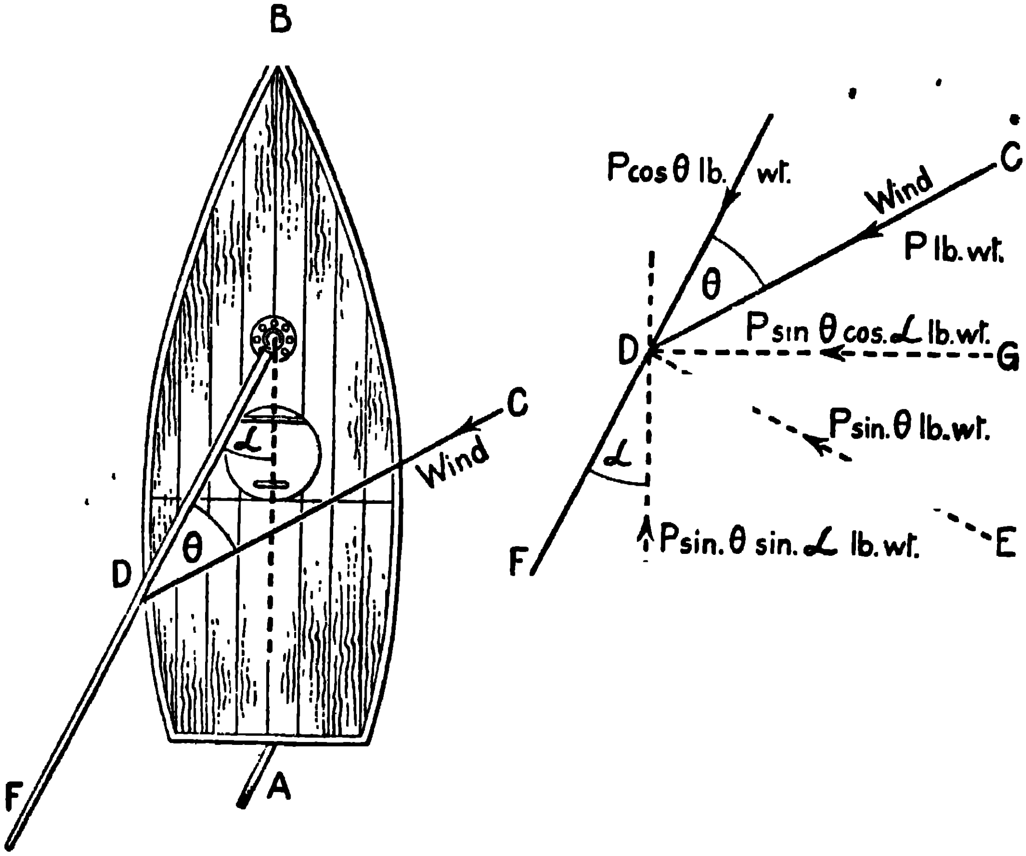


Fig. 37.

is equal to B when the aeroplane has a certain velocity, there will be no resultant force forwards or backwards, and according to Newton's First Law of Motion the aeroplane will continue forwards with uniform velocity.

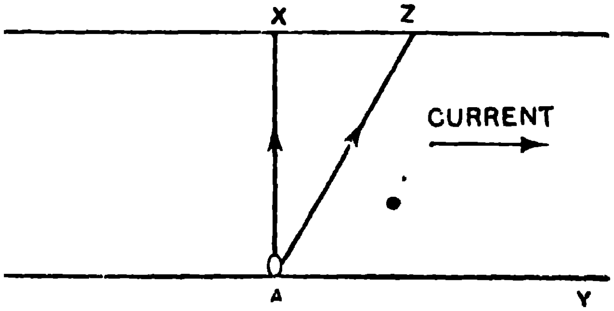


FIG. 38.

Fig. 36 (*b*) shows the plane when gliding with the engine not running. If it is tilted until R is almost vertical, B will become very small and L will be almost equal to R. L and W will then have a resultant in the direction of the glide which will be greater than B.

Fig. 37 illustrates a yacht sailing almost against the wind. As indicated in the diagram, the force exerted in the direction CD by the wind on the sail may be resolved into components, one acting along the sail and the other at right angles to it along ED. The former component will have no effect on the sail and the latter may be again resolved into components, one acting along HD, *i.e.* the centre line of the yacht, and the other at right angles to it along GD. The former component will drive the yacht forward, while the latter will cause a certain amount of drift to leeward.

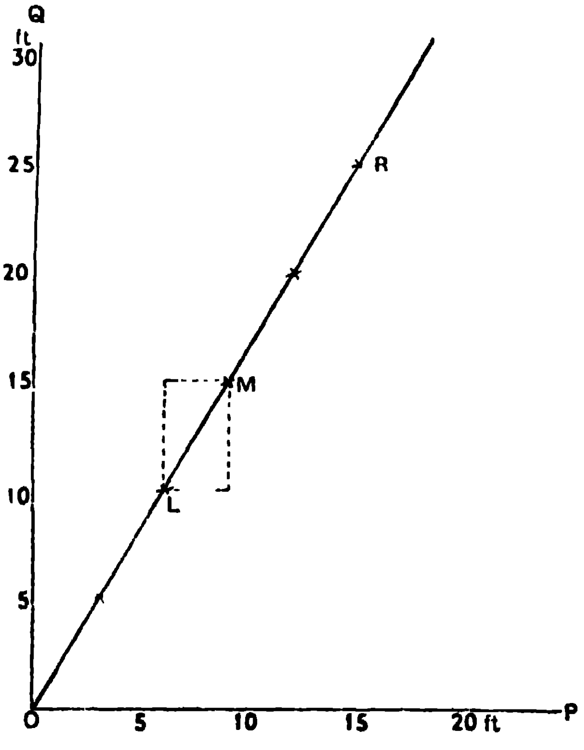


FIG. 39.

Parallelogram of Velocities

Velocities are vector quantities, and so may be compounded and resolved by the parallelogram theorem.

Suppose a boat is being rowed across a river from A (Fig. 38), the direction of rowing being directly across the river. The current will cause it to drift down the river as it crosses, so that it will actually move along some such line as AZ.

Suppose that the rowing would give it a velocity of 5 ft. per sec. in still water and that the current flows with a velocity of 3 ft. per sec. Then in each second it moves 5 ft. across and 3 ft. down the stream and, if OP and OQ in Fig. 39 represent the directions AY and AX, the crosses represent the positions of the boat at the ends of successive seconds, *i.e.* it travels in the direction OR. Further, LM represents the distance moved by the boat in 1 sec., *i.e.* its actual velocity, and LM is the diagonal of a parallelogram of sides 5 and 3 units long parallel to OQ and OP respectively.

QUESTIONS ON CHAPTER V

1. Find by drawing the magnitude and directions of the resultants in each of the following cases of two forces acting at a point.

1st Force	2nd Force	Angle Between Forces
100 lb.-wt.	150 lb.-wt.	60°
90 grm.-wt.	70 grm.-wt.	45°
600 dynes	450 dynes	150°
10 tons-wt.	12 tons-wt.	80°

2. Explain what is meant by (*a*) *a vector quantity*, (*b*) *a scalar quantity*, (*c*) *the resultant of two forces*.

State the parallelogram theorem for vector quantities and describe an experiment to prove it in the case of forces.

3. Calculate the magnitude of the resultant and the angle it makes with the larger force in each of the following cases, in which the two forces act at right-angles to one another.

(*a*) 12 lb.-wt. and 16 lb.-wt.
(*b*) 90 grm.-wt. and 120 grm.-wt.
(*c*) 17 pndl. and 11 pndl.

4. A steamer heading north would have a velocity of 15 ml. per hr. in still water, but it is in a current flowing to the east with a velocity of 6 ml. per hr. Find the direction in which it travels and its velocity in that direction.

5. Explain what is meant by *resolving a force into two components at right-angles*, and explain with the aid of a diagram why you should "keep the rope straight" in a tug-of-war.

6. Calculate the magnitudes of the two components at right-angles in each of the following cases:—

Force	Angle Made with Force by One Component
200 lb.-wt.	45°
150 grm.-wt.	30°
2 tons-wt.	60°

7. A lawn roller weighing 200 lb. is pulled by applying a force of 100 lb.-wt. to the handle which is inclined at an angle of 30° to the ground. Find (*a*) the force with which the roller is urged forward, (*b*) the force tending to lift it off the ground, (*c*) the actual force exerted by it on the ground.

8. A smooth roller weighing 500 grm. rests on a smooth surface sloping at an angle of 30° to the horizontal. Find the force, acting parallel to the slope, necessary to keep the roller from moving. (Note: *The forces acting on the roller are its weight vertically downwards, the applied force, and the reaction of the surface acting perpendicular to that surface.*)

9. State the theorem of the parallelogram of forces.

A small body rests on a smooth horizontal table and is acted on by the following forces: (*a*) 2 units to the east; (*b*) 3 units to the north-east; (*c*) 4 units at 30° to the west of north; (*d*) 8 units to the south. Find, graphically or otherwise, the magnitude and direction of the force necessary to produce equilibrium. [J.M.B.

10. State the "parallelogram of forces," and indicate how it may be proved experimentally.

A barge is being towed along the middle of a canal 40 ft. wide by men on either bank with ropes each 80 ft. long. Each rope is pulled with a force of 400 lb.-wt. What is the effective pull on the barge down the middle of the canal? [J.M.B.

11. What is meant by the *triangle of forces*?

A ping-pong ball of mass 2·25 grm. and suspended by cotton is blown to one side by a steady horizontal current of air so that the taut cotton makes an angle of 30° with the vertical. Draw a diagram of the arrangement indicating the forces acting on the ball, and find (*a*) the tension in the cotton, (*b*) the force of the air current on the ball. [J.M.B.

CHAPTER VI

LEVERS, MOMENTS, PARALLEL FORCES CENTRE OF GRAVITY

Levers

A **lever** is a stiff rod arranged so that it can turn around a point of support. See-saws, crow-bars, brake levers, and balance beams are common examples of levers. If a heavy boy and a lighter boy are see-sawing, the light one must sit further from the support than the other in order to make their weights balance. Similarly, a heavy stone may easily be moved by using a crow-bar as illustrated in Fig. 40, which also illustrates some terms used in connexion with levers. The support around which the lever turns is called its **fulcrum.** The force which is applied to it is called the **effort**, and the resistance overcome by it is the **load.** The distance between the fulcrum and the effort is the **effort arm**; that between fulcrum and load is the **load arm.**

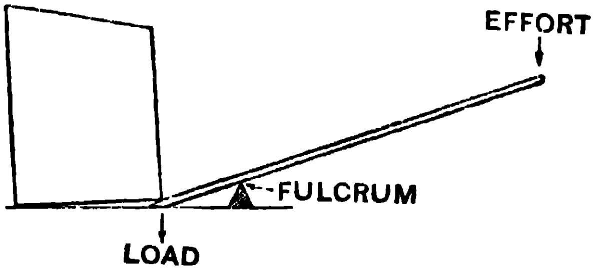


Fig. 40.

The law of levers may be investigated by setting up a half-metre scale to act as a lever, as shown in Fig. 41. A hole is bored through the middle of the scale so that it balances, but turns easily, on a smooth

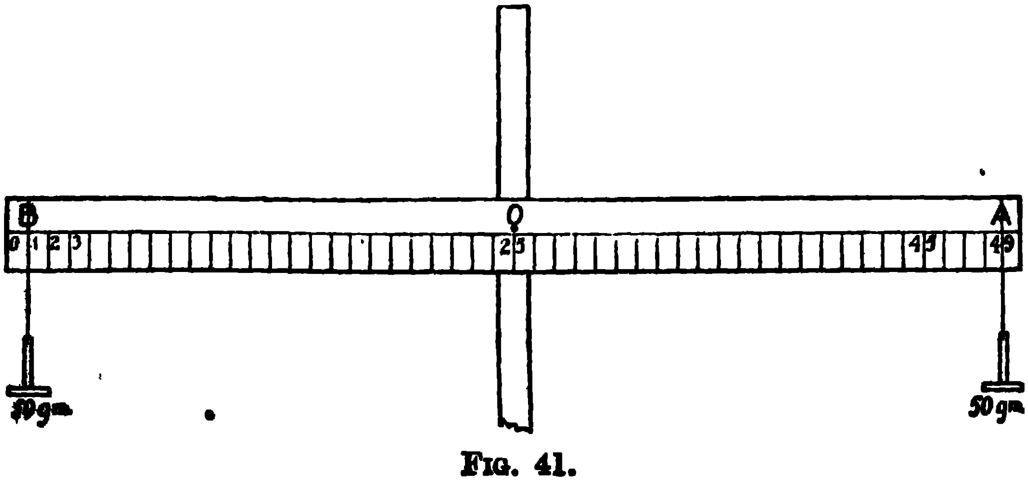


Fig. 41.

peg passing through the hole. A weight carrier of known weight is fitted on each arm in such a way that it can slide along the lever. Slotted weights can be placed on the carriers to apply varying forces to the lever. We may consider the weight on the right as the effort which raises the load on the left. By using various weights, sliding one or the other along the lever till balance is obtained, and then noting their distances from the fulcrum, a table such as the following may be obtained.

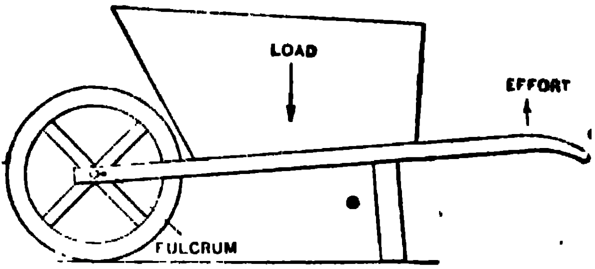


Fig. 42.

Effort	Effort Arm	Effort × Effort Arm	Load	Load Arm	Load × Load Arm
40 grm.	24·0 cm.	960	40 grm.	24·0 cm.	960
45 grm.	22·0 cm.	990	60 grm.	16·5 cm.	990
70 grm.	21·0 cm.	1470	100 grm.	14·7 cm.	1470
150 grm.	13·0 cm.	1950	80 grm.	24·4 cm.	1952

In each case it will be found approximately that

Effort × Effort arm = Load × Load arm.

Levers such as the above, which have the fulcrum between the effort and the load are frequently said to be of the *first class.*

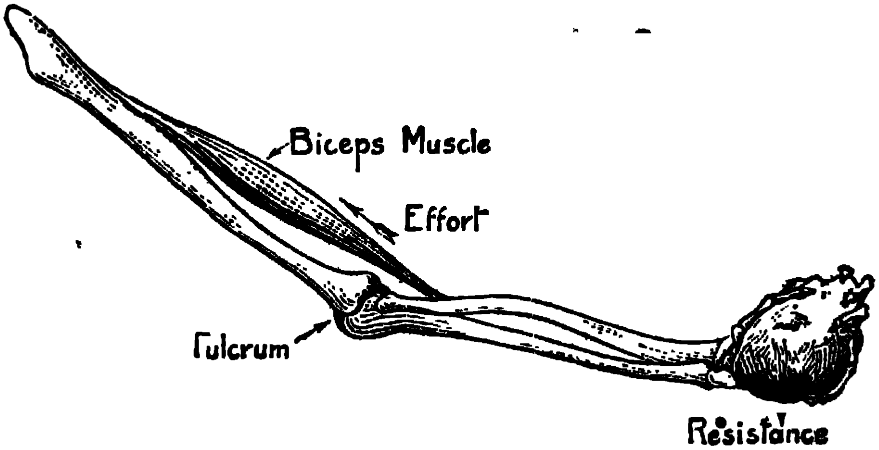


Fig. 43.

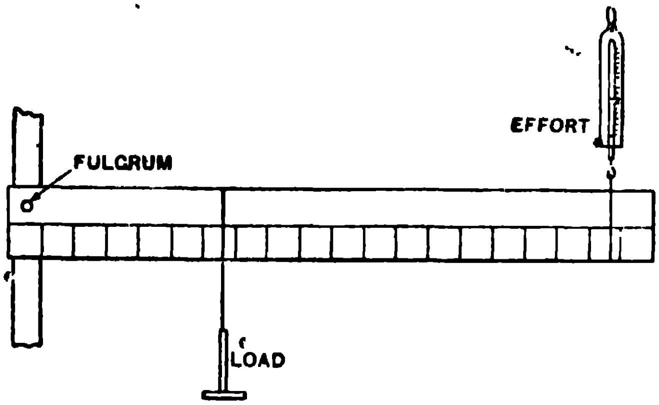


Fig. 44.

When the handles of a wheelbarrow are raised, as will be seen from Fig. 42, it constitutes a lever with the load between the fulcrum and the effort. This is often said to be a lever of the *second class.*

Fig. 43 shows that in raising a weight in the hand by means of the biceps muscle, the forearm acts as a lever with the effort between the fulcrum and load. This is usually called a lever of the *third class.*

By the method indicated in Fig. 44 it may be shown that levers of the second and third classes obey the same law as those of the first class.

A list should be made of common appliances such as pump-handles, scissors, nutcrackers, bicycle crank and gear-wheel, etc., in which the leverage principle is applied, noting in each case which class of lever is employed.

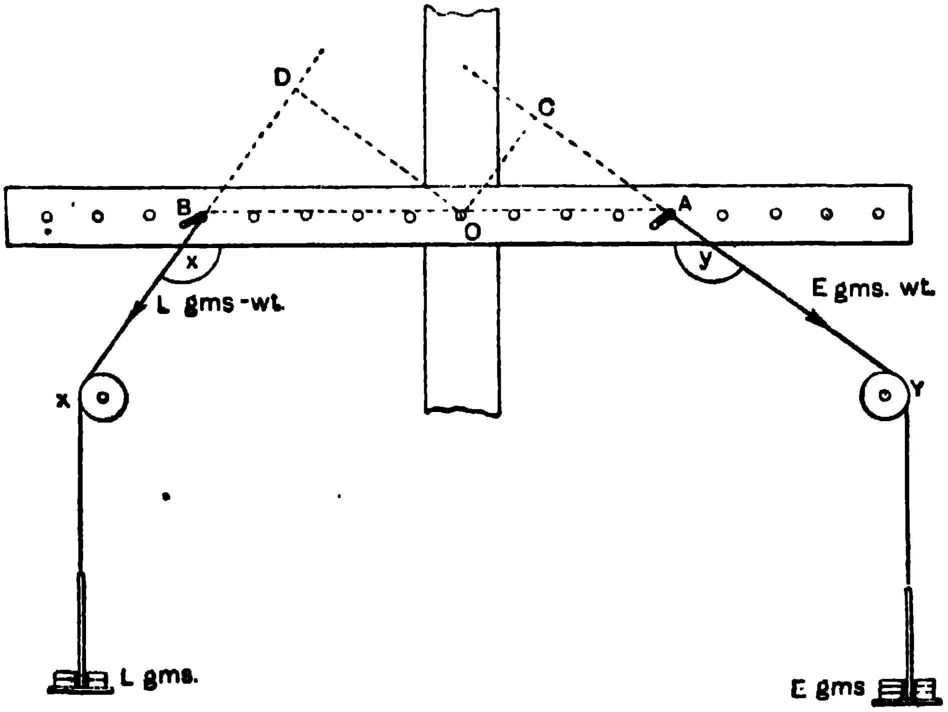


Fig. 45.

Moments

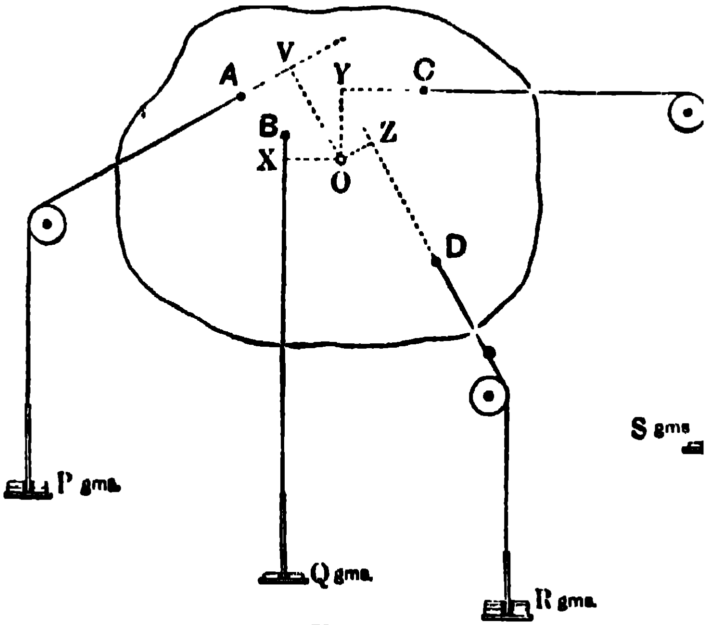


FIG. 46.

In the examples of levers which have been studied it will be noted that the effort and the load are forces which tend to turn the lever in opposite directions, and it is these turning tendencies which have been balanced. The magnitude of the tendency of a force to turn a body is called the *moment of the force around the point* (strictly, around the *axis*) *around which the body turns.*

From the condition we have found for balance of a lever it would appear that the moment of a force = force × distance from point around which the body turns. In the cases studied, however, we always dealt with forces at right angles to the lever. Fig. 45 shows how, by using a lever pierced by a number of holes into which pegs may be plugged, forces in varying directions may be applied to it. It will be found that balance may be obtained by altering the weights, or the angles of the strings, or the points at which they are attached to the lever. Usually it will be found that E × OA is not equal to L × OB. When balance has been obtained, measure the angles *x* and *y*, and on a sheet of paper construct to scale the figure XBOAY. Draw perpendiculars OC and OD from O to YA and XB produced. Measure OD and OC and verify that E × OC = L × OD. Thus, the moment of a force around a point is really measured by force × *perpendicular distance* from point around which body turns to line of action of the force.

This result is quite general and may be applied when more than two forces act on a body. Thus Fig. 46 represents a light board pivoted at O with weights attached to points A, B, C, D as shown. It comes to rest in such a position that

$$(P \times OV) + (Q \times OX) = (R \times OZ) + (S \times OY).$$

EXAMPLES.—(1) *The diagram* (*Fig.* 47) *represents a safety valve on a boiler. The opening at* O *has an area of* 4 *sq. in. What must be the* *value of* W *in order that the valve shall not open until the pressure inside the boiler is* 60 *lb. per sq. in. greater than that outside?*

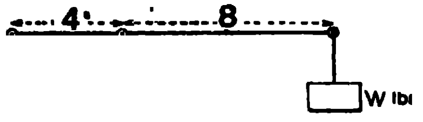


FIG. 47.

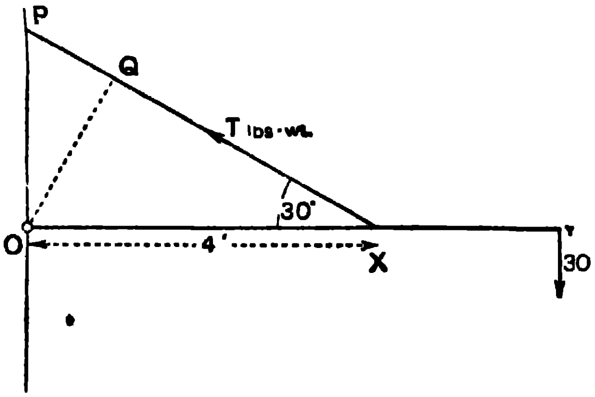


FIG. 48.

With required pressure, upward force on disc = 60 × 4 lb.-wt.

Taking this as effort and W as load:—

Load × load arm = effort × effort arm;

$$\therefore\ W \times 12 = 60 \times 4 \times 4;$$

$$W = \frac{60 \times 4 \times 4}{12} = 80.$$

(2) *A sign weighing* 30 *lb. hangs from the end of a pole* 6 *ft. long which is hinged to a wall. A wire, attached to the pole as indicated in the figure, maintains the pole in a horizontal position. Neglecting the weight of the pole, determine the tension (pull) in the wire.*

Let the tension be T lb.-wt. From O draw OQ perpendicular to PX (Fig. 48): then

$$OQ = OX.\sin 30^\circ$$

$$= 4 \times \tfrac{1}{2}\text{ ft.} = 2\text{ ft.}$$

Taking moments around O:—

$$T \times OQ = 30 \times 6;$$

$$\therefore\ T \times 2 = 30 \times 6;$$

$$\therefore\ T = \frac{30 \times 6}{2} = 90.$$

The tension = 90 lb.-wt.

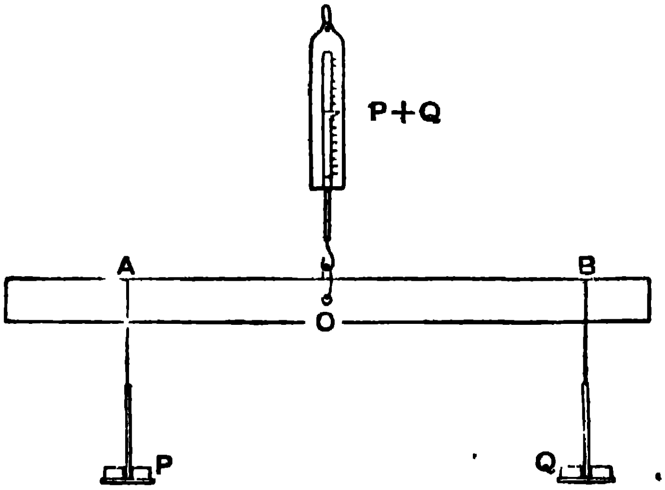


FIG. 49.

Parallel Forces

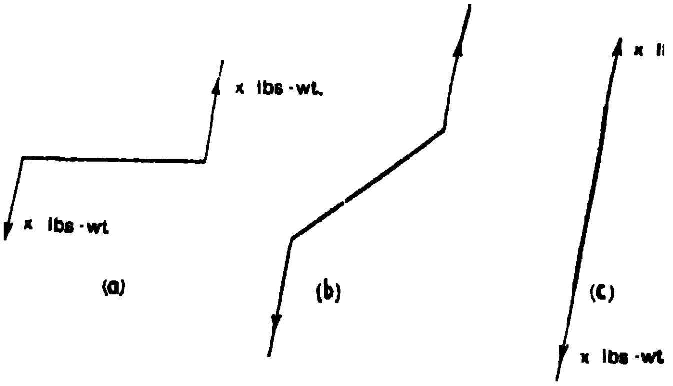


FIG. 50.

Suspend a light balanced lever from a spring balance and hang a weight on each arm as shown in Fig. 49. Adjust the positions of the weights until the lever is balanced and remains horizontal. It will be found that the reading of the spring balance equals the sum of the two weights P and Q. Clearly, the same result would be obtained if a vertical downward force of (P + Q) grm. were applied at O in place of the two weights, *i.e.* the resultant of the two parallel forces applied at A and B is equal to their sum and acts at O. Since the lever is not turning, the moments of P and Q around O are equal, *i.e.* taking clockwise moments as positive and anti-clockwise ones as negative, the sum of the moments around O is zero. **Hence, the resultant of two parallel forces acting in the same direction is equal to their sum and acts at a point around which the sum of their moments is zero.**

If a number of weights are hung at various points along the rod and their positions adjusted until balance is obtained, it will be found that the spring balance gives a reading equal to the sum of all of them, and that the sum of all the moments around O is zero.

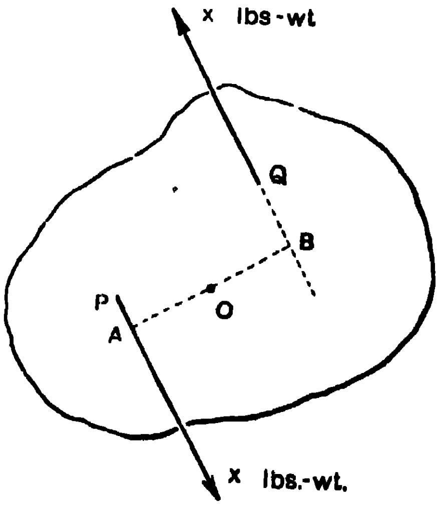


FIG. 51.

Couples

When two equal parallel forces act in opposite directions on a body they are said to constitute a couple. A couple can only exert a turning effect even if the body is not pivoted. Fig. 50 illustrates this. When position (*c*) is reached, the two forces, being just equal and opposite, will neutralise one another, and no further movement of the body will take place.

In Fig. 51 two forces forming a couple act at P and Q. Their respective moments around O are $x \times OA$ and $x \times OB$. As both tend to turn the body in the same direction their total turning effect is measured by $(x \times OA) + (x \times OB)$: that is:—

$$\text{Turning effect} = x\,(OA + OB) = x \times AB.$$

From the above we can say that **the moment of a couple is equal to one of the forces multiplied by the perpendicular distance between them.**

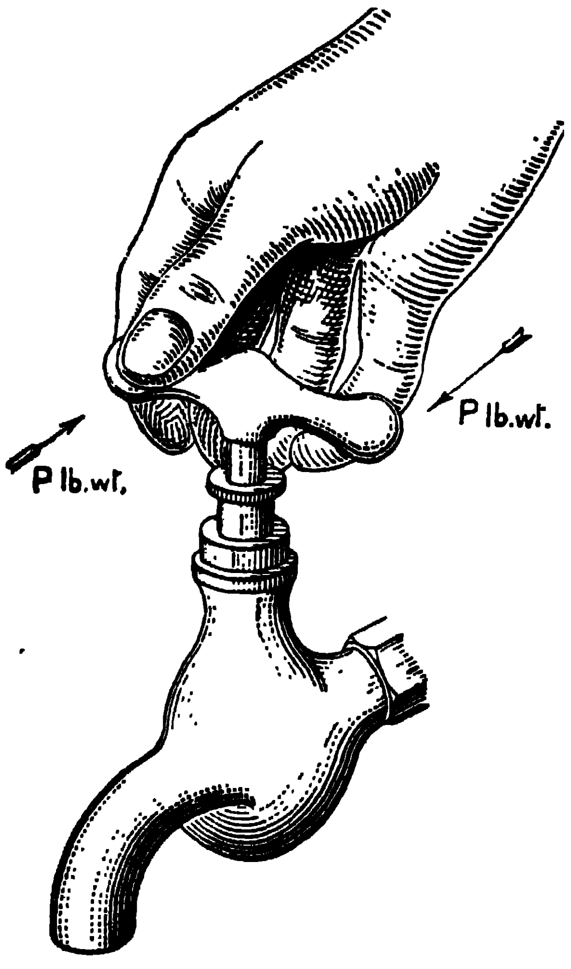

P lb.wt.
P lb.wt.

Fig. 52.

Many common objects where turning is the only movement required are arranged so that couples may be applied to them. Fig. 52 illustrates this in the case of a tap. Corkscrews, gimlets, and door keys similarly apply the same principle. The advantage is that strain or displacement due to a sideways pull which might be applied to a one-sided handle is avoided.

Centre of Gravity

Try to balance plates of various shapes—squares, triangles, etc.—cut out of sheet metal or cardboard on the point of a compass or knitting needle. It will be found in each case that there is one point, and only one point, at which the plate may be so supported. If in any case the required point cannot be found by trial, use the method illustrated in Fig. 53. Pierce several smooth holes through the plate near its edge. Support it by means of a smooth horizontal peg fitting loosely in one of the holes A, and suspend a plumb-line from the peg. Draw a line on the plate along the plumb-line. Repeat with one of the other holes B on the peg. The two lines so obtained will intersect at some point G. If now the third hole C is placed on the peg, the plumb-line will again pass through G. Try now if the plate will balance when supported at G.

These experiments may be explained as follows. The plate is made up of a number of particles, each of which has a weight, *i.e.* a vertical force acting downwards on it. Thus the whole plate is subject to a number of parallel forces acting vertically downwards. It has been shown above that such a set of forces has a resultant equal to their sum which acts through a fixed point. In this case the resultant must be equal to the whole weight of the body, and if an equal upward force is applied at the point where the resultant acts, the plate will be in equilibrium, *i.e.* it will balance. The reaction of the support provides the equal upward force.

The point through which the resultant of the weights of all the particles of a body acts is called its centre of gravity. In problems where the weight of a body has to be taken into account, that weight may be considered as a single force acting vertically downwards through the centre of gravity of the body.

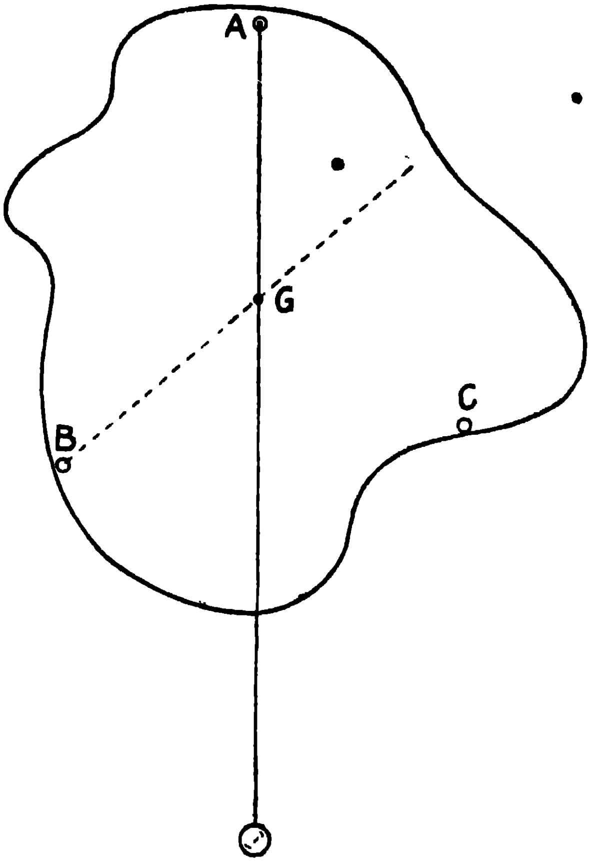


FIG. 53.

The method for finding the centre of gravity of a plate (page 60) depends on the fact that, if the centre of gravity G is in any position other than vertically below A, the weight of the plate acting through G will have a moment around A tending to rotate the plate. Thus, when the plate comes to rest, G must be vertically below A.

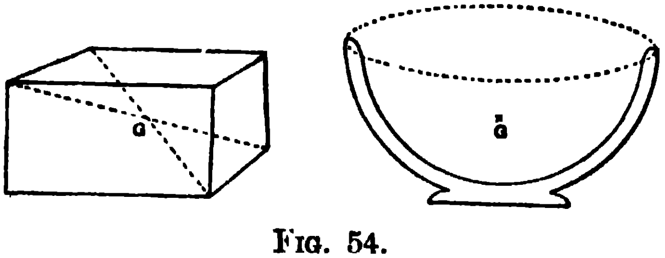


FIG. 54.

Flat plates only have been considered in the above, but it will be obvious that every body will have a centre of gravity. In the case of a brick the centre of gravity will clearly be inside it. In the case of a bowl it may be in the space inside and not in the material of the bowl. The following table should be noted.

BODY	POSITION OF CENTRE OF GRAVITY
Uniform rod.	Centre of rod.
Circular plate.	Centre of plate.
Plate a parallelogram (including square and rectangle).	Intersection of lines joining mid-points of opposite sides.
Triangular plate.	Intersection of medians.
Rectangular block (including cube).	Intersection of diagonals.
Sphere.	Centre of sphere.
Cylinder.	Mid-point of axis.

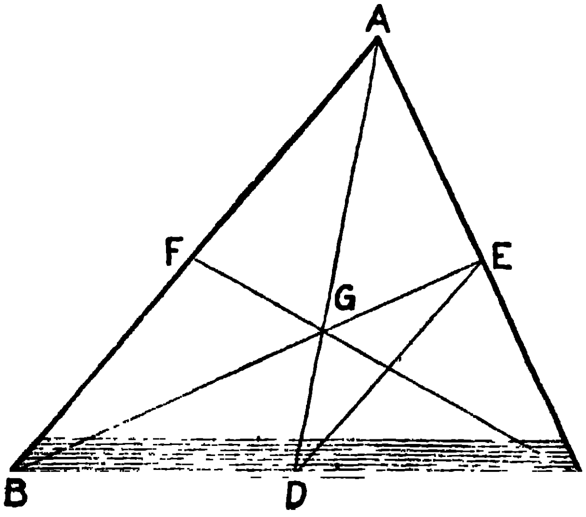


FIG. 55.

A number of the above cases can be justified on grounds of symmetry, *i.e.* the body may be divided up into a number of pairs of particles, the two members of any pair being at equal distances on opposite sides of the point named so that the sum of their moments about that point is zero.

A triangular plate may be considered to be made up of a number of very narrow rods all parallel to one side (see Fig. 55). Each of these rods has its centre of gravity at its mid-point. All these centres of gravity will lie along the median AD so that the weight of the plate may be considered to be made up of a number of weights acting at points along AD. The resultant of these would also act through some point on AD, *i.e.* the centre of gravity of the whole plate is a point on AD. Similarly it can be shown that it lies on each of the other medians and therefore it must be at their point of intersection. A similar method may be used in the case of the parallelogram.

The last paragraph illustrates the general principle that the position of the centre of gravity of a body can often be found if the centres of

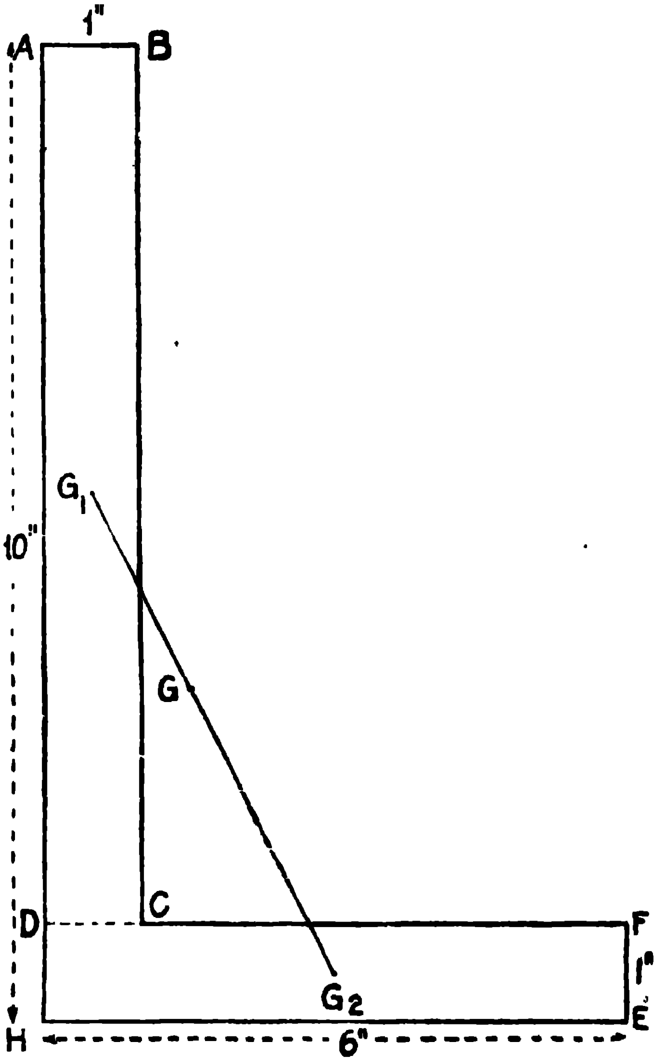


FIG. 56.

gravity of its various parts are first determined. In the L-shaped plate of Fig. 56, the centre of gravity of ABCD is at G_1, and that of EFDH is G_2. The areas of the two rectangles, and therefore their weights, are in the ratio $\frac{9}{6}$. Hence, if a point G is taken in G_1G_2, such that $9 \times G_1G = 6 \times G_2G$, the moments of the weights of the two parts around G will be equal and opposite and G will be the centre of gravity of the whole figure.

Since

$$9 \times G_1G = 6 \times G_2G;$$

$$\therefore \frac{G_1G}{G_2G} = \frac{6}{9} = \frac{2}{3}$$

hence to find the position of G, G_1G_2 must be divided into 5 parts and G_1G is equal to 2 of those parts.

EXAMPLES.—(1) *Fig.* 57 *represents a uniform circular plate with a circular hole punched out of it. It is required to find the centre of gravity of the plate.*

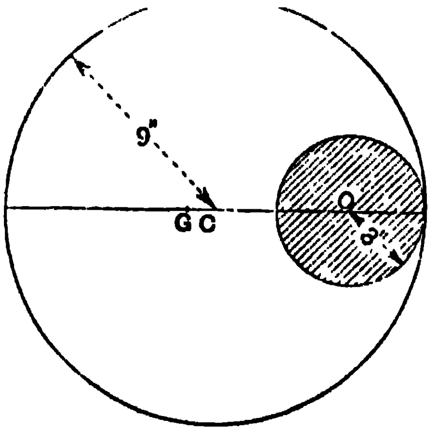


FIG. 57.

If the part removed were replaced, the centre of gravity of the whole plate would be at C and that of the shaded portion at O. Therefore the centre of gravity of the unshaded portion is on OC produced. Let it be at G.

Since the plate is uniform, weights of portions of it are proportional to their areas;

$$\frac{\text{Weight of shaded portion}}{\text{Weight of unshaded portion}} = \frac{\pi \times 3^2}{(\pi \times 9^2) - (\pi \times 3^2)}$$

$$= \frac{\pi \times 9}{\pi \times (81 - 9)} = \frac{1}{8};$$

$$\therefore 1 \times OC = 8 \times GC;$$

$$\therefore GC = \tfrac{1}{8}OC = \tfrac{1}{8} \times 6 \text{ in.} = \tfrac{3}{4} \text{ in.}$$

(2) *A uniform rod* 2 *ft. long is supported at a point* 4 *in. from one end and maintained in a horizontal position by hanging a* 100 *grm. mass* 2 *in. from that end. Find the weight of the rod.*

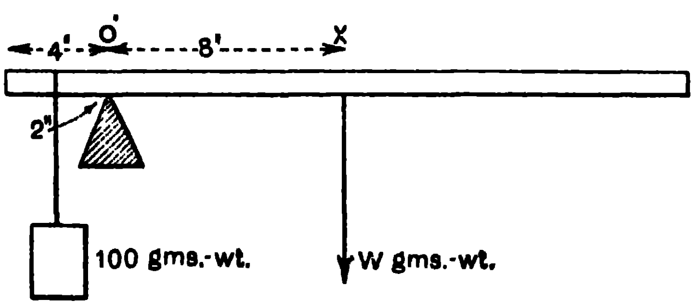


FIG. 58.

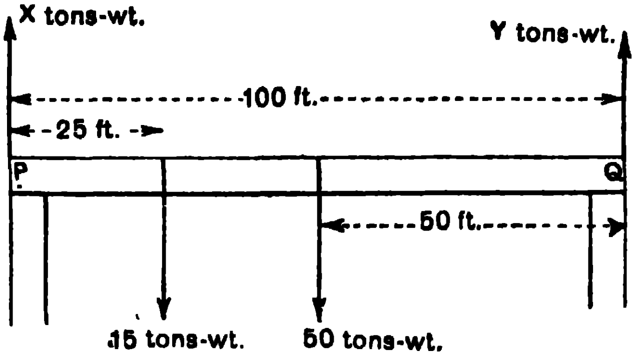


FIG. 59.

Let the weight of the rod be W grm-wt.

This will act vertically through the centre of gravity X which is half-way along the rod, *i.e.* 8 in. from the fulcrum. Taking moments around O,

$$8W = 2 \times 100;$$

$$\therefore \quad W = \frac{2 \times 100}{8} = 25.$$

The weight of the rod is 25 grm.-wt.

Note the principle of this example as a method by which the weight of a rod may be found experimentally.

(3) *A uniform bridge* 100 *ft. long weighs* 50 *tons. A lorry weighing* 15 *tons is* 25 *ft. from one end of it. Find the force exerted on each end support.*

The 50 tons-wt. of the bridge will act at its centre of gravity half-way along it.

The supports will exert vertically upward reactions equal to the forces on them. Let that of the support nearer to the lorry be X tons-wt. and the other Y tons-wt. Taking moments around P.

$$(15 \times 25) + (50 \times 50) - 100Y = 0;$$

$$\therefore 100Y = 2875; \quad \therefore Y = 28{\cdot}75.$$

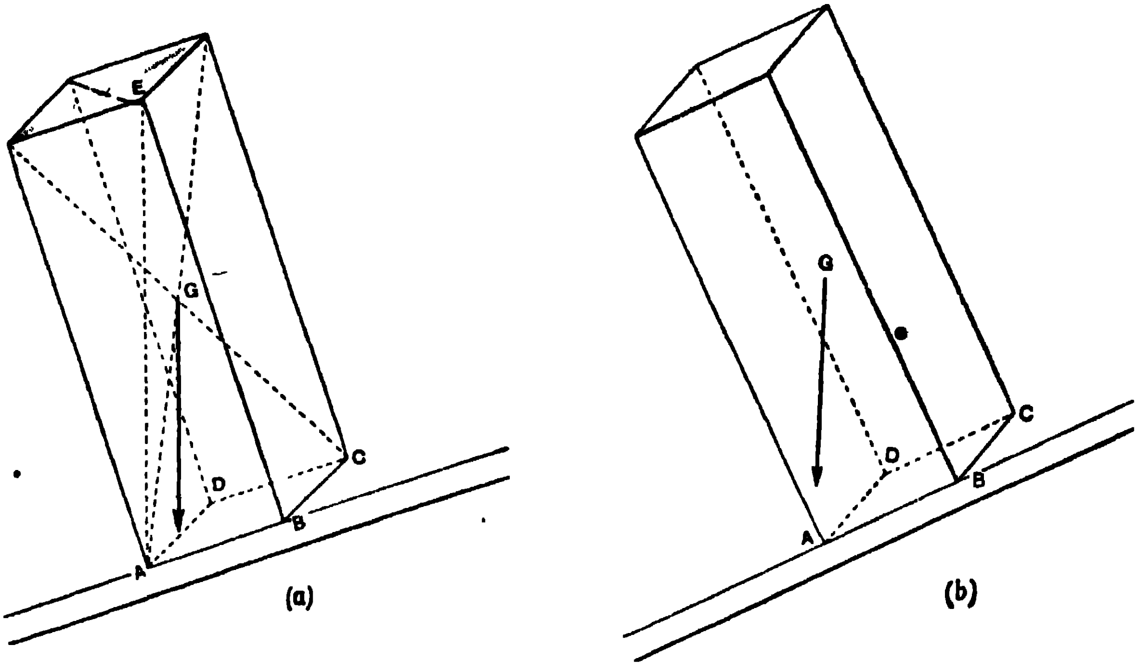


FIG. 60.

Taking moments around Q.

$$(15 \times 75) + (50 \times 50) - 100X = 0;$$

$$\therefore\ 100X = 3625; \quad \therefore\ X = 36{\cdot}25;$$

∴ Force on support nearer the lorry = 36·25 tons-wt.

Force on other support = 28·75 tons-wt.

(Check: X + Y must equal total weight of bridge and lorry.)

Stability of Bodies

Stand a rectangular block of wood on end on a drawing board and gradually tilt the board. When a certain angle is reached the block will topple over. Find the greatest inclination that can be given to the block without its toppling. By testing with a plumb-line show that EA [Fig. 60 (*a*)] is then vertical. This means that the vertical line from the centre of gravity G, along which the weight of the block acts, just falls within the base of support ABCD. If it is tilted a little more, (*b*), the vertical from G falls outside the base of support, and the weight of the block has a moment around AD tending to turn the block still

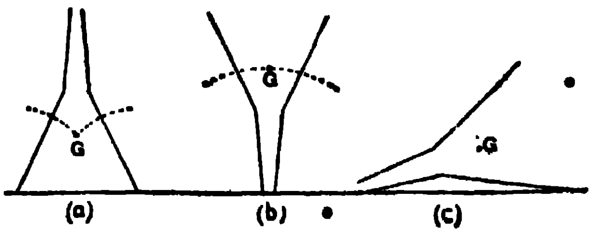


FIG. 61.

further and thus cause toppling. Thus it is seen that, for a body to be in equilibrium, the vertical line from its centre of gravity must fall within the base of support.

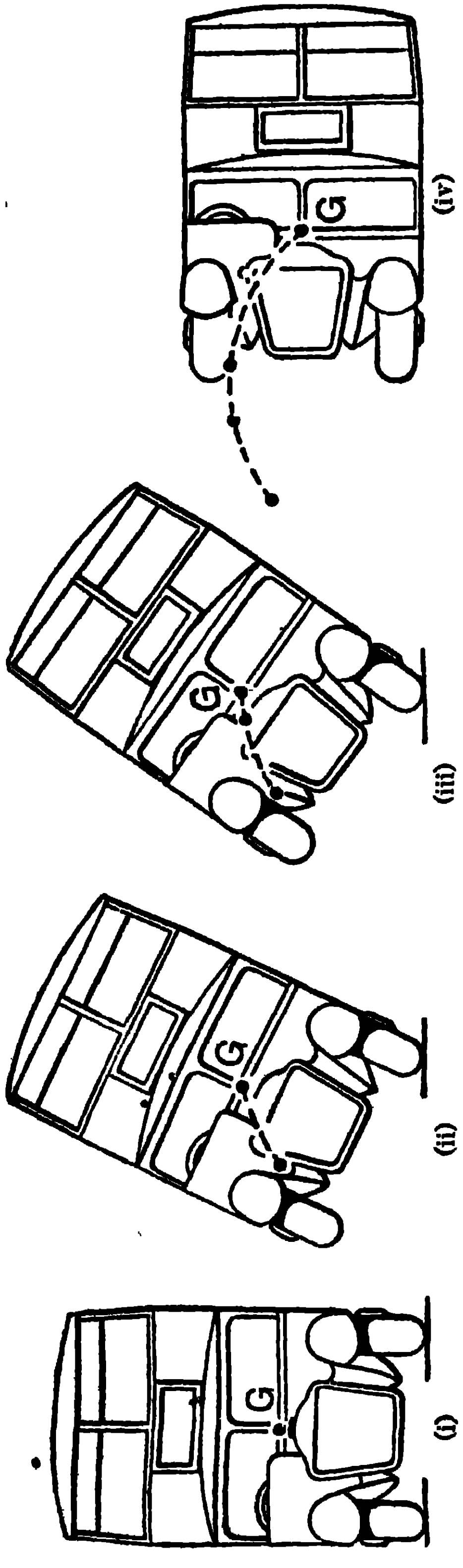
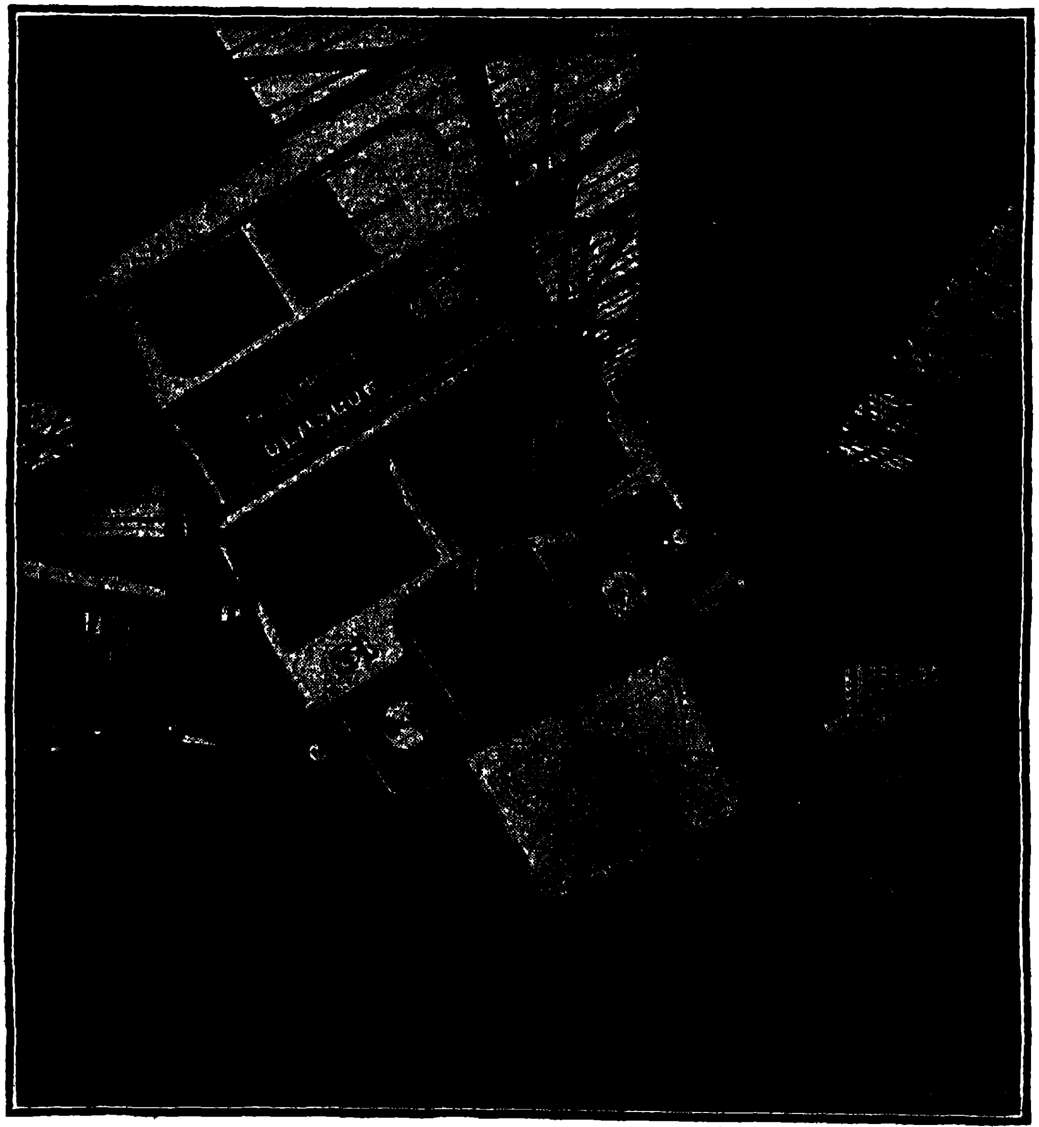


FIG. 62. SUCCESSIVE POSITIONS OF A BUS AS IT OVERTURNS.

Stand a funnel on its broad end on a horizontal table [Fig. 61 (*a*)]. Tilt it slightly and let go. It will return to its former position. Now balance it on its narrow end (*b*). A slight displacement when it is in this position will cause it to topple. Finally, lie it on its side (*c*). If it is turned slightly to one side, it will neither return to its former position nor move further when released, but will stay as it is. In each case the funnel is in equilibrium since the vertical line from the centre of gravity falls within the base of support. In the first case the equilibrium is said to be **stable**, in the second case it is **unstable**, and in the last case it is **neutral**.

The dotted lines in (*a*) and (*b*) indicate the path which will be followed by the centre of gravity as the funnel is tilted. In (*a*) small displacements raise the centre of gravity, while in (*b*) they lower it. These are the conditions for stable and unstable equilibrium. In (*c*) rolling the funnel will not alter the height of the centre of gravity, and so will not produce change in the condition of equilibrium. It should be noted in (*a*) that the centre of gravity will reach its highest position when it is vertically above the edge of the base. If tilted beyond this position the funnel will, of course, fall over.

From the above it is clear that a broad base of support and a low centre of gravity produce stable equilibrium. This has to be aimed at in vehicles such as buses. The picture below shows how a bus is tested to find if it can be tilted through a considerable angle before overturning. In testing, the upper deck is loaded with weights equal to a full upper deck of passengers but no weights are placed inside. Fig. 62 shows the various stages in the overturning of a bus, and illustrates how the lowness of the centre of gravity allows it to turn through a very considerable angle before the centre of gravity reaches its highest position and further turning is dangerous.



Courtesy Chas. H. Roe. Ltd.

STABILITY TEST OF A BUS.

QUESTIONS ON CHAPTER VI

1. Explain the terms *lever, fulcrum, load arm, effort arm.* State the law of levers and explain why, with the aid of a lever, a heavy stone may be moved by a small effort.

2. Explain: (i) Why levers of the third class are less common than those of other classes in mechanical contrivances; (ii) why scissors for cutting cloth may have blades much longer than the handles, but shears for cutting metal have short blades and long handles; (iii) why a nut can be cracked more easily with nut-crackers than with the fingers.

3. Draw a diagram of a system of rods and levers suitable for operating the brake on the back wheel of a bicycle. Indicate dimensions on the levers which would enable a force of 1 lb.-wt. applied to the brake handle to cause a force of 100 lb.-wt. to be exerted on the brake.

4. Assuming a balanced lever, fill in the spaces in the following table:—

EFFORT	EFFORT ARM	LOAD	LOAD ARM
50 grm-wt.	65 cm.	125 grm.-wt.	
20 lb.-wt.		600 lb.-wt.	4 in.
	8 in.	190 grm.-wt.	30 in.
5 grm.-wt.	75 cm.		2·5 cm.

5. Explain what is meant by (*a*) the moment of a force, (*b*) a couple, (*c*) the moment of a couple.

Show that rotation is the only kind of motion that can be caused by a couple.

6. How could you find the approximate weight of a body weighing about 200 grm. if you were supplied with a half-metre scale, pierced by a hole near one end, and a spring balance reading up to 20 grm.?

7. A uniform rod 4 ft. long has weights of 200 grm. and 20 grm. hung from its ends, and then balances when supported at a point 8 in. from the larger weight. What is the weight of the rod?

8. A metal plate of uniform thickness is in the form of a square of side 4 in. with an isosceles triangle of height 3 in. on one side. Where is its centre of gravity?

How could you find the centre of gravity experimentally?

9. Find the centre of gravity of a plate of uniform thickness in the form of a square of 4 in. side with a square of 1 in. side cut out from one corner.

10. Explain each of the following: (*a*) Why you are easily knocked over when standing on one foot. (*b*) Why you place your feet wide apart when receiving a charge from an opponent at football. (*c*) Why standing passengers are sometimes allowed on the lower deck of a bus but not on the upper deck. (*d*) Why the toys consisting of a light paper cone mounted on half of a spherical lead bullet will not lie down.

11. The base of a cylindrical chimney 50 ft. high and 6 ft. in diameter, uniformly built, sinks a little on one side. How many degrees from the vertical may the chimney tilt without being in danger of falling ?

12. Describe and explain the principle of the action of a common steelyard. [L.U.

13. Define the *centre of gravity*. The centre of gravity of a boxwood metre scale, which has some holes bored in it, is at the 49 cm. mark. Being supplied with a 50-grm. weight, and using the scale as a lever, describe how you can determine its weight.

Give an example in illustration, if this weight is found to be 100 grm. [L.U.

14. If you were provided with a pound weight, a 12-in. ruler, and some string, how would you make use of these to find the weight of an ordinary garden spade ?

Give a numerical example to illustrate your answer, taking 5 lb. for the weight of the spade and 3 ft. for its length.

Explain *fully* the method you adopt. [L.U.

15. Distinguish between the three forms of equilibrium: stable, unstable, and neutral, giving one example of each.

Draw diagrams to show the forces acting upon (*a*) a cyclist rounding a bend, (*b*) a glider in flight. State what each force shown represents. [L.U.

16. Define *the moment of a force about a point.*

A uniform bar AB, 4 ft. long and 10 lb. in weight, is supported in a horizontal position by the hooks from two spring balances hanging vertically from 2 nails. If the points of support are 8 in. and 6 in. respectively from the ends A and B of the bar, find the readings of the spring balances when a weight of 2 lb. is hung from A. What weight hung from this end will make the reading of the further balance zero ? [J.M.B.

17. A uniform beam 18 ft. long weighing 2 tons and supported at its ends carries a weight of 7·5 tons at a distance of 6 ft. from one end. Find the reactions of the supports. Make a diagram showing clearly all the forces acting on the beam. [J.M.B.

CHAPTER VII

WORK, ENERGY, POWER

Work

When we see men carrying bricks up ladders to builders we realise that they are doing work. If we wish to compare the work done by one man with that done by another we might think it sufficient to count the bricks each has carried. A little thought will show that this may not give a true comparison. If one man has carried 100 bricks to a height of 40 ft. and the other has also carried 100 bricks but only to a height of 20 ft., the former has done more work than the latter.

Evidently there are two factors which must be considered when estimating an amount of work done. When climbing with a load of bricks a man would be exerting a certain force to overcome the weight of the bricks and his own weight. The magnitude of this force has something to do with the work done, but also the distance through which the bricks are raised must be taken into account.

Let us suppose that one unit of work is done when a mass of 1 lb. is raised a distance of 1 ft. If it is raised another foot, another unit of work will be done, and so on, one unit of work being done for each foot raised. Thus, when a mass of 1 lb. has been raised a height of 5 ft., 5 units of work have been done. Note that the force used will be 1 lb.-wt. whether the distance raised is one foot or five. If another mass of 1 lb. is raised 5 ft. another 5 units of work will be done, so in raising a mass of 2 lb., which would require a force of 2 lb.-wt. to be used, to a height of 5 ft., 10 units of work will be done. Similarly raising a mass of 6 lb., requiring a force of 6 lb.-wt., to a height of 8 ft. would mean that 48 units of work were done. This leads to the conclusion that

Work done = Force used × Distance moved by point of application.

The unit of work evidently depends on the units in which the force and the distance are measured.

Work done when a force of		acts through a distance of		is	
	1 lb.-wt.		1 ft.		1 ft.-lb.
	1 pndl.		1 ft.		1 ft.-pndl.
	1 grm.-wt.		1 cm.		1 grm.-cm.
	1 dyne		1 cm.		1 erg.

British engineers frequently use the foot-pound as their unit of work but the erg is the unit mostly used in scientific work. As the erg is a very small unit, a larger unit equal to 10^7 ergs is often used. This unit is called a **joule**.

Since 1 lb.-wt. = 32 pndl. and 1 grm.-wt. = 980 dynes, it follows that 1 ft.-lb. = 32 ft.-pndl., and 1 grm.-cm. = 980 ergs.

In the above cases the weight of the body has been considered because the force used was equal to the weight, but it is the actual force used and not necessarily the weight of the body which must be taken into account in calculating work done.

EXAMPLES.—(1) *Find the work done in raising* 1 *cwt. of coal to the surface from the bottom of a mine* 300 *ft. deep. Answer in both gravitational and absolute units.*

(*a*) *In gravitational units,*

$$\text{Force used} = 112 \text{ lb.-wt.};$$

$$\therefore \text{ Work done} = 112 \times 300 \text{ ft.-lb.} = 33{,}600 \text{ ft. lb.}$$

(*b*) *In absolute units,*

$$\text{Force used} = 112 \times 32 \text{ pndl.};$$

$$\therefore \text{ Work done} = 112 \times 32 \times 300 \text{ ft.-pndl.}$$

$$= 1{,}075{,}200 \text{ ft.-pndl.}$$

(2) *A mass of* 1000 *grm. is given an acceleration of* 40 *cm.* (*per sec.*)^2. *What work has been done when it has moved* 250 *cm.?*

In absolute units,

$$\text{Force} = \text{mass} \times \text{acceleration} = 1000 \times 40 \text{ dynes};$$

$$\therefore \text{ Work done} = 1000 \times 40 \times 250 \text{ ergs} = 10{,}000{,}000 \text{ ergs}.$$

$$\text{and Work done} = \frac{10{,}000{,}000}{980} \text{ grm.-cm.}$$

$$= 10{,}204 \text{ grm.-cm. (to nearest unit).}$$

Energy ✓

A person is generally described as energetic if he has the capacity for performing a large amount of work. In mechanics the term **energy** denotes *that which gives ability to do work.* Thus, when the spring of a toy engine is wound up, it can perform a certain amount of work in driving the engine a certain distance along the rails. It is said to possess an amount of energy numerically equal to the amount of work it can do in unwinding. Since energy is measured in this way, the units of energy are given the same names as those of work.

Energy may be stored in a body in a number of different ways. The spring just mentioned possesses energy when it is in a condition of being wound up. The steam in the boiler of an engine possesses energy, which enables it to do work when admitted to the cylinders, because it is in a condition of high pressure. In a grandfather clock the energy to perform the work of moving the wheels, etc., is provided by a weight which has to be raised from its lowest possible position in order to make the clock go, *i.e.* the weight possesses energy when in certain positions. Energy due to the mechanical condition or to the position of a body is called **potential energy.** The potential energy of a raised body is easily calculated. If it is allowed to fall the force acting will be its weight, and the distance through which it can act is its height above the ground. Hence the work that can be done by it equals its weight × its height, and this measures its potential energy.

On the bed of a swiftly running stream sand and stones may often be observed being rolled along by the water. Evidently the water must possess energy to enable it to do the work of moving these bodies, and, as it would not move them if it were still, it possesses this energy because of its motion. The work of driving the machinery of a wind mill is performed by particles of air hitting the sails. Here, too, it is clear that the air possesses energy because it is in motion. In modern turbine engines steam rushes into the turbines, and by hitting against the blades causes them to rotate. Again we have an example of energy possessed because of the motion of the particles. Energy due to motion is called **kinetic energy.**

The kinetic energy of a moving body may be calculated as follows:—

Let a body of mass 100 grm. have a velocity of 50 cm. per sec., and let it be brought to rest with uniform loss of velocity in 5 sec.

Acceleration while being brought to rest $= -\frac{50}{5}$ cm. (per sec.)^2;

$$\therefore \text{ Force applied} = \frac{100 \times 50}{5} \text{ dynes.}$$

Average velocity while being brought to rest $= \frac{50}{2}$ cm. per sec.;

$$\therefore \text{ Distance moved} = \frac{50 \times 5}{2} \text{ cm.;}$$

∴ Work done in losing velocity

$$= \frac{100 \times 50}{5} \times \frac{50 \times 5}{2} \text{ ergs} = \tfrac{1}{2}(100 \times 50^2) \text{ ergs,}$$

that is, the kinetic energy of the moving body is given by the expression:—

Kinetic energy = $\frac{1}{2}$ mass × square of velocity.

This will be found to be the case if the symbols m, v, and t are used for the mass, velocity, and time, and the working carried out as above, showing that the result is generally applicable.

Note carefully that weight × height gives the potential energy of a raised body in *gravitational* units, but $\frac{1}{2}$ mass × velocity^2 gives the kinetic energy of a moving body in *absolute* units.

EXAMPLE.—*What will be the kinetic energy of a rock weighing* 50 *lb. after it has been falling for* 5 *sec.?*

$$\text{Acceleration} = 32 \text{ ft. (per sec.)}^2;$$
$$\therefore \text{ Velocity after 5 sec.} = 32 \times 5 \text{ ft. per sec.};$$
$$\therefore \text{ Kinetic energy} = \tfrac{1}{2} \times 50 \times (32 \times 5)^2 \text{ ft.-pndl.},$$
$$\textit{i.e.} \text{ Kinetic energy} = 640{,}000 \text{ ft.-pndl.}$$

Conservation of Energy

In mechanics potential energy and kinetic energy are the two forms most often dealt with, but it will be shown later that heat and light are also forms of energy. Energy may also be stored in electrical and chemical forms. One form of energy may be transformed into another.

When a stone is projected upwards it possesses kinetic energy. As it rises it loses velocity and so loses kinetic energy, but owing to its increasing height, it gains potential energy. At its highest point it is stationary for a moment during which it has no kinetic energy but considerable potential energy. As it falls again it loses this potential energy, but as it gains velocity it gains kinetic energy.

A whole series of energy transformations takes place in a steam engine. The fuel has much energy stored in a chemical form. During its burning in the furnace this chemical energy is liberated and transformed to heat. The heat causes the production of steam in the boiler and is transformed into potential energy due to the compressed state of the steam. The steam expands into the cylinders, losing potential energy owing to its loss of pressure, but setting the pistons, etc., in motion, thus imparting kinetic energy to them.

The law of conservation of *energy* states that during transformation of energy from one form to another, the total amount of energy is unchanged, *i.e.* the amount of the new form which appears is equal to the amount of the old form which has disappeared. This is readily verified in the case of a falling body. In Example (1), page 73, it was shown that a mass of 50 lb. after falling for 5 sec. would have gained 640,000 ft.-pndl. of kinetic energy.

$$\text{Its average velocity during the five sec.} = \tfrac{160}{2} \text{ ft. per sec.};$$

$$\therefore \text{ Distance fallen} = \frac{160 \times 5}{2} \text{ ft.} = 400 \text{ ft.};$$

$$\therefore \text{ Loss of potential energy} = 50 \times 400 \text{ ft.-lb.}$$

$$= 50 \times 400 \times 32 \text{ ft.-pndl.}$$

$$= 640{,}000 \text{ ft.-pndl.},$$

so its gain of kinetic energy is equal to its loss of potential energy.

It will be shown in later chapters that similar relationships exist for transformations between mechanical forms of energy and heat and electrical energy.

Power

We are familiar with such terms as "a twelve horse-power motor car" or "a 300 horse-power locomotive." If a high-powered steam crane and a low-powered one raise equal loads it will be found that the former raises its load more rapidly than the latter, *i.e.* the former is capable of doing work at a faster rate than the latter. When we speak of **power** in mechanics we refer to the rate at which work is done or the rate of expenditure of energy, and the power of a machine is measured by the number of units of work it can do in one unit of time.

The power of a small motor, such as a "Meccano" clockwork or electric motor, may be determined as follows. Tie a light string to a drum fixed to the driving axle. Pass the string over a pulley and attach a weight pan to its free end. Load the pan until the motor can only just raise it steadily. Place a metre scale upright by the side of the pan and measure the distance by which the motor can raise the pan in a fixed time, say 10 sec.

Weigh the pan and its contents. Calculate the power as below.

$$\text{Mass raised} = 255 \text{ grm.}$$

$$\text{Height raised} = 97 \text{ cm.}$$

$$\text{Time} = 10 \text{ sec.};$$

$$\therefore \text{ Work done} = 255 \times 97 \text{ grm.-cm.};$$

$$\text{Power} = \frac{255 \times 97}{10} = 2473{\cdot}5 \text{ grm.-cm. per sec.}$$

The power developed by a boy's leg muscles may be approximately determined by timing him while he runs as fast as possible up a flight of stairs. The height of the flight is measured and the boy weighed and the calculation made as above. If weight and height are taken in pounds-weight and feet the power will be given in foot-pounds per second.

When steam engines were introduced they were largely employed for doing work formerly performed by horses, and it was important to know how the rate of working of a given engine would compare with that of a horse. After experiments, James Watt came to the conclusion that a generous estimate of the rate of working of a good cart horse was 550 ft.-lb. per sec. Hence an engine is said to be of **one horse-power** when it can do work at that rate. When electric power is used, it is more convenient to measure rate of working in units derived from the metric system. Such a unit is the **watt**, which is a rate of working of 10^7 ergs per sec., *i.e.* 1 joule per sec. One horse-power is equal to 746 watts. As the watt is a rather small unit, the **Kilowatt**, which equals 1000 watts, is often used.

EXAMPLE.—*A steam crane raises a load of* 10 *tons to a height of* 30 *ft. in half a minute. At what power is it working? Give the answer in horse-power and in Kilowatts.*

$$\text{Force used} = 10 \times 2240 \text{ lb.-wt.}$$

$$\text{Work done} = 10 \times 2240 \times 30 \text{ ft.-lb.}$$

$$\text{Power} = \frac{10 \times 2240 \times 30}{30} \text{ ft.-lb. per sec.}$$

$$= \frac{10 \times 2240}{550} = 40{\cdot}7 \text{ h.p.}$$

$$40{\cdot}7 \text{ h.p.} = \frac{40{\cdot}7 \times 746}{1000} = 30{\cdot}4 \text{ kW.}$$

Engines are often rated as being of a certain **brake horse-power.** This refers to the method by which their horse-power is measured. The engine is made to rotate a cylinder around which a rope is wound, as shown in Fig. 63. The loading of the rope with the weight W makes it act as a brake retarding the rotation of the cylinder. The force exerted by the spring balance tends to pull the cylinder round in the direction of its rotation, and thus reduces the effect of W. W is increased until the engine can just rotate the cylinder steadily giving

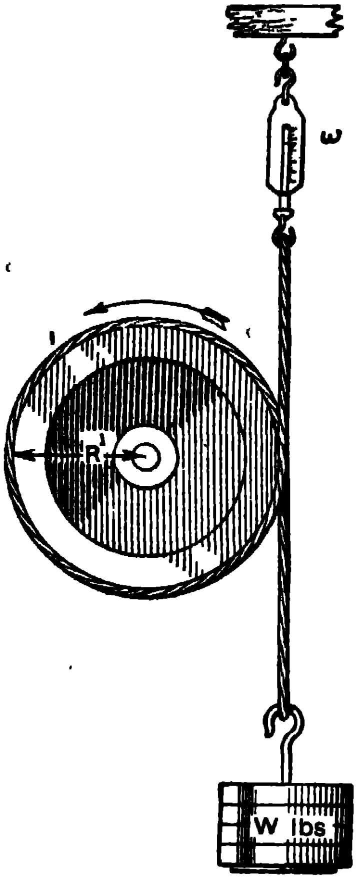


FIG. 63.

a steady reading on the spring balance. The number of rotations made by the cylinder in a given time is then counted.

Suppose $W = 133$ lb. and $w = 1{\cdot}75$ lb.

Then the force exerted on the drum by the rope = 131·25 lb.-wt. Let the radius of the drum be 6 in.

Then its circumference is $2 \times \frac{22}{7} \times 6$ in., and this is the distance through which the force acts during each revolution of the drum.

If the drum revolves 200 times per minute,

Work done per minute

$$= 200 \times 2 \times \tfrac{22}{7} \times 6 \times 131{\cdot}25 \text{ ft. lb.};$$

∴ Rate of working

$$= \frac{200 \times 2 \times \frac{22}{7} \times 6 \times 131{\cdot}25}{60 \times 550}$$

$$= 30 \text{ horse-power.}$$

You could determine your own horse-power when turning a handle by fitting up a similar arrangement with a handle attached to the cylinder.

QUESTIONS ON CHAPTER VII

1. Distinguish between *force*, *work*, *energy*, and *power*. State the absolute units in both the British and Metric systems in which each is measured.

What is meant by *a horse-power*, *a watt?*

2. Explain the terms *potential energy* and *kinetic energy*, giving two examples of each.

What is meant by the *law of conservation of energy?* Illustrate the law by considering a stone of mass 2 lb. projected vertically upwards with the initial velocity of 128 ft. per sec., dealing particularly with its potential and kinetic energies at the beginning of its flight and at the ends of successive periods of 2 sec. each until it returns to the level from which it was projected.

3. A cage containing a truck of coal, the total weight being 15 cwt., is raised from the bottom of a mine shaft 300 ft. deep to the surface in 1 min. Calculate (*a*) the work done, (*b*) the average horse-power at which the hauling engine worked.

4. A cistern 60 ft. deep and 8 ft. in diameter contains water to a depth of 30 ft. A pumping engine empties it in 1 hr. Find the work done and the horse-power at which the engine worked. (NOTE: *All the water has not to be raised the same height but the work done may be calculated from the average height the water has to be raised.*)

5. To maintain a speed of 30 ml. per hr. in a train the engine has to exert a pull of 1500 lb.-wt. How much work does the engine do in 1 min., and what is the power in Kilowatts at which it is working ?

6. A car weighing 1000 Kilog. travelling along a level road is accelerated for 2 min. at the rate of 15 cm. (per sec.)^2. Find its gain of kinetic energy during those two minutes.

7. How much work is done against gravity by a man weighing 11 st. in climbing a mountain 6000 ft. high ? If his actual climbing time is 3 hr., what is the approximate horse-power he develops while climbing ?

8. What is meant by the *transformation of energy?* Illustrate your answer by reference to *either* (*a*) a steam engine in which coal is used as fuel, *or* (*b*) a petrol engine. [L.U.

9. Explain the terms *momentum*, *potential energy*, and *kinetic energy*. Illustrate your answer by reference to the action of a pile driver. [L.U.

10. What do you understand by *force*, *work*, and *energy?*

Enumerate as many forms of energy as you can, and show how it is possible for the energy of an electric power station to be derived from solar-energy. [L.U.

11. Define *kinetic energy* and *potential energy*. What is the source of energy in (*a*) a water mill, (*b*) a wind mill ?

A car has a velocity of 30 ml. per hr. when it begins to ascend a hill without help from its engine. To what height will it rise before coming to rest, if a third of its energy is lost in friction ? [J.M.B.

12. Explain the meaning of *acceleration* of a body.

A motor car starts from rest to move down a hill without its engine being used. If the time for each 100 yd. is measured for about 600 yd., explain how you would use the observations to test whether the acceleration was uniform.

What changes occur in the energy of the car during its motion ? [J.M.B.

CHAPTER VIII

MACHINES, FRICTION

The age in which we live is frequently called the machine age, and there are few forms of work to-day in which machinery is not used. Although many machines are exceedingly intricate and there is an immense variety of them, they all consist of combinations of a relatively few simple mechanisms, some of the more common of which will be dealt with in this chapter.

Mechanical Advantage

A machine may be described as any arrangement which enables a force applied to it at one point to overcome a resistance (force) at another point. As in the case of the lever, the force applied may be described as the effort, and the force overcome as the load. Machines are usually designed so that a small effort may overcome a large load, and the ratio $\frac{\textbf{Load}}{\textbf{Effort}}$ **is called the mechanical advantage** of the machine. In some cases a machine may be useful although the load is less than the effort because it enables forces to be applied conveniently where they are wanted. In that case the mechanical advantage is less than 1 and might be described as mechanical disadvantage.

Efficiency

In the working of a machine the point at which the effort is applied will move, and the point at which the load is overcome will also move. Thus, work is done on the machine by the effort and by the machine on the load. While the load may be many times as great as the effort, the work done by the machine cannot exceed that done on the machine, for, by the law of conservation of energy, the energy gained by the load cannot exceed that lost by the effort. In a perfect machine the work done by the machine would be just equal to that done on it, and this is true of any machine if the work done against friction and gravitation in moving its own parts is included in the work done by it. The *useful* work done, *i.e.* the work done to the load, is always less than that done by the effort, since some of the latter is used in moving parts of the machine. The ratio

$$\frac{\text{Useful work done by the machine}}{\text{Work done on the machine}}$$

measures the **efficiency** of the machine. The above fraction multiplied by 100 gives the percentage efficiency. A perfect machine would have an efficiency of 1. Actual machines have efficiencies less than 1, in many cases much less.

Velocity Ratio

If a machine has very light, freely moving parts, its efficiency may be very nearly 1. In that case, approximately,

$$\text{Work done by machine} = \text{Work done on machine};$$

$$\therefore \text{Load} \times \text{Distance moved} = \text{Effort} \times \text{Distance moved};$$

$$\therefore \frac{\text{Load}}{\text{Effort}} = \frac{\text{Distance effort moves}}{\text{Distance load moves}}.$$

The second ratio in the above equation is called the **velocity ratio** of the machine, because the two distances are moved in the same interval of time so that they are proportional to the velocities of effort and load.

The above equation shows that, for a perfect machine,

$$\text{mechanical advantage} = \text{velocity ratio}.$$

From this relation the maximum mechanical advantage a machine can possibly have may often be calculated from the dimensions of its parts.

If the machine is not perfect,

$$\begin{aligned}\text{Efficiency} &= \frac{\text{Work done on load}}{\text{Work done by effort}}\\ &= \frac{\text{Load} \times \text{Distance load moves}}{\text{Effort} \times \text{Distance effort moves}}\\ &= \text{Mechanical advantage} \times \frac{1}{\text{Velocity ratio}}\\ &= \frac{\text{Mechanical advantage}}{\text{Velocity ratio}}.\end{aligned}$$

This enables the efficiency to be calculated if the mechanical advantage and the velocity ratio are determined.

As a rule, the efficiency of a machine varies with the load. If the load is small compared with the forces needed to move the parts of the machine, the useful work done is only a small part of the total work, and the efficiency is low. Thus it may be anticipated that the efficiency of a machine will increase as its load increases. To a certain extent this is true, but there is often a limit above which, owing to frictional causes, further increase of load causes a decrease in efficiency. It is clearly important in a given case to determine loads at which the efficiency of a machine approaches its maximum so that it may be worked in the most economical way.

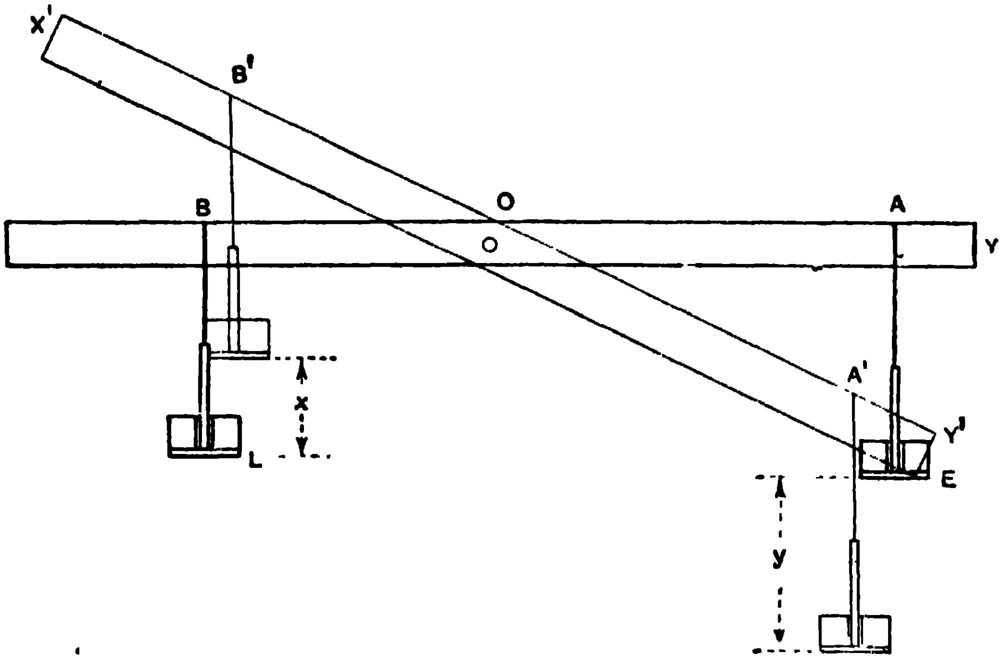


Fig. 64.

The **lever**, one of the simplest machines, has already been partly dealt with in Chapter VI. It was shown that, for a light lever,

$$\text{Load} \times \text{Load arm} = \text{effort} \times \text{Effort arm};$$

$$\therefore \frac{\text{Load}}{\text{Effort}} = \frac{\text{Effort arm}}{\text{Load arm}},$$

$$\text{i.e. Mechanical advantage} = \frac{\text{Effort arm}}{\text{Load arm}}$$

Balance masses E and L on a lever XY. Tilt it to position X′Y′ measuring the vertical heights x and y through which L and E move. It will be found that

$$\frac{y}{x}=\frac{\mathrm{OA}}{\mathrm{OB}},\ i.e.\ \frac{\text{Distance effort moves}}{\text{Distance load moves}} : \frac{\text{Effort arm}}{\text{Load arm}},$$

$$i.e.\ \text{Velocity ratio} = \text{Mech. advantage.}$$

$$\text{Hence efficiency} = \frac{\text{Mech. advantage}}{\text{Velocity ratio}} = 1.$$

From this it appears that a lever is a perfect machine. This is approximately the case if it is pivoted at its centre of gravity on an almost frictionless fulcrum.

Experiments should be made with a fairly heavy lever not pivoted at its centre of gravity. Using various loads and efforts, in each case adjust their positions until the effort just raises the load. Measure the velocity ratio as above and also measure the effort and load to find the mechanical advantage. Try to explain any discrepancies between your results and the theoretical result above. Note particularly that, if the centre of gravity is on the effort side of the fulcrum, the weight of the lever itself helps to overcome the resistance of the load, so there is an advantage in using a heavy lever in such cases.

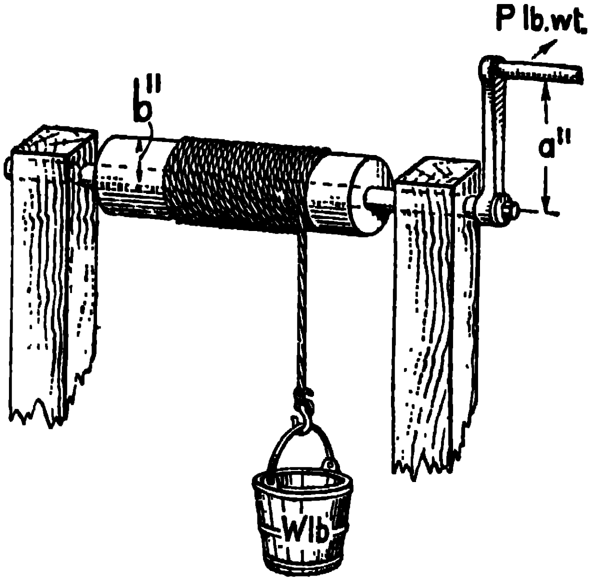


FIG. 65. WINDLASS.

The Windlass and Wheel and Axle

These are machines which utilise the lever principle. From Fig. 65 it is clear that the windlass shown is equivalent to a lever with effort and load arms of lengths a and b respectively. The rope will make one complete turn round the barrel, and therefore the load will be raised a distance equal to the circumference of the barrel, while the handle is making one complete revolution.

Hence the velocity ratio

$$= \frac{\text{Circumference of circle described by handle}}{\text{Circumference of barrel}} = \frac{2\pi a}{2\pi b} = \frac{a}{b};$$

$$\therefore \text{ The theoretical mechanical advantage} = \frac{a}{b},$$

and the length of the handle must be greater than the radius of the barrel to give real mechanical advantage.

Experiments should be made with a windlass to determine its actual mechanical advantage by arranging the crank horizontally and finding the mass which must be hung from it to raise a given load. Owing to friction it will probably be less than the theoretical value. The efficiency at various loads may be calculated from

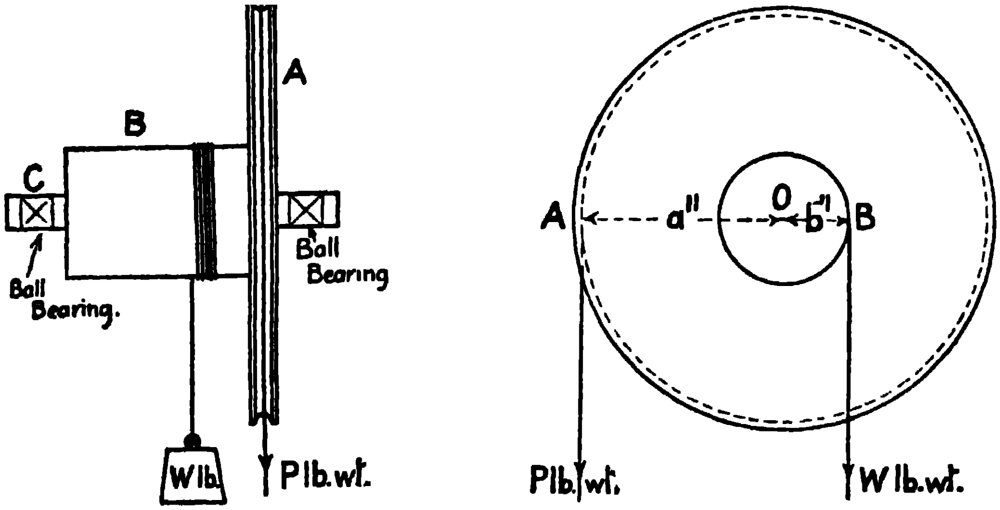


FIG. 66. WHEEL AND AXLE.

$$\text{Efficiency} = \frac{\text{Mech. advantage}}{\text{Velocity ratio}} = \frac{\text{Mech. advantage}}{a/b}$$

$$= \text{Mech. advantage} \times \frac{b}{a}.$$

The variation of efficiency with load can be shown by plotting efficiency against load in a graph.

In the **wheel and axle** shown in Fig. 66, ropes are wound in opposite directions around the wheel A and the axle B, so that a pull which unwinds rope from A will wind more rope on B and so raise the load. again we have the equivalent of a lever with effort arm a and load arm *b*, so that, as with the windlass, the theoretical mechanical advantage is a/b. Consideration of the lengths of rope wound and unwound at the same time will show that the velocity ratio is a/b. Actual mechanical advantage and efficiency may be investigated as in the case of the windlass. Note the mounting on ball-bearings to reduce friction.

EXAMPLES.—(1) *A windlass has a crank* 1 *ft. long and a barrel of* 6 *in. diameter. Assuming an* 80 *per cent. efficiency, calculate the effort necessary to raise a load of* 60 *lb.*

Let the required effort be x lb.-wt.

$$\text{The mech. advantage} = \frac{60}{x}.$$

$$\text{The velocity ratio} = \tfrac{12}{3} = 4.$$

$$\text{Percentage efficiency} = \frac{\text{Mech. adv.}}{\text{Velocity ratio}} \times 100;$$

$$\therefore\ 80 = \frac{60}{x} \times \frac{100}{4};\quad \therefore\ 80x = 60 \times 25;$$

$$\therefore\ x = \frac{60 \times 25}{80} = 18{\cdot}75.$$

Effort required = 18·75 lb.-wt.

(2) *In a wheel and axle mechanism the wheel has a diameter of* 3 *ft. and the axle a diameter of* 8 *in. A load of* 1 *cwt. can be raised by an effort of* 32 *lb.-wt. What is the percentage efficiency for a load of* 1 *cwt., and how much "waste" work is done in raising* 1 *cwt. a height of* 20 *ft.?*

$$\text{Mech. advantage} = \tfrac{112}{32} = 3{\cdot}5$$

$$\text{Velocity ratio} = \tfrac{36}{8} = 4{\cdot}5;$$

$$\therefore\ \text{Percentage efficiency} = \frac{3{\cdot}5 \times 100}{4{\cdot}5} = 77{\cdot}8 \text{ per cent.};$$

$$\therefore\ \text{Waste work} = 22{\cdot}2 \text{ per cent. of work done by effort.}$$

To raise load 20 ft., effort must move $20 \times 4{\cdot}5 = 90$ ft.;

$$\therefore\ \text{Work done by effort} = 32 \times 90 \text{ ft.-lb.};$$

$$\therefore\ \text{Waste work} = \frac{32 \times 90 \times 22{\cdot}2}{100} = 639{\cdot}4 \text{ ft.-lb.}$$

Pulleys

(*a*) Single Fixed Pulley

Fig. 67 (*a*) shows the simplest way of using a pulley. Since rope simply passes over the pulley from one side to the other, the effort P and the load W will move equal distances and the velocity ratio will be 1. This means that a perfectly balanced frictionless pulley would have a mechanical advantage of 1. Testing with various loads will show that mechanical advantage is slightly less than 1, but in spite of slight mechanical disadvantage the arrangement is often useful as it enables the operator to use the weight of his body for raising the load.

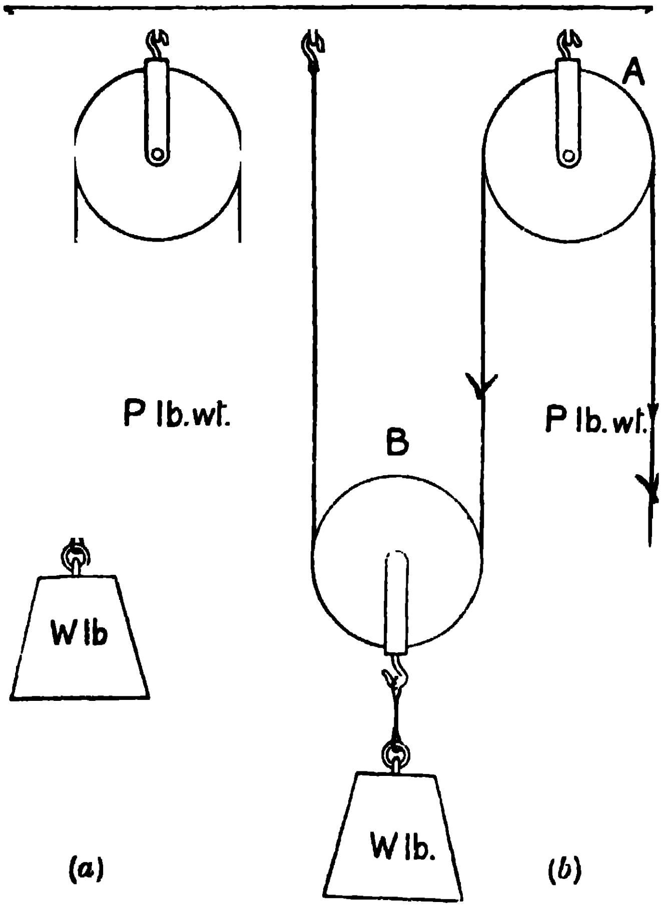


Fig. 67.

(*b*) One Fixed and One Movable Pulley.—This arrangement is shown in Fig. 67 (*b*). If the load, and therefore pulley B, rises 1 ft., each of the two sides of the loop supporting B is shortened by 1 ft., and therefore 2 ft. of rope passes over A and the effort moves 2 ft. Hence the velocity ratio is 2. The mechanical efficiency, however, particularly for small bodies, is considerably less than 2 as the effort has to raise the movable pulley as well as the load.

The following table was obtained for such a system. In each case efficiency has been calculated from

$$\frac{\text{Mechanical advantage}}{\text{Velocity ratio}},$$

and the velocity ratio has been taken as 2.

Load	Effort	Mech. Advantage	Efficiency
0 grm.-wt.	16 grm.-wt.	0	0
20 ,,	26 ,,	·769	·385
40 ,,	36 ,,	1·111	·556
60 ,,	46 ,,	1·304	·652
80 ,,	56 ,,	1·429	·715
110 ,,	72 ,,	1·528	·764
140 ,,	87 ,,	1·609	·805
170 ,,	102 ,,	1·677	·838

Loads have been plotted against corresponding efficiencies in Fig. 68, which shows clearly the increase in efficiency as the load increases, and that the efficiency tends to become almost constant with high loads.

(c) A Block and Tackle is shown in Fig. 69. If the lower block rises 1 ft., each of the 4 lengths of rope between the blocks is shortened by 1 ft., and so 4 ft. pass into the free length to which P is applied and the velocity

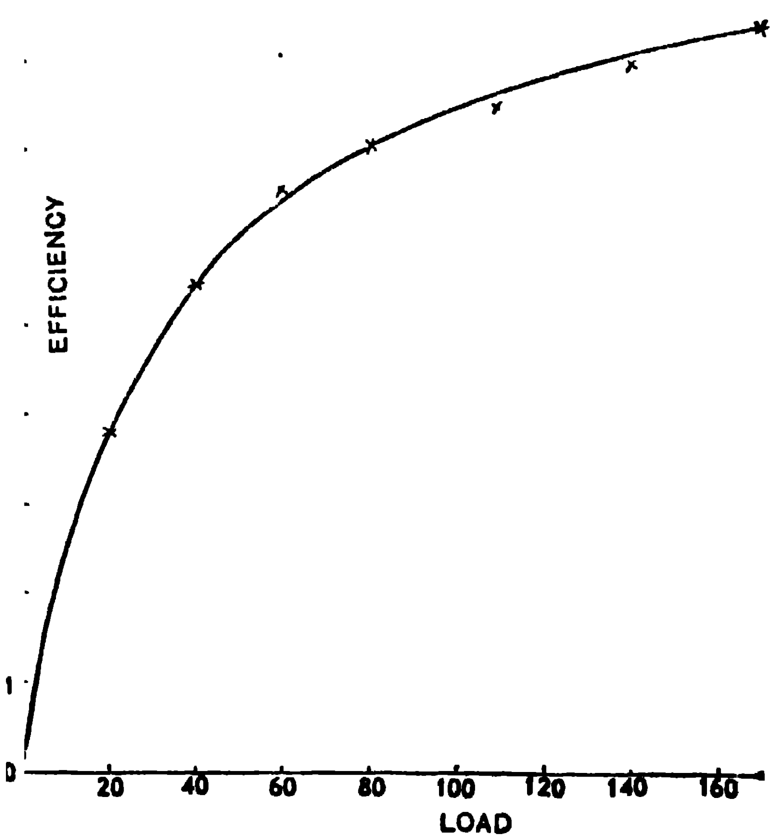


Fig. 68.

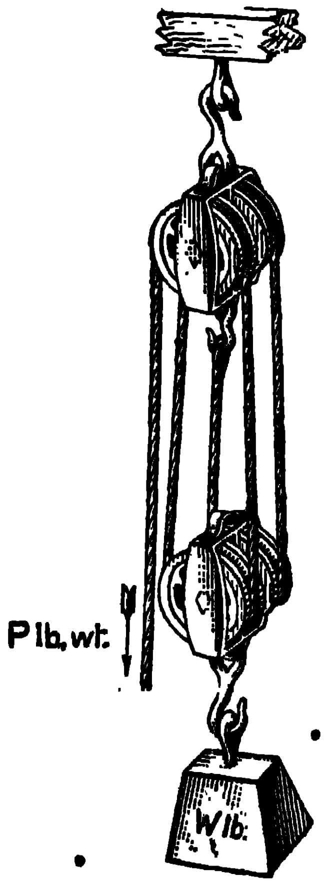


Fig. 69.

ratio is 4. As in the last case, the weight of the lower block reduces the mechanical advantage and efficiency. This should be investigated as in the last paragraph and similar results will be obtained.

The number of pulleys in each block may be more than two, but the number of lengths of rope connecting the two blocks will always give the velocity ratio.

(*d*) WESTON'S DIFFERENTIAL PULLEY (Fig. 70*a*) differs from the block and tackle in having the two pulleys in the upper block in one piece so that they turn together, one having a larger radius than the other. An endless chain is wound round the pulleys as shown in Fig. 70 (*b*). Projections in the grooves of the pulleys interlock with the chain, preventing slipping. This introduces a large amount of friction, but the friction has a use in this particular machine.

Suppose the chain to be pulled in the direction indicated until the upper pulleys have made a complete revolution. A length of chain equal to the circumference of A, *i.e.* $2\pi R$ in., will have passed over A and the effort P will have moved that distance. This length of chain will have been taken from the loop supporting C, but a length $2\pi r$ in. will have been returned to the loop over B. Therefore the part of the

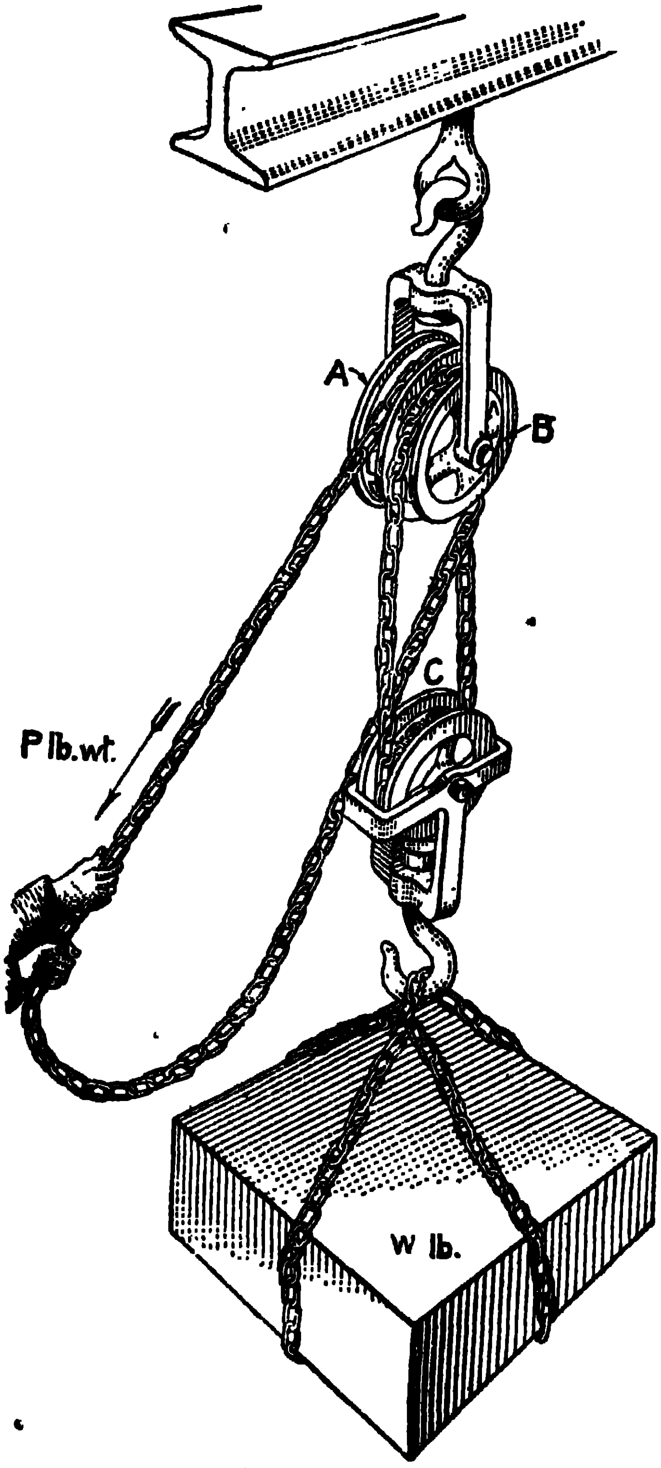


FIG. 70 (*a*).

chain forming the loop will have been shortened by $(2\pi R - 2\pi r)$ in. $= 2\pi(R - r)$ in., and C will have risen half that distance, *i.e.* $\pi(R - r)$ in. Therefore the velocity ratio is $\frac{2\pi R}{\pi(R - r)} = \frac{2R}{R - r}$. If the difference between R and r is small, the above fraction will have a high value, *e.g.* if $R = 10$ and $r = 9\frac{3}{4}$ the velocity ratio will be $\frac{20}{\frac{1}{4}} = 80$.

Owing to the great friction between the wheels and the chain, the mechanical advantage never approaches the velocity ratio in this machine, in fact its efficiency is always less than 50 per cent., but this means that the frictional resistance is always greater than the load which will not therefore run down again if the chain is released. Even with its small efficiency a big mechanical advantage can be obtained and very heavy loads raised with little effort. It will be noted that the high velocity ratio shows that the load rises very slowly compared with the rate at which the effort moves. The relation between mechanical advantage and velocity ratio shows that this is always the case when a big mechanical advantage is obtained. Engineers express this fact in the statement, "What is gained in force is lost in speed."

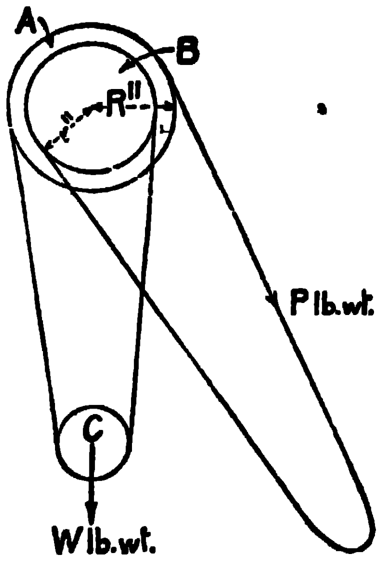


FIG. 70 (*b*).

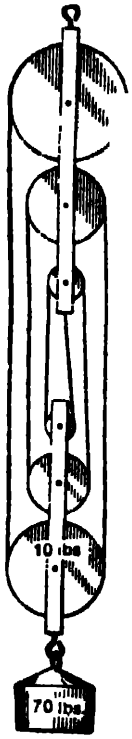


FIG. 71.

EXAMPLES.—(1) *A block and tackle has* 3 *pulleys in each block. The lower block weighs* 10 *lb. Assuming that friction is negligible, what effort must be employed to raise a load of* 70 *lb.?*

Fig. 71 shows 6 lengths of string between the blocks;

$\therefore$ Velocity ratio $= 6$;

$\therefore$ Mech. advantage when wt. of block is included in load $= 6$;

$\therefore$ Effort required $= \frac{80}{6} = 13{\cdot}3$ lb.-wt.

(2) *The pulleys in the fixed block of a Weston Differential Pulley have radii of* 8 *in. and* 7½ *in. Taking its efficiency as* 40 *per cent., what is the greatest load which can be raised by a man weighing* 11 *st.?*

$$\text{Velocity ratio} = \frac{2 \times 8}{\frac{1}{2}} = 32.$$

$$\text{Since efficiency} = 40 \text{ per cent.,}$$

$$\text{Mech. advantage} = \tfrac{40}{100} \times 32 = 12{\cdot}8.$$

$$\text{Greatest effort the man can exert} = 11 \text{ st.-wt.;}$$

$$\therefore \text{ Greatest load he can lift} = 11 \times 12{\cdot}8 = 140{\cdot}8 \text{ st.}$$

$$= 17{\cdot}6 \text{ cwt.}$$

The Inclined Plane

You probably know from experience that, when a heavy body is to be raised, it is easier to push it or roll it up a sloping surface than to lift it directly. Workmen loading barrels on to a lorry usually roll them up a sloping plank, and it is believed that the huge blocks of stone in the upper parts of the Egyptian Pyramids were raised by hauling them up sloping ramps temporarily built alongside the pyramids.

Since the **inclined plane** gives mechanical advantage it may be regarded as a machine.

Fig. 72 shows that if the effort is applied in a direction parallel to the plane, it moves through the length, AB, of the plane while the weight of the body is being overcome through a distance equal to the height, CB, of the plane. Therefore:—

$$\text{Velocity ratio} = \frac{\text{Length of plane}}{\text{Height of plane}} = \frac{1}{\sin\theta},$$

where θ is the angle the plane makes with the horizontal. Fig. 73 indicates the manner in which the actual mechanical advantage may be determined.

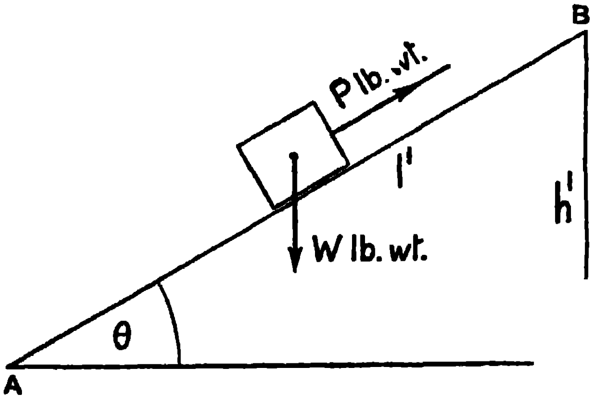


FIG. 72.

If the effort is applied horizontally to the body, it moves a horizontal distance equal to the base, AC, of the plane while the weight is being overcome through the height, CB. Therefore, in that case,

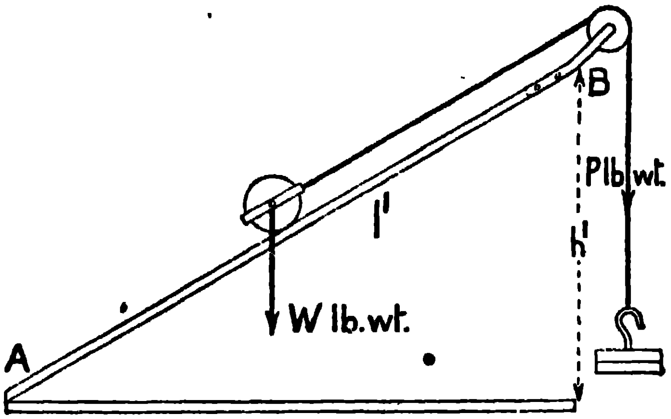


FIG. 73.

Velocity ratio

$$= \frac{\text{Base of plane}}{\text{Height of plane}} = \frac{1}{\tan \theta}.$$

EXAMPLE.—*A barrel weighing 2 cwt. is raised from the ground to a platform 4 ft. high by rolling it up a plank, the effort being applied in a horizontal direction. Assuming that resistance due to friction, etc., is negligible, calculate the length of plank necessary to allow the barrel to be raised by a force of* 64 *lb.-wt.* (Fig. 74).

$$\text{Mechanical advantage} = \frac{224}{64}.$$

$$\text{Velocity ratio} = \frac{AC}{4 \text{ ft.}}.$$

Since friction, etc., negligible,

$$\frac{AC}{4 \text{ ft.}} = \frac{224}{64}; \qquad AC = \frac{224}{64} \times 4 = 14 \text{ ft.}$$

$$AB = \sqrt{AC^2 + BC^2} = \sqrt{14^2 + 4^2} = \sqrt{16 + 196 \text{ ft.}}$$

$$= \sqrt{212} = 14{\cdot}56 \text{ ft.}$$

$$= 14 \text{ ft. } 7 \text{ in. (approx.).}$$

The Screw Jack

This is illustrated in Fig. 75. You have probably seen similar machines being used to lift a car so that its wheels may be changed. If the *pitch* of the thread, *i.e.* the vertical distance between consecutive threads, is x in., the load will rise x in. while the effort moves through a distance equal to the circumference of the circle traced out by the lever arm, *i.e.* $2\pi \times$ Length of lever arm. Therefore:—

Velocity ratio

$$= 2\pi \times \frac{\text{Length of lever arm}}{\text{Pitch of screw}}.$$

Thus a long lever and small pitch will give a very high velocity

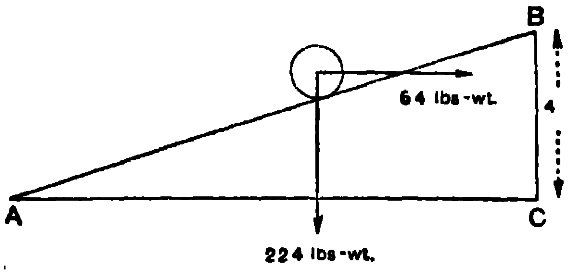


FIG. 74.

ratio and therefore a big mechanical advantage. There is usually considerable friction between the screw and the thread in which it works, and this reduces the mechanical advantage. For the jack to be useful the frictional resistance must be sufficient to prevent the load from pushing the screw back when the effort is removed.

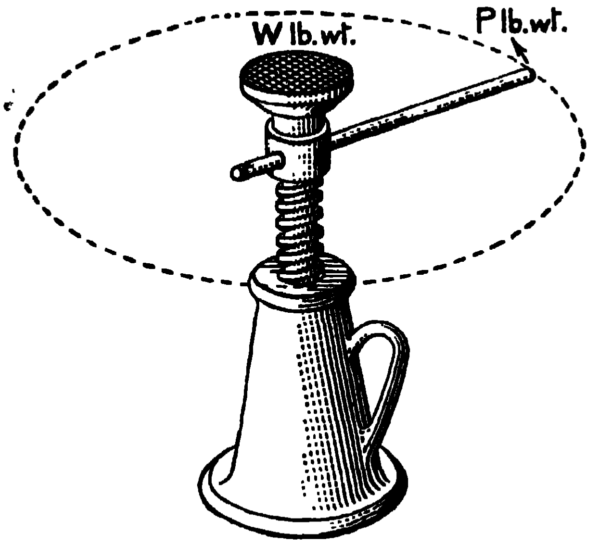


FIG. 75. SCREW JACK.

The mechanical advantage of such a jack may be determined by placing a load on it and attaching a cord to the end of the lever arm. The cord should pass over a pulley so placed that the cord exerts a horizontal pull perpendicular to the lever. The weight with which the cord must be loaded to start the lever moving can then be found.

To find the velocity ratio, first measure the length of the lever arm from the axis of the screw. If the pitch of the screw is not easily measured directly measure the distance by which the load is raised by a number, say 10, of complete revolutions of the lever, and calculate its rise per revolution.

EXAMPLE.—*Assuming a* 40 *per cent. efficiency, what must be the length of the lever of a screw jack, with screw pitch* $\frac{5}{8}$ *in. in order that an effort of* 28 *lb.-wt. may raise a load of* 1 *ton?*

$$\text{Mechanical advantage} = \tfrac{2240}{28} = 80.$$

$$\text{Efficiency 40 per cent.;}$$

$$\therefore \text{ Velocity ratio} = \frac{80 \times 100}{40} = 200.$$

Let the length of the lever be x in.

$$\text{Then velocity ratio} = \frac{2\pi x}{\frac{5}{8}}$$

$$= 2 \times \tfrac{22}{7} \times \tfrac{8}{5}x = \tfrac{352}{35}x;$$

$$\therefore \tfrac{352}{35}x = 200; \quad \therefore x = \frac{200 \times 35}{352} = \tfrac{875}{44} = 19{\cdot}9;$$

$$\therefore \text{ Length of lever must be approx. 20 in.}$$

Gears

Gears are devices frequently used in machines to obtain a suitable velocity ratio for rotating parts. In some arrangements a pulley on one shaft is driven by a belt passing round a pulley of different diameter on another shaft. In Fig. 75 (*a*), B will rotate more times than A so, if the shaft of B is directly driven by an engine and load is applied to the shaft of A, a high velocity ratio and therefore a high mechanical advantage is obtained, while if the engine drives A and the load is applied to the shaft of B, velocity ratio and mechanical advantage will be low but there is a gain of speed. It is clear that the velocity ratio in such an arrangement is given by the ratio of the radii of the pulleys.

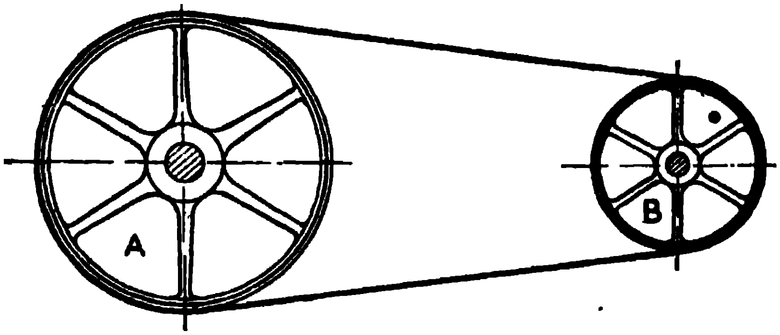


FIG. 75 (*a*).

In belt and pulley gears there is usually a certain amount of slipping of the belt on the pulleys. Where this must be avoided toothed wheels are used. Fig. 75 (*b*) shows two such wheels on separate shafts and clearly a high or low velocity ratio will be obtained by applying the direct drive to B or A. With such gears the velocity ratio is given by the ratio of the numbers of teeth on the two wheels. By arranging a train of such wheels as indicated in Fig. 75 (*c*) a very large difference between the rotational velocities of the first and last wheels in the train can be obtained. Such trains of wheels are used in clocks and watches, and by arranging separate trains driven from the same first wheel different velocities for the hour and minute hands are obtained.

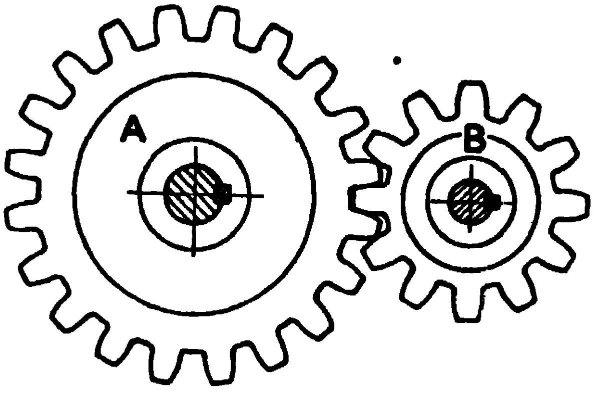


FIG. 75 (*b*).

Trains of cog wheels are also used in the gearbox of a motor car.

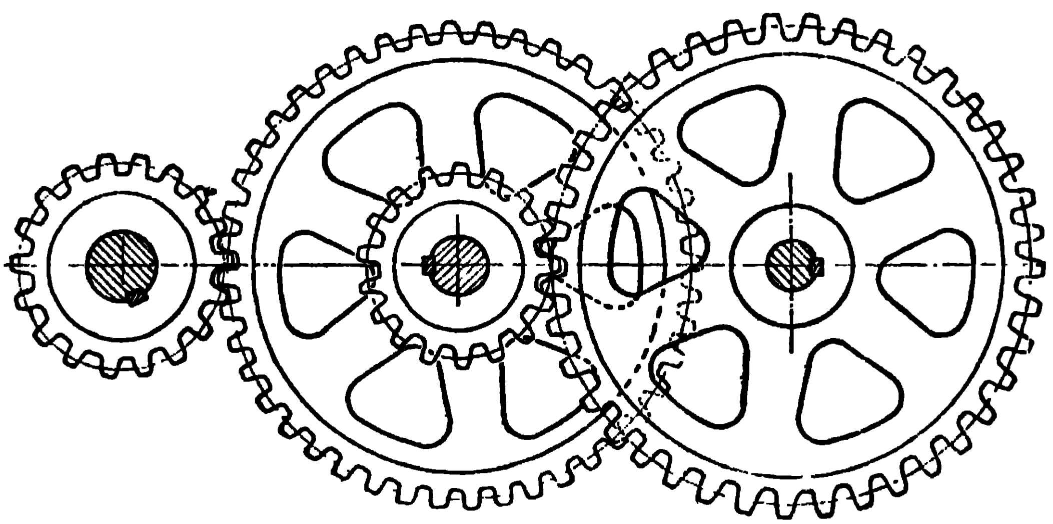


Fig. 75 (c).

Fig. 75 (*d*) shows the essential features of a four-speed gearbox. A is driven directly from the engine. M, the mainshaft, which drives the car wheels, is not in one piece with this but revolves freely in a bearing inside B. B and C are always in mesh, driving the lay-shaft, N. The wheels E, G, and K can slide along grooves in M, their positions being controlled by movements of the gear lever. With the gear lever in the low gear position, K meshes with L, as shown. M is then driven through B, C, L, and K. Since C has more teeth than B, N revolves more slowly than A, and since K has more teeth than L, M will revolve more slowly than N. Thus A is revolving much more rapidly than M, giving a high velocity ratio and a high mechanical advantage, which is useful when the inertia of the car has to be overcome on starting or when a big force is needed for going uphill. When there is less load to overcome the gear lever is

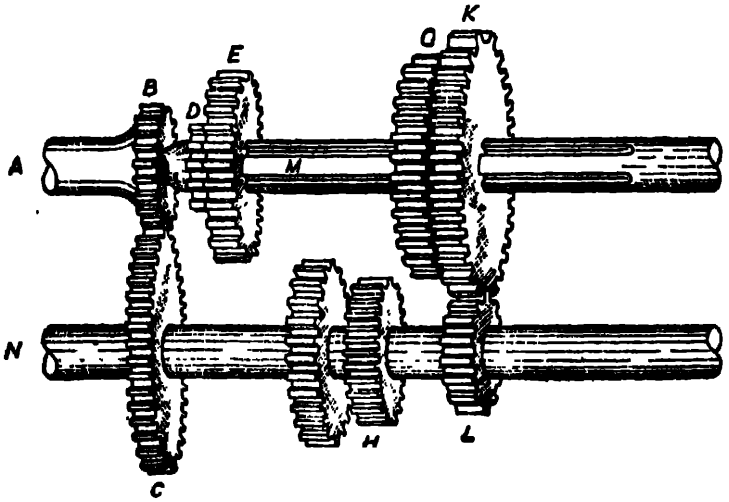


Fig. 75 (*d*).

moved to the next position. This disengages K and L and brings G and H into mesh. As will be seen, this increases the speed at which M revolves, reducing velocity ratio and mechanical advantage but increasing the speed of the car. The change to the third position engages E and F, which again reduces mechanical advantage and increases the speed. The speed of M, however, is still less than that of A. In top gear position the projections at D engage in recesses in B, so that M rotates at the same speed as A. This position is used on the level or downhill, where little load has to be overcome and speed is of more importance than mechanical advantage. For reversing, a wheel on N drives a wheel on a separate shaft which in turn drives one on M which is thus made to rotate in the opposite direction to A.

A bicycle is said to have a gear equal to

$$\text{Ht. of back wheel} \times \frac{\text{No. of cogs on sprocket wheel}}{\text{No. of cogs on hub wheel}}.$$

This multiplied by π gives the distance the bicycle travels during each revolution of the pedals, for (π × ht. of back wheel) = circumference of back wheel, and the fraction gives the number of times it turns during each pedal revolution. Since the cogs are evenly spaced, the fraction is equal to the ratio of the circumferences of the sprocket and hub wheels. Thus the distance moved by the bicycle illustrated during each pedal revolution is $2\pi d \times \frac{2\pi b}{2\pi c} = \frac{2\pi bd}{}$. The distance the effort moves in the same time is

2π × length of crank

Velocity ratio

$$= 2\pi a \Big/ 2\pi bd \quad \frac{ac}{bd}.$$

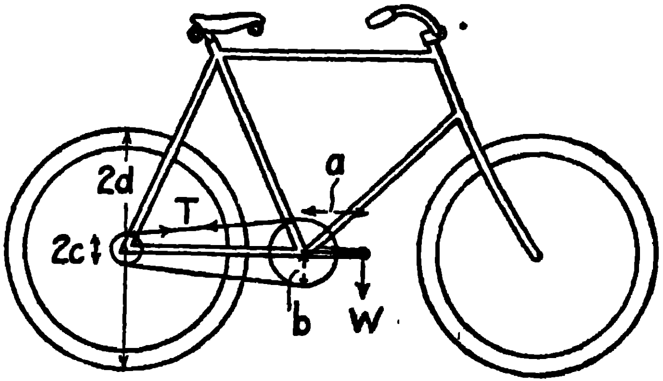


FIG. 76. MODERN BICYCLE.

It will be noted that the velocity ratio, and therefore the mechanical advantage, is considerably less than 1. In this case, however, the "load"—mainly frictional and air resistances—is usually small, so mechanical advantage can be sacrificed to securing a rapid movement of load compared with that of the effort.

Friction

A number of references to **friction** have been made in this chapter. It is generally known that the term refers to forces which resist the motion of one surface over another surface in contact with it. Friction arises because no surface is perfectly smooth, and the roughnesses of the one surface catch on those of the other. The reactions between these roughnesses act as forces parallel to the surfaces in contact.

It has been seen that friction reduces the efficiency of machines and often causes much of the work done by the effort to be wasted, but that in some cases friction plays a useful part. Actually, life would be very difficult if friction did not exist. Walking would be impossible for the forces exerted on the feet would cause them to slip backwards instead of the reactions to them propelling the body forwards. Any body placed on a sloping surface would slide down, since there would be nothing to counteract that component of its weight acting parallel to the surface. Brakes would not stop cycles and motor cars as there would be no force exerted in the direction necessary for stopping the motion of the wheels.

It is obviously important to engineers that the laws governing friction should be understood. As an example of this it may be pointed out that the earliest locomotives had cogged driving wheels which engaged projections on the rails, as it was not realised that frictional forces would be sufficient to prevent the wheels from slipping on the rails. Similar arrangements are still used on very steep mountain railways, and railway engineers need to know what conditions will make this necessary.

Laws of Friction

Obtain a block of wood about 6 in. × 4 in. × 3 in. Plane one of the 6 in. × 4 in. and one of the 6 in. × 3 in. surfaces as smooth as possible, and leave the others somewhat rough. Place the block with its planed 6 in. × 4 in. surface on the bench and attach a dynamometer as shown in Fig. 77. Gradually increase the pull on the dynamometer until the block just moves, and try to note the reading when the block is on the point of sliding. Note also that the reading falls as sliding begins. Repeat several times and note that the same reading is always obtained just before the sliding begins. Pull the block from the other end and note that the same reading is obtained. Repeat the experiment with the block on the planed 6 in. × 3 in. surface, and the same reading will be obtained once more. With the block resting on one of its unplaned surfaces a higher reading will be obtained.

From these observations the following deductions may be made:—

(1) The friction always acts in such a direction that it opposes the motion.

(2) Until motion takes place it adjusts itself so as to be equal to the force tending to produce motion.

(3) In any given case there is a limiting value to the frictional force.

(4) When motion is taking place the friction is less than the limiting value mentioned in (3).

(5) The limiting friction is independent of the area of the faces in contact.

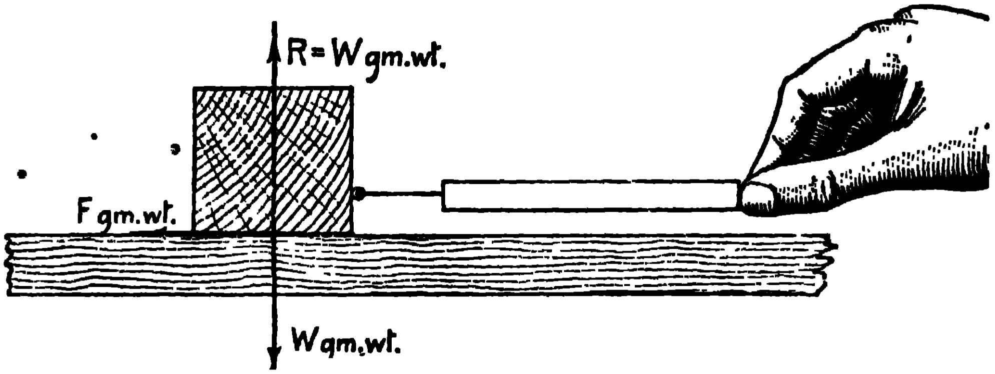


FIG. 77.

(6) It does depend on the nature of the faces.

With regard to points (2) and (3) it should be noted that when any pull less than that which caused motion was given to the block, there must have been an equal and opposite force acting to prevent motion. Therefore, as the pull increased, the friction must have increased at the same rate until a point was reached beyond which the friction could not increase and motion began when the pull exceeded that limit.

Coefficient of Friction

Weigh the block of wood. Let it rest on one of its planed faces, and place a weight on it. The total weight of the block and weight is called the **normal reaction** of the block, because it measures the force exerted on the bench in a direction perpendicular to it, and hence, by Newton's Third Law, the reaction in an opposite direction of the bench on the block (W and R in Fig. 77). Find the limiting friction as before.

Repeat a number of times, varying the weight on the block. It will be found that, as the normal reaction increases, the limiting friction increases. Tabulate your results in the form—

Normal Reaction	Limiting Friction	$\frac{\text{Limiting Friction}}{\text{Normal Reaction}}$

It will be found that a constant value is obtained for the fraction in the last column, showing that the limiting friction is directly proportional to the normal reaction. This constant fraction is called **the coefficient of friction** between the two faces in contact. It depends partly on the two substances concerned and partly on the extent to which the two faces have been smoothed. For two metal surfaces it may vary from ·15 to ·30. Between leather and metal it may be as high as ·6. The high value in the latter case is one of the reasons for using leather belting in machinery.

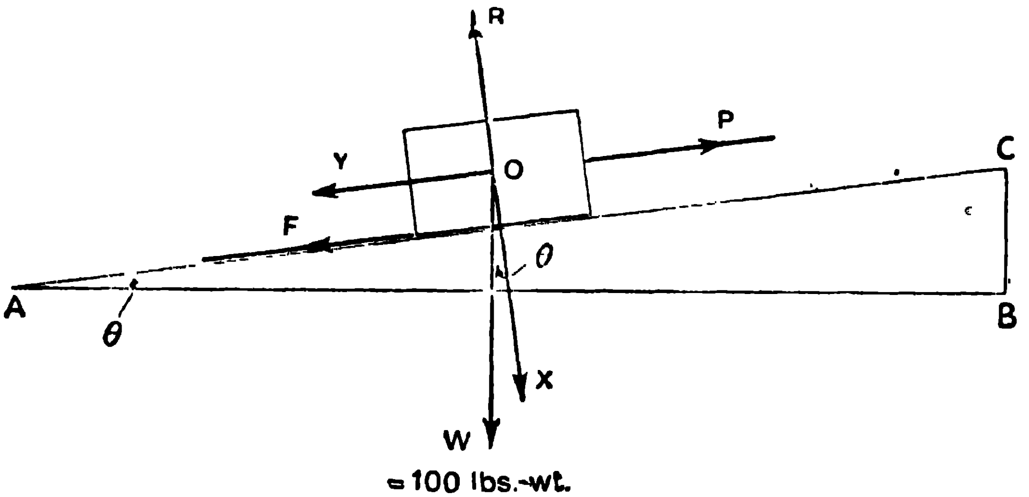


Fig. 78.

Lubrication

Where friction is a nuisance it is often decreased by **lubrication**, *i.e.*, introducing oil or grease between the surfaces sliding over one another. The lubricant forms a film on each surface, and to some extent keeps their roughnesses apart. Where the presence of oil or grease is undesirable, graphite or black lead is often used, as this will form a highly polished coating on the surfaces.

Another device for lessening friction is the **ball** or **roller bearing.** In such bearings the surfaces do not slide but roll over one another, and rolling friction is much less than sliding friction.

Example.—*A plank, 8 ft. long, has one end 1 ft. higher than the other. What force parallel to the plank will be necessary to push a box up it, the box and its contents having a mass of 100 lb. and the coefficient of friction between it and the plank being 0·3?* (See Fig. 78.)

The normal reaction R = X, which is the component of the wt. of the box acting perpendicularly to the plank.

$$\angle WOX = \angle CAB = \theta; \ \therefore R = X = 100 \cos \theta = 100 \times \frac{AB}{AC} \text{ lb.-wt.}$$

$$AB = \sqrt{AC^2 - BC^2} = \sqrt{64 - 1} = \sqrt{63} = 7{\cdot}94 \text{ ft.};$$

$$\therefore R = 100 \times \frac{7{\cdot}94}{8} \text{ lb.-wt.};$$

$$\therefore \text{Limiting friction, F,} = {\cdot}3 \times 100 \times \frac{7{\cdot}94}{8} = 29{\cdot}8 \text{ lb.-wt.}$$

But Y, the component of the weight of the box acting parallel to the plank, also acts down the slope.

$$Y = 100 \sin \theta \text{ lb.-wt.} = 100 \times \frac{BC}{AC} \text{ lb.-wt.}$$

$$= 100 \times \tfrac{1}{8} = 12{\cdot}5 \text{ lb.-wt.};$$

$\therefore$ Total force, P, required to move box upwards

$$= 29{\cdot}8 + 12{\cdot}5 = 42{\cdot}3 \text{ lb.-wt.}$$

QUESTIONS ON CHAPTER VIII

1. Give the meanings of the terms *mechanical advantage, velocity ratio,* and *efficiency* as applied to a machine, and explain the relation between them.

How would you measure the velocity ratio and the mechanical advantage of a screw jack?

2. What is the velocity ratio of the pulley arrangement shown in Fig. 79? If, when W = 100, the effort P needed to raise it is 30 lb.-wt., what is its mechanical advantage and efficiency for the given load?

How would you expect its efficiency to vary with the load? Give reasons for your answer.

3. Draw diagrams of a windlass and of a wheel and axle. Explain the working of each, and from your diagrams deduce their velocity ratios. For what definite purposes have you seen such arrangements being used.

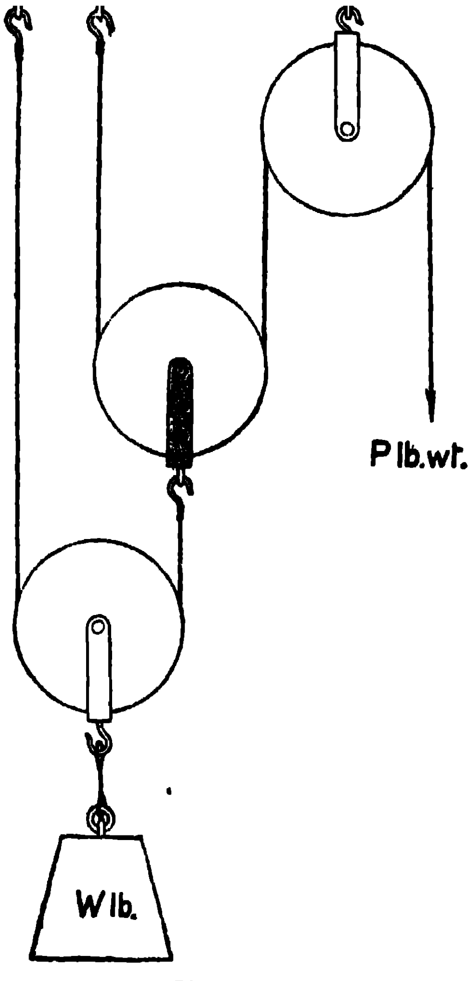


FIG. 79.

4. A pulley system has three pulleys in the upper block and two in the lower one. Draw a diagram showing how the rope is arranged, and from it deduce the velocity ratio of the system.

If its efficiency is 60 per cent., what effort will be needed to raise a load of 2 cwt. with it, and what would be the maximum load a man weighing 12 st. could raise with it?

5. Draw a diagram of Weston's differential pulley and deduce its velocity ratio.

If the pulleys in the fixed block have diameters of 6 in. and 5¾ in., and the efficiency is 20 per cent., what is the mechanical advantage?

6. A pulley system with two pulleys in each block is used to pull a truck which, with its load, weighs 5 tons up inclined rails which rise 1 ft. to each 10 ft. of rails, the rope attached to the truck being parallel to the rails. Assuming 75 per cent. efficiency of the pulley system and neglecting friction in the bearings of the truck, calculate the effort required. Also calculate the work done by the effort and the work done on the truck when the latter travels 10 ft. along the rails.

7. What is the pitch of the thread of a screw jack if 12 complete turns of the handle raise the head 9·6 in.? Assuming an efficiency of 40 per cent., what length must the handle be to give a mechanical advantage of 30?

8. Explain the meaning of *sliding friction*. Give three examples in which friction is troublesome and three in which it is useful.

Mention three ways of reducing the effects of friction in machines.

9. State the law of friction between two surfaces, and define any terms you use in the statement.

The following results were obtained with a block of wood on a wooden surface:—

Wt. of block and load ...	450	500	550	600	650 grm.
Force required to make block slide	90·5	99·5	110	121	129 grm-wt..

Show that they approximately agree with the law, and obtain a value for the coefficient of friction between the surfaces. What difference would you expect to be made to the results by (*a*) resting the block on another face which was smaller than, but as smooth as, the one used, and (*b*) using a rougher surface.?

10. A body weighing 100 lb. is on a surface inclined at 30° to the horizontal, the coefficient of friction between the body and the surface being 0·3. What force acting parallel to the surface must be applied to prevent the body from sliding down the slope ?

11. Explain what is meant by *mechanical advantage* and *velocity ratio* of a machine.

Describe how a velocity ratio of 4 can be obtained with (*a*) a lever, (*b*) a system of pulleys, (*c*) an inclined plane, (*d*) a wheel and axle. [L.U.

12. Describe the principle of the action of a simple lever. In an ordinary water pump the lever is bent at an angle at the fulcrum and is heavy at the handle end. Explain the advantages gained by these features. [L.U.

13. Describe, with the aid of clear diagrams, two machines suitable for raising from ground level a case weighing 4 cwt., provided with a hook, when the available force does not exceed 200 lb.-wt.

What work must be done by this force in lifting the case 2 ft. if the efficiency of the machine is (*a*) 100 per cent., (*b*) 90 per cent. ? [J.M.B.

14. Explain the terms *velocity ratio*, *mechanical advantage*, and *efficiency* as applied to a machine. Calculate the value of each of

these quantities in the case of a wheel and axle, the radii being respectively 4 in. and 1 in., and a load of 20 lb. being raised by a force of 6 lb. applied in the usual way. [J.M.B.

15. A simple machine intended for hoisting loads has its mechanism totally enclosed, and hanging from it are two ropes marked A and B. Describe how you would determine—

(*a*) To which rope the load should be attached.

(*b*) The velocity ratio of the machine.

(*c*) The effort required to raise a load of 100 lb.

(*d*) The efficiency of the machine when used to raise a load of 100 lb. [J.M.B.

16. Explain carefully (*a*) *the moment of a force about a point*, (*b*) *the principle of moments.*

A bucket weighing 40 lb. is drawn out of a well by a chain wound round a cylinder of diameter 4 in. The crank by which the cylinder is turned describes a circle of radius 2 ft. Give a diagram of the arrangement and find the force that must be applied.

How much work is done in 10 revolutions? [J.M.B.

CHAPTER IX

ELASTICITY

The word elastic is probably connected in the reader's mind with fabric containing rubber threads which has the property of being easily stretched and of springing back to its original length when released. Non-scientific people usually think mainly of the ease with which it is stretched when thinking of "elastic." Actually this is an indication that the fabric referred to is not a very elastic substance. For instance, no one would speak of plasticine as being an elastic substance, though a roll of it may easily be pulled out to a greater length. The difference between the "elastic" and the plasticine is the tendency of the former to recover its original length after being stretched.

The difference between elastic and non-elastic substances may also be illustrated by dropping a ball of plasticine and a glass marble from a height of a yard or two on to a hard floor. The marble will rebound into the air, but the plasticine ball will not. Examination will show that the part of the plasticine ball which came into contact with the floor is flattened, but there is no indication of flattening on the marble. If the marble is dropped again on to a hard surface which has been smeared with ink, a considerable area of its surface will be marked by the ink. This indicates that the part of the marble which struck the surface was flattened but that it recovered its shape again. The rebounding of the marble is due to this.

These illustrations indicate that, when an elastic body is subjected to forces which tend to alter its shape or size, opposing forces are set up which tend to restore it to its original condition. This was shown in the investigation of the stretching of a spiral spring on page 10. The greater the opposing forces and the more complete the restoration of the original condition, the more elastic the body is said to be. Thus glass is much more elastic than india-rubber. Much more force is needed to stretch a glass rod than to stretch equally an india-rubber cord of the same length and thickness, and the glass rod will spring back to its original length much more strongly than the rubber cord.

From very early times use has been made of elastic forces. The restoring forces set up when a bow is bent propel the arrow, and stones are flung from catapults by elastic forces. A long list of present-day applications of elasticity could be made including all uses of springs and the absorption of shocks by the inflated tyres of cycles and motor cars.

Stress and Strain

In the study of elasticity the term **stress** is used to denote the force exerted on the body, and the term **strain** to denote the resulting effect. In defining exactly what is meant by stress and strain in any particular case consideration must be given to the kind of effect being produced.

In connexion with the lengthwise stretching of wires, it is evident that the amount of extension caused by a given force will depend on the thickness of the wire. If two wires of the same length and material but of different thicknesses are to be given equal extensions, we should expect a greater force to be required for the thicker one than for the thinner one. We therefore define stress in that case as the force exerted per unit area of cross-section, or—

$$\text{Stress} = \frac{\text{Force}}{\text{Area of cross-section}}$$

We should also expect the extension to depend on the original length of the wire, for the force would be transmitted all along the wire and each unit of it would undergo a certain extension. Hence we define the strain as the extension per unit of original length, or—

$$\text{Strain} = \frac{\text{Extension}}{\text{Original length}}$$

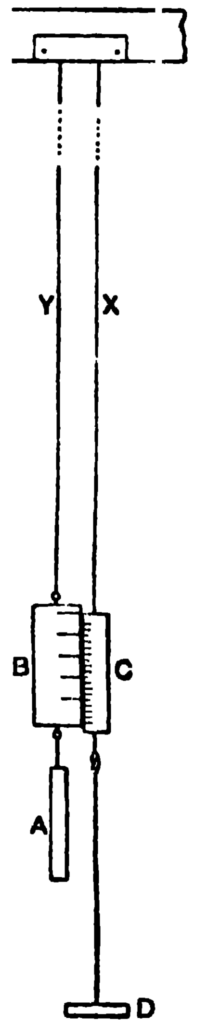


FIG. 80

Hooke's Law

Robert Hooke (1635-1703) stated that for elastic substances **strain is proportional to stress**, $\frac{\text{stress}}{\text{strain}}$ is constant. This constant is called **the modulus of elasticity** of the substance for the particular type of **strain** being considered. In the case of stretching the constant is **known as Young's Modulus** for the substance.

Young's Modulus

For the substance of a wire, this can be determined by the arrangement shown in Fig. 80. A long wire, X, of the material to be tested, and another wire, Y, of about the same length are clamped to a beam on the ceiling. Y is kept taut by a fixed weight, A, and carries a metal plate B with a scale of millimetres engraved on it. X has attached to it a plate C with a vernier scale which slides alongside B. Below C a weight carrier D is fixed.

The length of X is measured as is its diameter, the latter measurement being taken by a screw gauge. A weight is hung on D after a vernier reading has been taken. The movement of the vernier enables the extension of X to be measured. Weights are added step by step and the extension produced by each load is measured. In this way a set of results as below can be obtained.

Material: Copper. Length of X = 2 m.

Diameter of X = ·9 mm.; ∴ Area of cross-section = ·00644 sq. cm.

LOAD	STRESS = $\frac{\text{LOAD}}{\text{AREA OF X SECT.}}$	EXTENSION	STRAIN = $\frac{\text{EXTENSION}}{\text{ORIGINAL LENGTH}}$	YOUNG'S MODULUS = $\frac{\text{STRESS}}{\text{STRAIN}}$
	GRM. PER SQ. CM.			GRM. PER SQ. CM.
1 kilog.	$\frac{1000}{\cdot00644} = 1\cdot553 \times 10^5$	·22 cm.	$\frac{\cdot22}{200} = \cdot0011$	$14\cdot1 \times 10^8$
2 ,,	$3\cdot106 \times 10^5$	·45 ,,	·00225	$13\cdot9 \times 10^8$
3 ,,	$4\cdot659 \times 10^5$	·67 ,,	·00335	$13\cdot9 \times 10^8$
4 ,,	$6\cdot212 \times 10^5$	·90 ,,	·0045	$13\cdot9 \times 10^8$
5 ,,	$7\cdot764 \times 10^5$	1·12 ,,	·0056	$13\cdot9 \times 10^8$
6 ,,	$9\cdot317 \times 10^5$	1·34 ,,	·0067	$13\cdot9 \times 10^8$

It will be seen that a constant value is found for Young's Modulus. If a graph of extension against load is drawn as in Fig. 81, it will be found to be a straight line showing the regular increase of extension as the load is increased.

EXAMPLE.—*Young's modulus for steel is* 2×10^{12} *dynes per sq. cm. What extension will be produced in a steel wire,* 1 *mm in diameter and* $2\frac{1}{2}$ *m. long, by hanging a mass of* 3 *kilog. from it?*

Force applied = 3000 grm.-wt. = 3000 × 980 = 294 × 10^4 dynes.

Area of cross-section $= \frac{22}{7} \times (\cdot 05)^2 = \frac{\cdot 055}{7}$ sq. cm. $= \frac{55}{7 \times 10^3}$ sq. cm.;

$$\therefore \text{ Stress} = \frac{294 \times 10^4}{\frac{55}{7 \times 10^3}} \text{ dynes per sq. cm.}$$

$$= \frac{294 \times 7 \times 10^7}{55} \text{ dynes per sq. cm.}$$

Let the extension produced be x cm.

$$\text{Then strain} = \frac{x}{250};$$

$$\frac{294 \times 7 \times 10^7}{55} \Big/ \frac{x}{250} = 2 \times 10^{12};$$

$$\frac{294 \times 25 \times 7 \times 10^8}{55x} = 2 \times 10^{12};$$

$$x = \frac{294 \times 25 \times 7 \times 10^8}{55 \times 2 \times 10^{12}} = \frac{5145}{11 \times 10^4} = \frac{467 \cdot 7}{10^4} = \cdot 04677;$$

∴ Extension would be approximately ·047 cm.

Limits of Elasticity

When the stress on a body is steadily increased a stage is reached when Hooke's Law no longer applies. The following figures were obtained by loading a wire until it broke.

Load in lb.	5	10	15	20	25	30	35	40	45	50
Extension in cm.	·56	1·12	1·68	2·24	2·80	3·36	4·12	5·25	8·05	Broke

The graph obtained by plotting them is shown in Fig. 82. Above M the line is no longer straight, showing that extension is no longer proportional to load but that equal additions of load produce increasing extensions. Under the conditions represented by M, the wire is said to have reached its **elastic limit.** If loaded beyond that limit it will not completely recover when the load is removed, but will have been permanently stretched to a certain extent.

When the wire is near breaking point, as at N, the graph is nearly horizontal, showing that a very small addition to the load will cause a very large extension. When in this state a small pull with the fingers will pull the wire out considerably. It is then said to have passed its **yield point.**

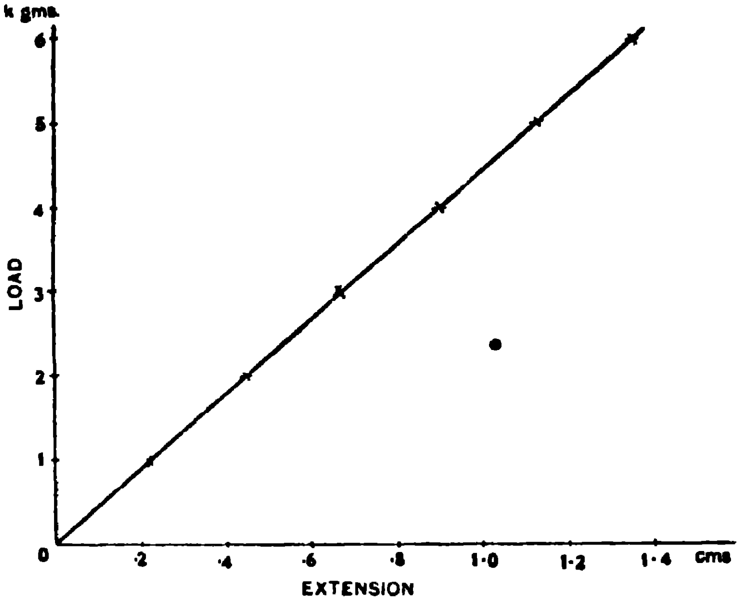


FIG. 81.

QUESTIONS ON CHAPTER IX

1. Explain, illustrating your answer by examples, what you mean by a substance being elastic. How could you show by experiment that iron is more elastic than wood?

Give three common examples of the application of the elastic properties of substances.

2. State Hooke's Law, and define any terms you use in the statement.

Explain what is meant by Young's Modulus of elasticity for a substance, and state how you would determine it for a given metal.

3. Explain what you mean by the *limit of elasticity* and the *yield*

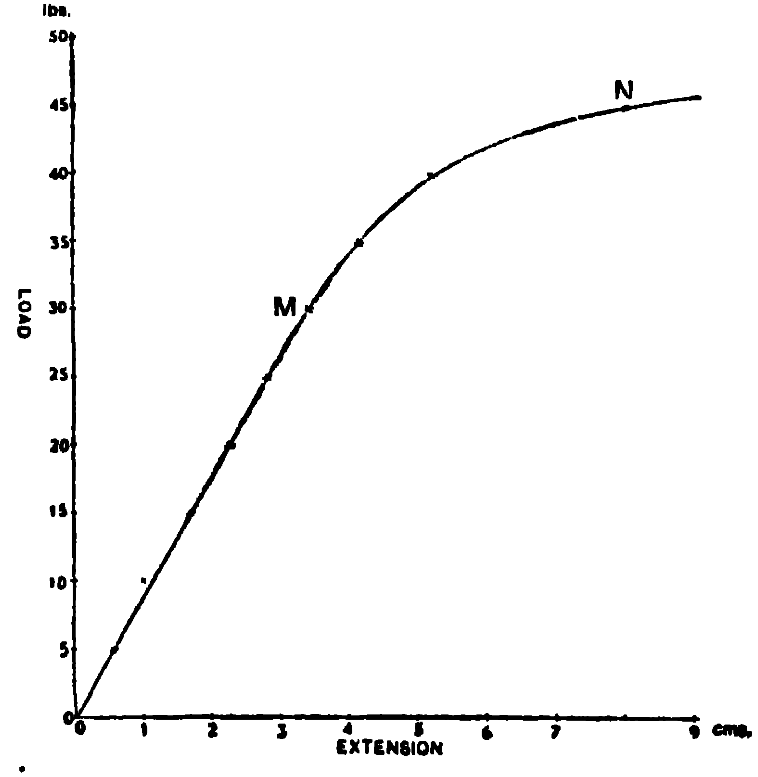


FIG. 82.

point in connexion with a wire, and draw a graph to illustrate your answer.

4. A wire 2·8 m. long is stretched by ·5 *mm.* when a weight of 9 kilog. is hung on it. Its diameter was 2 mm. Calculate (*a*) the stress, (*b*) the strain, (*c*) Young's Modulus for the substance of the wire.

5. If Young's Modulus for steel is 2×10^{12} dynes per sq. cm., what force would be necessary to stretch a steel wire 3 m. long and 2 mm. in diameter by ·5 cm.? Answer in kilograms-weight.

6. A certain type of wire is found to pass its elastic limit when the strain exceeds $\frac{1}{100}$. It has a diameter of ·04 in., and Young's Modulus for its substance is 12×10^6 pndls. per sq. in. What is the greatest weight that can be hung from a length of it without causing permanent stretching?

Would that maximum load vary with the length of wire used? Give reasons for your answer.

CHAPTER X

PRESSURE IN LIQUIDS

A body always exerts a force equal to its weight on any surface supporting it. The effect of this on the surface depends on the area over which it is spread. When you stand on loose sand your feet sink deeply into it, but if you lie down on it your body sinks in very little. When you are standing, the whole of your weight is exerted on the area covered by your feet so that a big force is exerted on each square inch of the area supporting you, and considerable movement of the sand below your feet is caused. When you lie down the total weight supported by the sand is the same as before, but it is distributed over a much greater area, and each square inch of the sand surface experiences a much smaller force than before. In considering the actions of forces on surfaces we must, therefore, think of force per unit area rather than of total force. This force per unit area is called the **pressure on the surface.**

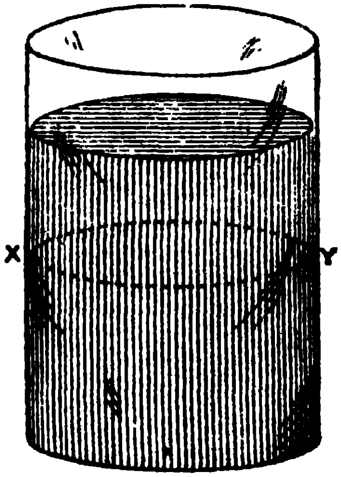


FIG. 83.

Pressure Within a Liquid

If a point within a liquid is considered it will be clear that there must be equal upward and downward pressures at that point. If we think of a thin layer, XY (Fig. 83), below the surface of some liquid in a vessel, the weight of the liquid above must be exerting a downward force on that layer. But the layer does not move, so an equal upward force must also be acting on it.

Not only are there upward and downward pressures at a point in a liquid, but there are pressures in all directions. Thus if a hole is punched in the side of a deep can filled with water, the water spurts out horizontally from it, showing the existence of sideways pressure.

The pressures in all directions at a given depth in a liquid are equal. This can be shown as indicated in Fig. 84. To a tube, bent as shown, attach the head of a thistle-funnel by a short length of stout rubber

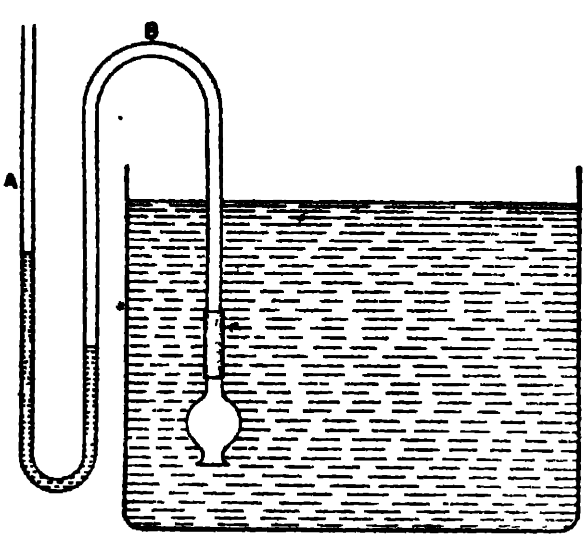


FIG. 84.

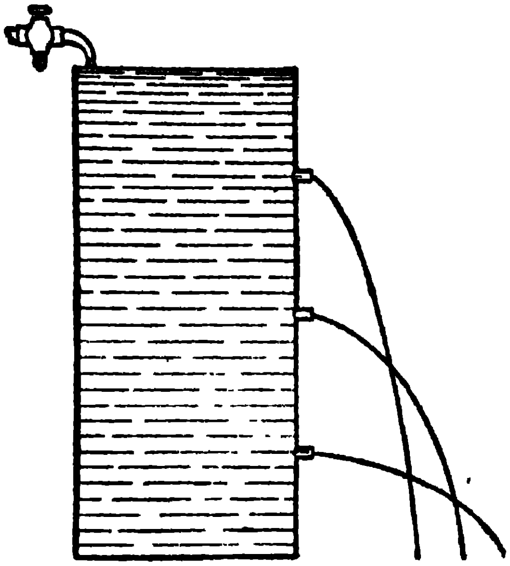


FIG. 85.

tubing. Tie a thin sheet of rubber tightly across the mouth of the funnel, and put a little water in the lower bend of the tube. When the funnel is lowered into water, the pressure of the water will push the rubber inwards. This will make the air in B press on the water in the tube and drive more of it up into A. Turn the mouth of the funnel in various directions, always keeping the centre of the rubber at the same level. It will be found that the water is always driven to the same level in A. If the funnel is placed at various depths in the water, the level in A will vary.

Depth and Pressure

Bore holes in the side of a deep tin can (Fig. 85). Place it under the tap and keep it full of water. The water will spurt out much further horizontally from holes near the bottom of the can than from those near the top. If two holes are made at the same level, water will spurt to equal distances from them. This shows that pressure increases as depth increases.

The relation between depth and pressure may be found as follows. Into a flat-bottomed tube about 1 in. wide put sufficient sand or lead shot to make it float upright. Weigh the tube and its contents, and float in water in a glass jar (Fig. 86). Measure the depth to which the bottom of the tube sinks below the surface of the water. Repeat several times using various weights. Measure the diameter of the tube and calculate the area of its bottom. When the tube is floating, the upward force of the water on the bottom of it must be equal to the weight of the tube and its contents. Hence, that weight, divided by the area of the bottom, must give the pressure at the depth to which the tube sank. Tabulate your results as follows:—

Diameter of tube = 2 cm.

Area of bottom of tube = $\pi \times (\text{radius})^2 = \frac{22}{7} \times 1^2$ sq. cm. = 3·14 sq.cm.

WEIGHT	PRESSURE = $\frac{\text{WEIGHT}}{\text{AREA}}$	DEPTH	• $\frac{\text{PRESSURE}}{\text{DEPTH}}$
16·3 grm.	5·19 grm. per sq. cm.	5·1 cm.	1·02
·41·8 „	13·3 „ „ „ „	13·3 „	1·00
55·4 „	17·7 „ „ „ „	17·6 „	1·01
64·1 „	20·4 „ „ „ „	20·5 „	·995
77·9 „	24·8 „ „ „ „	24·8 „	1·00

An approximately constant value of $\frac{\text{Pressure}}{\text{Depth}}$ is found, showing that the pressure is directly proportional to the depth. Also, with water it is found that this constant value is approximately 1, *i.e.* it is numerically equal to the value of the density of water.

By repeating the experiment, using other liquids whose densities are known, it can be shown that $\frac{\text{Pressure}}{\text{Depth}}$ = Density is true in each case. This result may also be written—

Pressure = Depth × Density.

Remembering that the density of water is 62·5 lb. per cub. ft. it follows that the pressure at a depth of 10 ft. in water is 625 lb. per sq. ft., at 20 ft. it is 1250 lb. per sq. ft., and so on. Since the pressure increases with depth, the thrust on a reservoir dam increases with depth: hence dams are much thicker at the base than at the top. You will also realise that, as a diver descends into water, air must be pumped into his suit up to a pressure equal to that of the surrounding water. At a comparatively small depth this pressure becomes greater than the body can bear, and for deep sea diving, metal suits must be used which will

LEAD SHOT OR SAND

FIG. 36.

withstand the pressure of the water without being inflated to an equal pressure.

Example.—*A tube whose area of cross-section is 5 sq. cm. is loaded with shot till its total weight is 32 grm. To what depth will it sink in water, and what is the density of a liquid to which it sinks to a depth of 7·5 cm.?*

(*a*) Pressure to support tube $= \frac{32}{5} = 6{\cdot}4$ grm.-wt. per sq. cm.

$$\text{Depth} = \frac{\text{Pressure}}{\text{Density}} = \frac{6{\cdot}4}{1} = \underline{6{\cdot}4 \text{ cm.}}$$

$$(b)\ \text{Density} = \frac{\text{Pressure}}{\text{Depth}} = \frac{6{\cdot}4}{7{\cdot}5} = \underline{{\cdot}85 \text{ grm. per c.cm.}}$$

Columns of Liquids

If a liquid is poured into a set of communicating vessels it rises to the same height in each, as shown in Fig. 87, although they may have very different diameters and shapes. The pressure must be the same at all points along the horizontal tube in the apparatus illustrated or the liquid would be moving from one part of it to another. This emphasises the fact that the only factors which need be taken into account in connection with the pressure of a column of liquid are its vertical height and its density.

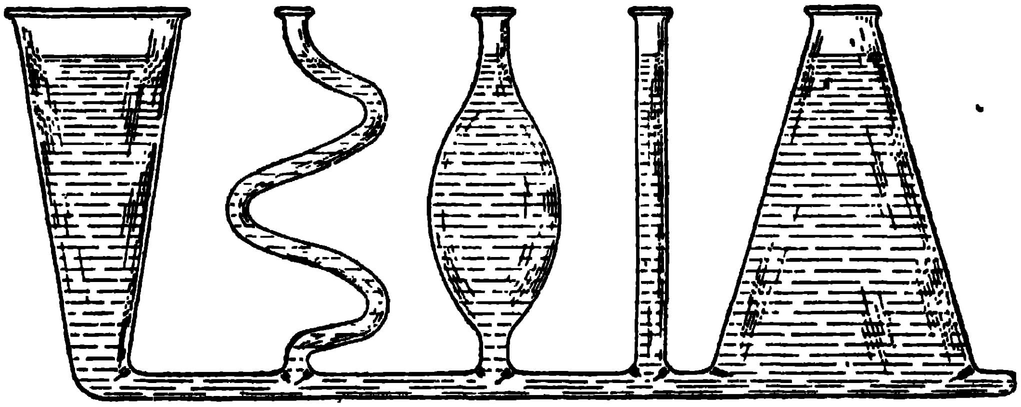


Fig. 87.

The principle that **balanced columns of the same liquid will always rise to the same height** ("Water finds its own level") has many applications. Fig. 88 shows a surveyor's water-level. However the instrument may be tilted, sighting along the two water surfaces will always give a true horizontal line.

Gauges to show the height of water in boilers or petrol in tanks can be made by connecting narrow vertical glass tubes to them. The liquid will always stand at the same height in the gauge as in the vessel. The same principle is applied in the water supply to a town. If possible, the supply reservoir is made on high ground. Where this is not convenient, the water is pumped to the top of a high tower to give a "head." Water will then rise in the connected pipes to any point which is not higher than the "head."

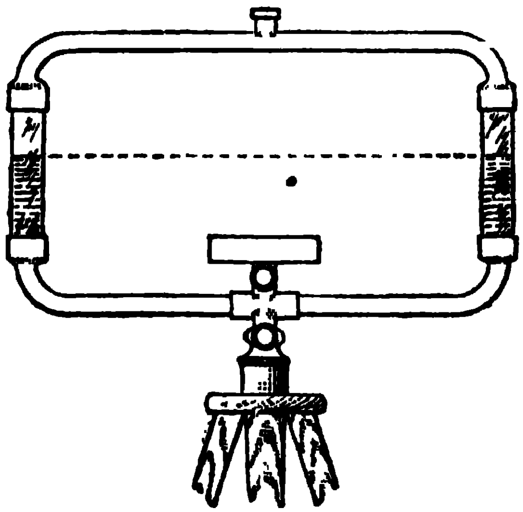


FIG. 88. SURVEYOR'S WATER LEVEL.

Fig. 89 illustrates an **artesian well**. A, B, and C are saucer-shaped beds of rock, A and C being impervious to water while B is porous. Water soaking in at the ends of B fills it up to the level XY. If a hole is bored through A in the position shown, water will spout up from it, and if pipes are fitted to the hole, it will rise in them to the level XY.

Experiments with U-Tubes

Into a U-tube pour sufficient mercury to fill the bend. Pour water into limb A (Fig. 90) until it is nearly full. This will push the mercury

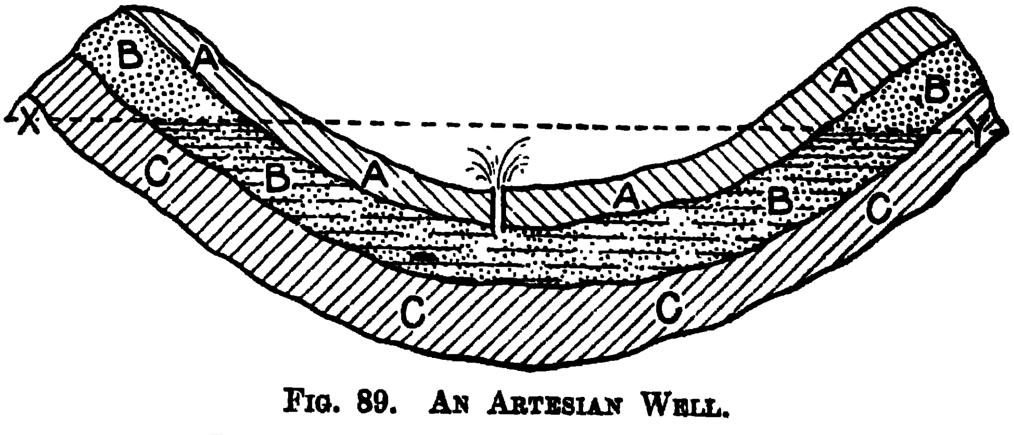


FIG. 89. AN ARTESIAN WELL.

A = Impervious stratum. B = Water-laden chalk.
C = Impervious stratum.

round to the right, but it will not rise to the same height as the water in A. The mercury in A will balance that in B up to Y at the same level as X. Therefore the column of mercury of height h_1 balances the column of water of height h_2. Measure these heights. Since the columns balance they have equal pressures;

$$\therefore\ h_1 \times \text{density of mercury} = h_2 \times \text{density of water};$$

$$\therefore\ \frac{\text{Density of mercury}}{\text{Density of water}} = \frac{h_2}{h_1},$$

$$\textit{i.e.}\ \text{Relative density of mercury} = \frac{\text{Ht. of water column}}{\text{Ht. of mercury column}}.$$

To find the relative density of other liquids, such as paraffin, the balancing columns may be separated by mercury, as in Fig. 91. Pour the mercury in first. Then pour some paraffin into A. Next, carefully pour water into B until the two mercury surfaces are at the same level. The paraffin column is then balancing the water column, and as above,

$$\text{Relative density of paraffin} = \frac{\text{Ht. of water column}}{\text{Ht. of paraffin column}}.$$

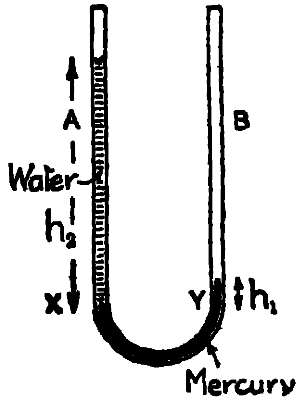


FIG. 90.
SPECIFIC GRAVITY OF MERCURY.

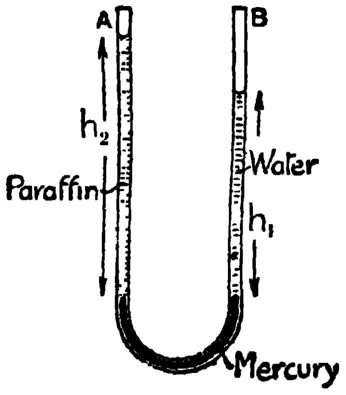


FIG. 91.
SPECIFIC GRAVITY OF PARAFFIN.

Since height and density completely determine the pressure of a column (page 109), the results of these experiments will not be affected by inequalities in the bore of the tube, nor will they be affected by the tubes not being upright so long as the heights are measured vertically.

EXAMPLE.—*A column of water* 50 *cm. high balances a column of alcohol* 62·5 *cm. high or a column of glycerine* 40 *cm. high. Calculate the specific gravities of alcohol and glycerine.*

Also calculate the height of a glycerine column which will balance a column of alcohol 50 *cm. high.*

$$\text{Specific gravity of alcohol} = \frac{50}{62{\cdot}5} = \underline{{\cdot}8.}$$

$$\text{Specific gravity of glycerine} = \tfrac{50}{40} = \underline{1{\cdot}25.}$$

Let x cm. be height of required glycerine column.

$$\text{Then } 1{\cdot}25 \times x = 50 \times {\cdot}8;$$

$$\therefore x = \frac{50 \times {\cdot}8}{1{\cdot}25} = 32;$$

$$\therefore \text{Ht. of glycerine column} = \underline{32 \text{ cm.}}$$

The Hydraulic Press * ✓

This machine is illustrated in principle in Fig. 92. It consists of two connected cylinders of different bores filled with water and fitted with tight pistons. If a mass of 20 lb. is supported on the piston in A, the pressure on the water surface in A is 20 lb.-wt. per sq. ft. Since pressures at points at the same level in a liquid are equal there is a pressure of 20 lb.-wt. per sq. ft. at the surface of the water in B. Therefore there is a total force of 200 lb.-wt. on the piston in B, and the 20 lb. mass on A will support a 200 lb. mass on B. Clearly, by making A very narrow and B very wide, a small force applied to the piston in A can raise a large mass on the piston in B.

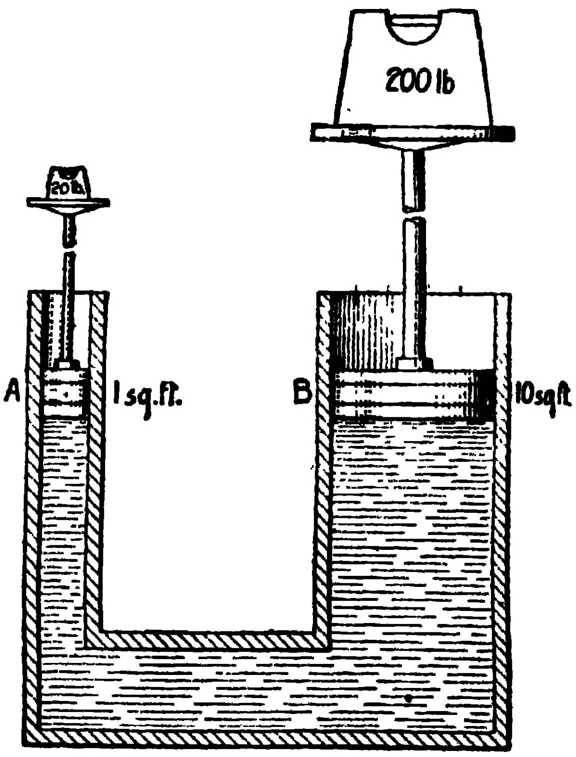


FIG. 92.

It will follow from the above example, that for a hydraulic press,

$$\text{Mechanical advantage} = \frac{\text{Area of large piston}}{\text{Area of small piston}}.$$

It can be shown that the velocity ratio has the same value, for to transfer 1 cub. ft. of water from A to B, the plunger must descend 1 ft. But an additional cubic foot of water in B will only raise the level $\frac{1}{10}$ ft.;

$$\text{Velocity ratio} = \frac{1}{\frac{1}{10}} = \frac{10}{1} = \frac{\text{Area of large piston}}{\text{Area of small piston}}.$$

From these figures the principle of work also follows, for a force of 20 lb.-wt. acting through a distance of 1 ft. overcomes a load of 200 lb.-wt. through a distance of $\frac{1}{10}$ ft.;

$$\therefore \text{ Work done on the machine} = 20 \times 1 \text{ ft.-lb.} = 20 \text{ ft.-lb.}$$

$$\text{Work done by the machine} = 20\cancel{0} \times \frac{1}{1\cancel{0}} \text{ ft.-lb.} = 20 \text{ ft.-lb.}$$

Hydraulic machinery is much used to-day in presses, jacks, and lifts. Braking systems on many cars are also actuated by pressure transmitted through a liquid contained in a cylinder, the brake lever operating a piston in the cylinder. By connecting all the brakes by pipes to the same cylinder, even pressure on all of them is obtained.

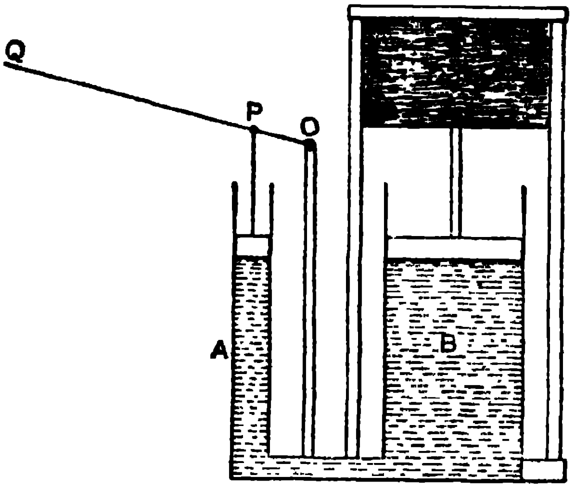


FIG. 93.

EXAMPLE.—*The diagram illustrates a baling press.* A *has a diameter of* 6 *in.,* B *a diameter of* 24 *in.* OP *is* 8 *in., and* OQ *is* 2 *ft. Assuming* 70 *per cent. efficiency, find* (*a*) *the velocity ratio,* (*b*) *the mechanical advantage,* (*c*) *the force exerted on the bale when an effort of* 10 *lb.-wt. is applied at* Q.

$$\text{For lever OQ, velocity ratio} = \frac{OQ}{OP} = \frac{\cancel{24}}{8} = 3.$$

$$\text{For the hydraulic press, velocity ratio} = \frac{\pi \times 12^2}{\pi \times 3^2} = \frac{\cancel{12} \times \cancel{12}}{\cancel{3} \times \cancel{3}} = 16;$$

$\therefore$ When the piston in B moves 1 in., that in A moves 16 in. and

$$Q \text{ moves } 16 \times 3 = 48 \text{ in.};$$

$$\therefore \text{ Velocity ratio} = \underline{48.}$$

$$\text{Mechanical advantage} = \frac{48 \times 7\cancel{0}}{10\cancel{0}} = \underline{33{\cdot}6;}$$

$\therefore$ Force exerted on bale by effort of 10 lb.-wt. at Q

$$= 10 \times 33{\cdot}6 = \underline{336 \text{ lb.-wt.}}$$

QUESTIONS ON CHAPTER X

1. Explain what is meant by (*a*) *the pressure on a surface*, (*b*) *the pressure at a point in a liquid.*

Describe an experiment to show that equal pressures are exerted in all directions at a point in a liquid.

2. How could you show experimentally that the pressure at a point in a liquid (*a*) varies with the depth of the liquid, and (*b*) is the same at all points at the same depth?

3. Describe an experiment to show that the pressure at a point in a liquid is equal to the depth of the point multiplied by the density of the liquid.

4. A flat-bottomed tube, 4 sq. cm. in cross-section, sinks to a depth of 8 cm. in water and to a depth of 10 cm. in alcohol. What is the weight of the tube and its contents, and what is the specific gravity of alcohol? To what depth would the tube sink in salt water of density 1·02 grm. per c.cm.?

5. Describe the method of measuring the specific gravity of a iquid by means of a U-tube, explaining how the result would be alculated. Need the tube be of uniform bore? Give reasons for our answer.

6. Mercury is placed in the bend of a U-tube which is of uniform ore. Water is then poured into one limb until a column of water 0 cm. high is obtained. What will be the difference between the levels of the two mercury surfaces?

How high a column of alcohol, of specific gravity 0·8, would have to be placed in the other limb to bring the mercury surfaces to the same level once more?

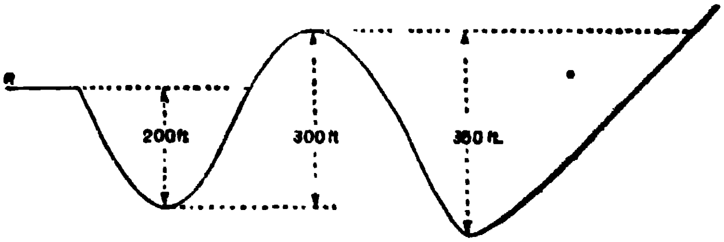


FIG. 94.

7. Describe three cases in which the principle that "water finds its own level" is applied.

Fig. 94 shows a section, with vertical scale exaggerated, of the country between a town and its water reservoir. The reservoir is at R and the town is built on the slope XY. Make a drawing of the section and mark on it the path along which the water main must be laid so that as much as possible of it lies on the surface of the ground. Mark also the highest point on the slope XY at which a house can be served with water from the reservoir.

8. Explain the principle of the hydraulic press, and make a sketch of a press suitable for pressing loose materials into tight bales.

Assuming perfect efficiency, explain how the mechanical advantage of the machine you draw could be calculated.

9. Water is poured into a glass U-tube of uniform cross-section 1 sq. cm. until each limb (30 cm. high) is half filled. Explain why the level of the water is the same in both limbs.

20 c.cm. of paraffin (specific gravity 0·8) are carefully poured into one limb. What is the final height, above the bottom of the tube, of the surface of separation between the water and the paraffin? [J.M.B.

CHAPTER XI

PRESSURE IN GASES. BAROMETER. BOYLE'S LAW

" Streamlining " to reduce the effects of air resistance has played a big part in the efforts to increase the speed of aeroplanes, motor cars, and locomotives, and this indicates the importance of pressure in gases as well as in liquids.

Illustrations of Air Pressure

The fact that the atmosphere exerts a pressure is quickly realised if you try to wave a large sheet of cardboard in the air.

Place a little water in a tin can and heat it until steam issues freely, driving out the air from the can. Remove the flame and cork up the can quickly. Hold it under the cold-water tap. The steam inside will condense and the can will collapse under the external pressure of the atmosphere. This shows that the pressure of the atmosphere has a considerable magnitude.

A number of other illustrations of the existence of air pressure can be given. Completely fill a tumbler with water. Place a sheet of paper over its mouth and press it closely to the rim. The tumbler may then be inverted and the paper will remain in position, preventing the water from running out. The upward pressure of the air on the paper supports the weight of the water.

If a sheet of thin rubber is tied over the wide end of a funnel and the air sucked out through the narrow end, the air pressure outside will push the rubber inwards. This will happen no matter in what direction the funnel is pointing, indicating that the air exerts a pressure in all directions.

This can also be shown by means of the rubber " suckers " used for fixing hooks on shop windows. When the " sucker " is pressed closely to the glass, the air is pressed out from its concave surface and the pressure of the air outside holds it in position. The sucker will stick on the wall, the floor, the ceiling, or on a board facing any direction.

Simple Barometer

For collecting gases the arrangement shown in Fig. 95 is often set up. Since the pressure in a liquid is the same at all points at the same level, the pressure at A is equal to that at B which is due to the air above it. Hence, the water in the jar is supported by the pressure of the air on the surface of the water in the trough.

If a tube two or three yards long, closed at the upper end, is used instead of the jar, it too will remain filled with water when inverted in the trough. If, however, a tube about a yard long is carefully filled with mercury, all air bubbles being tapped out as it is filled, and then inverted in the same way in a bowl of mercury, the mercury will fall several inches when the mouth is opened, forming a column of mercury about 30 in. in height above the surface of the mercury in the bowl, as in Fig. 96 (*a*).

Since all air was removed from the tube, the space at the top must be a vacuum in which there cannot be any pressure. Hence the column of mercury must have a pressure just equal to the atmospheric pressure which is supporting it.

FIG. 95.

As 30 in. = 2·5 ft. and the density of mercury is 13·6 × 62·5 lb. per cub. ft., we can say—

$$\begin{aligned}\text{Pressure of atmosphere} &= \text{Pressure of mercury column}\\ &= 2{\cdot}5 \times 13{\cdot}6 \times 62{\cdot}5 \text{ lb.-wt. per sq. ft.}\\ &= \frac{2{\cdot}5 \times 13{\cdot}6 \times 62{\cdot}5}{144} \text{ lb.-wt. per sq. in.}\\ &= 14{\cdot}76 \text{ (approx. 15) lb.-wt. per sq. in.}\end{aligned}$$

To express this pressure in Metric units, 30 in. = 76 cm. (approx.);

$$\begin{aligned}\therefore \text{ Pressure of atmosphere} &= 76 \times 13{\cdot}6 \text{ grm.-wt. per sq. cm.}\\ &= 1034 \text{ (approx.) grm.-wt. per sq. cm.}\end{aligned}$$

Thus the arrangement described may be used for measuring the pressure of the atmosphere. Any instrument for doing this is called a **barometer**.

If simple barometers are set up with tubes of different bores, the mercury columns will all have the same height because they all have the same pressure —that of the atmosphere—and the pressure of a liquid column depends on height and density only.

If the tube is tilted, mercury moves along it giving a longer column, but the surface remains at the same level, as in Fig. 96 (*b*). This again emphasises the fact that it is vertical height which must be considered in connexion with pressure of liquids.

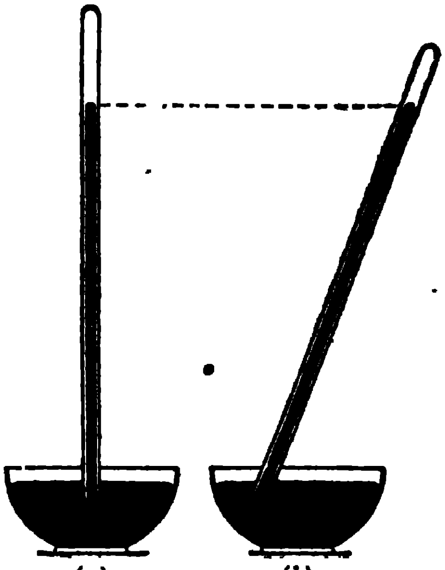

(*a*) (*b*)

FIG. 96. SIMPLE BAROMETER.

If a simple barometer is left set up and measurements of the height of the column are made from day to day, it will be found to vary somewhat, showing that the pressure of the atmosphere varies. If that pressure falls, it will be unable to support as tall a column of mercury as before, and some will run out of the tube until the pressures balance once more. Conversely, if the atmospheric pressure rises, more mercury will be forced into the tube to restore balance.

That the height of the column does depend on the pressure of the air on the mercury in the bowl may be shown by setting up a barometer so that it passes through a cork fixed in a bell-jar as shown in Fig. 97. The jar is stood on the plate of an air pump and air pumped out of it. As the air is removed, the column falls, and as air is re-admitted to the jar, it rises again.

For many purposes we do not need to know the value of the pressure of the air in force units as calculated on page 116, but only to compare the pressure at one time with that at another time. For this reason pressures are often given by simply stating the heights of the mercury columns they will support. The statement " a pressure of 75 cm." means " a pressure that will support a column of mercury 75 cm. high."

It is interesting to calculate the length of tube required for a water barometer. For equal pressures,

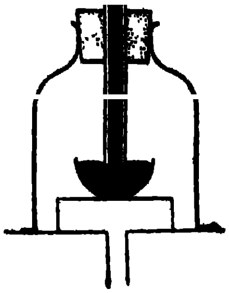


FIG. 97.

$$\frac{\text{Ht. of water column}}{\text{Ht. of mercury column}} = \text{Density of mercury} = 13{\cdot}6;$$

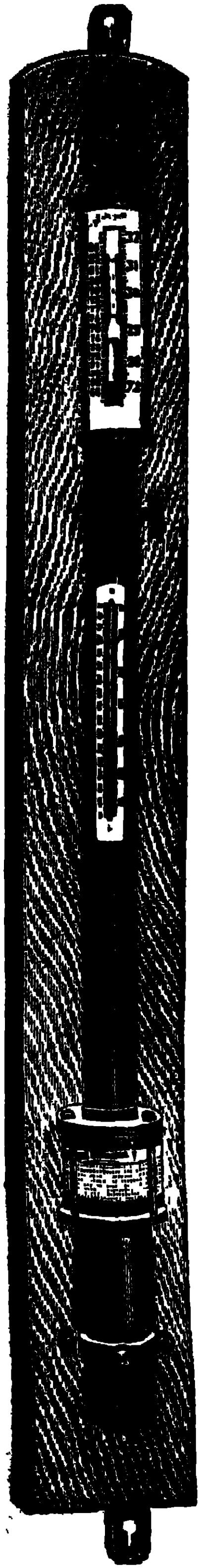


FIG. 98. FORTIN BAROMETER.

∴ The height of a water column which can be supported by the pressure of the atmosphere is 13·6 times that of the mercury column = 30 × 13·6 in. = 34 ft. or 76 × 13·6 cm. = 1034 cm. = 10·34 m.

A water barometer would be more sensitive than a mercury barometer since the movement for any given change of pressure would be 13·6 times as great, but the instrument would be inconveniently long.

The Fortin Barometer

This barometer, which is often used in laboratories, is shown in Fig. 98. The tube is enclosed in a metal case which carries a scale at the top. A vernier is attached so that the height of the column may be read to $\frac{1}{10}$ mm. The tube fits into a glass cistern fitted to the bottom of which is a wash-leather bag. Fig. 99 shows the details of the construction of the cistern. Above the mercury is a pointed piece of ivory. The scale at the top of the tube measures heights above the point of this piece of ivory, so the surface of the mercury in the cistern should always be at the level of that point when a reading is taken. The screw at the bottom, carrying a plunger pressing on the wash-leather bag, enables the mercury to be adjusted until its surface just touches the ivory point. It should be noted that, since the scale really measures the length of the mercury column, the tube should be vertical when a reading is taken, so that length and height are the same.

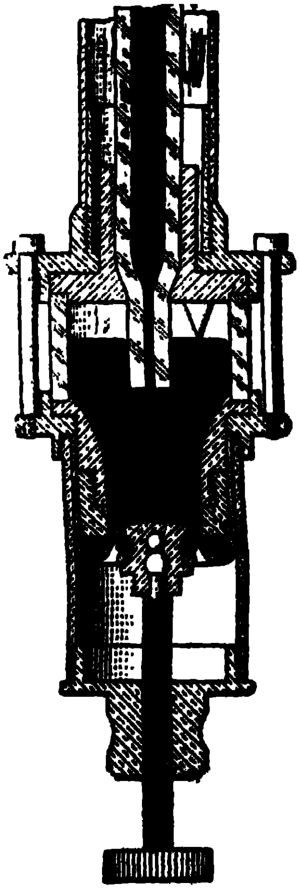


FIG. 99.

Aneroid Barometers

These barometers contain no liquid, and are frequently used for household purposes and in cases where they have to be carried about and liquid might be spilt from an ordinary barometer. Fig. 100 shows the working parts of such a barometer. A is a metal box with strong sides and a thin springy corrugated top which has been partially evacuated. From the middle of the top rises a projection connected with a series of levers pivotted at B, C, and D. S is a strong spring. At E a thread is attached which winds round an axle carrying the pointer GH. F is a hair-spring also attached to the axle.

The pressure of the atmosphere tends to bend the lid of A inwards. If this pressure becomes less, the springiness of the lid, together with the pull of S tends to raise B. This will raise C and cause E to move to the right, unwinding some of the thread from the axle and causing the pointer to move in an anti-clockwise direction, looking from above. Increase of atmospheric pressure will depress B, thus slackening the thread; the hair-spring will then cause the axle to rotate in a clockwise direction until the thread is taut once more. The pointer travels over a circular scale which has been graduated by comparison with a mercury barometer.

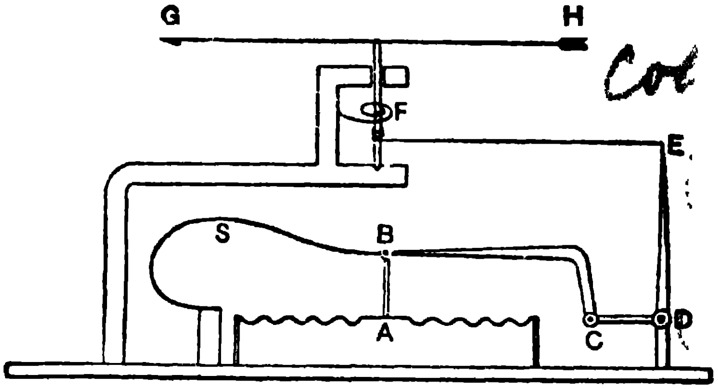


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FIG. 100. ANEROID BAROMETER.

Uses of Barometers

In addition to measuring atmospheric pressures in laboratories, barometers are used in connexion with weather forecasting. Details regarding this will be found in books on meteorology, but it may be mentioned that, generally, high atmospheric pressures are connected with dry conditions of the air, and low pressures with moist conditions. A rapid fall of pressure indicates the approach of a gale.

It must be remembered that the motion of winds is also responsible for changes in atmospheric pressure so that it does not always follow that a decrease in barometer height means coming rain.

Barometers are also used for measuring altitudes. The higher one goes the less air there is above one, and so the lower the atmospheric pressure becomes. From the reduction of pressure the height attained can be calculated. The altimeters of aeroplanes are aneroid barometers graduated to give direct readings of altitudes to which they are carried.

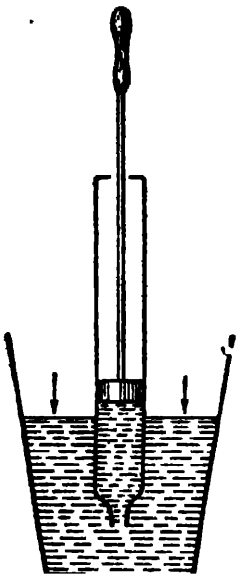

Fig. 101.

Applications of Air Pressure

(1) The Garden Syringe (Fig. 101).—The nozzle is dipped in water. When the tight-fitting piston is drawn upwards it sweeps the air out of the barrel so that there is no air pressure below it. The air pressure on the surface of the water in the pail then forces water up the barrel. When the piston is pressed in again it will drive the water out.

(2) The Bicycle Pump.—This is a modified syringe. The piston is made of leather and is cup-shaped as shown at A (Fig. 102). The connector screws on to the valve B which is inside the tyre. This valve has a tight-fitting rubber tube slipped over it. When the piston is raised, the pressure of the air in the tyre presses the rubber tube to the valve, closing the side opening at C. Thus air cannot flow back from the tyre to the barrel and the pressure below A is reduced. The pressure of the air above A then forces the flexible edge of the piston away from the sides of the barrel allowing air to flow downwards past it. As A descends again, the pressure of air below it presses its edge outwards, preventing escape upwards. Thus the pressure in the barrel and connection increases until it is sufficient to press the rubber away from the opening at C and the air is pushed between the valve and the rubber tube into the tyre.

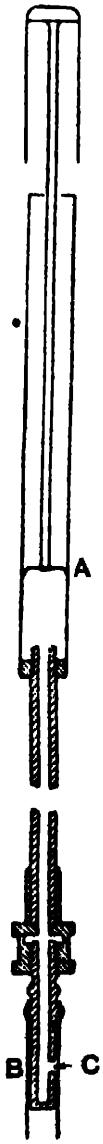

Fig. 102.

(3) The Common Pump.—This is sometimes called the lift pump, and also depends on air pressure. At the bottom of the barrel and in the piston there are valves which will open upwards but not downwards. When the piston makes an upstroke [Fig. 103 (*b*)] the air pressure above keeps its valve closed so that air is swept out of the barrel, reducing the pressure below the piston. Thus, as in the syringe, the pressure of air on the water in the well forces it up into the barrel. The downstroke [Fig. 103 (*c*)] tends to compress the water in the barrel, closing the lower valve which prevents the water from flowing back into the well, and opening the piston valve so that the piston can pass down through the

water. On the next upstroke [Fig. 103 (*d*)] the pressure of the water above the piston will close its valve so that it lifts that water till it runs out of the spout. At the same time water will be pushed into the barrel from the well as before.

From the calculation on page 120 it is clear that, if the surface of the water in the well is more than 34 ft. below the barrel, water will only rise to a height of 34 ft. in the pipe and the pump will not work. It is interesting to recall that the failure of a pump made in the seventeenth century for a deep well belonging to the Duke of Milan led to the recognition of the pressure of the air. Previously the working of pumps had been explained by saying that " Nature abhors a vacuum," *i.e.* it was believed that the water came into the barrel to prevent a vacuum being formed there. Gallileo was called in to investigate the Milan pump. He suggested that perhaps the working was due to air pressure which had a limited value and therefore could not support a column of water of more than a certain height. One of his pupils, Torricelli, then performed for the first time the experiment of setting up a simple mercury barometer showing that a vacuum could exist and that the pressure of the atmosphere had a limited value. For this reason the space at the top of the mercury barometer is still referred to as the Torricellian vacuum.

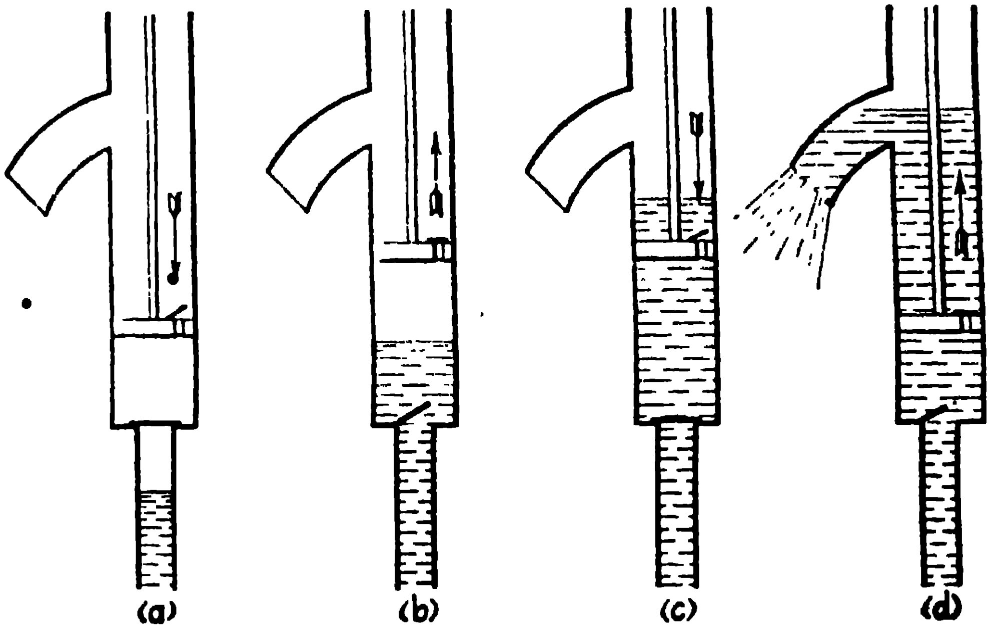

FIG. 103. COMMON OR LIFT PUMP.

(4) THE FORCE PUMP (Fig. 104).—This is a modification of the common pump which enables water to be raised to greater heights. The piston is solid, and a side tube, with an outward opening valve, leads out from near the bottom of the barrel. On the upstroke of the piston, water is forced up into the barrel as in the case of the common pump, outside pressure keeping the valve in the side tube closed. The downstroke closes the valve at the entrance to the barrel and forces the water up the side tube. The air cushion in the chamber connected to the side tube steadies the flow of water up the tube. During the downstroke of the piston this air cushion is compressed, and its pressure keeps the water flowing during the piston's upstroke. Pumps of this type are used in fire-engines so that water may be forced to heights of 100 ft. or more.

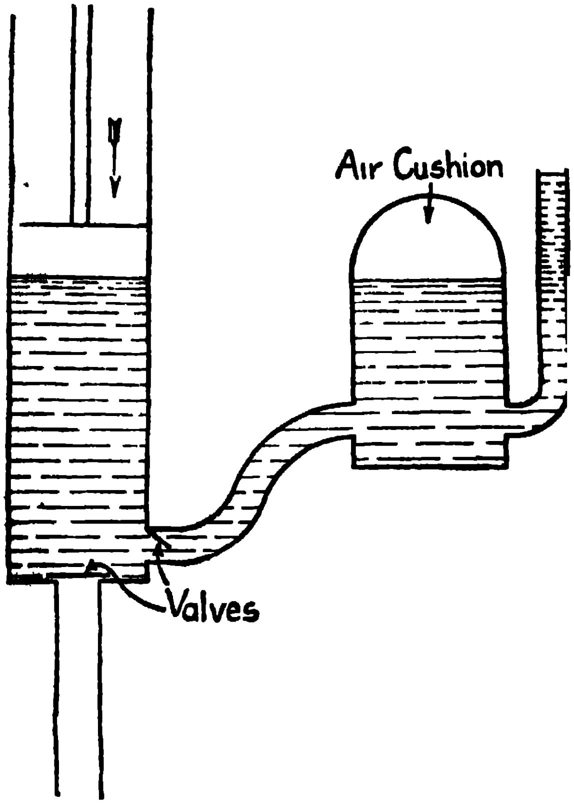

FIG. 104. FORCE PUMP.

The height to which water may be forced up the side tube depends only on the mechanical strength of the pump, but since the water is pushed into the barrel by air pressure, the barrel must not be more than 34 ft. above the water in the well.

EXAMPLE.—*Fig.* 105 *represents the lever and piston of a force pump, the fulcrum of the lever being at* B. AB *is* 6 *in. and* BC *is* 3 *ft. The area of the piston is* 24 *sq. in. To what height above the pump may water be raised by a force of* 4 *st.-wt. applied at* C?

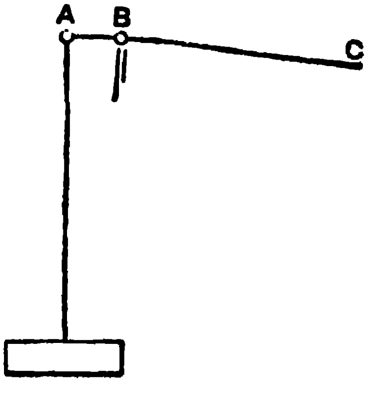

FIG. 105.

Mech. advantage of lever $= \frac{36}{6} = 6$;

Force exerted on piston

$$= 56 \times 6 = 336 \text{ lb.-wt.};$$

$$\text{Pressure on water in barrel} = \frac{336 \times 144}{24} = 2016 \text{ lb.-wt. per sq. ft.};$$

$\therefore$ Water can be raised inside tube till pressure is 2016 lb.-wt. per sq. ft.

$$\text{Height} = \frac{\text{Pressure}}{\text{Density}} = \frac{2016}{62{\cdot}5} = 32{\cdot}3 \text{ ft.}$$

(5) **The Siphon.**—This is an arrangement, shown in Fig. 106, for running liquid out of vessels. At A there is a pressure up the tube equal to the atmospheric pressure on the surface of the liquid. There is also a downward pressure equal to that of a column of liquid of height h_1, so that the resultant pressure up the tube at A is atmospheric pressure — pressure of column of liquid of height h_1. Similarly the resultant pressure up the tube at B is atmospheric pressure — pressure of column of liquid of height h_2. Since h_2 is greater than h_1, the resultant upward pressure at A is greater than that at B, so that liquid is driven from A to B and runs out at B, more liquid being driven into the tube by the pressure of the atmosphere.

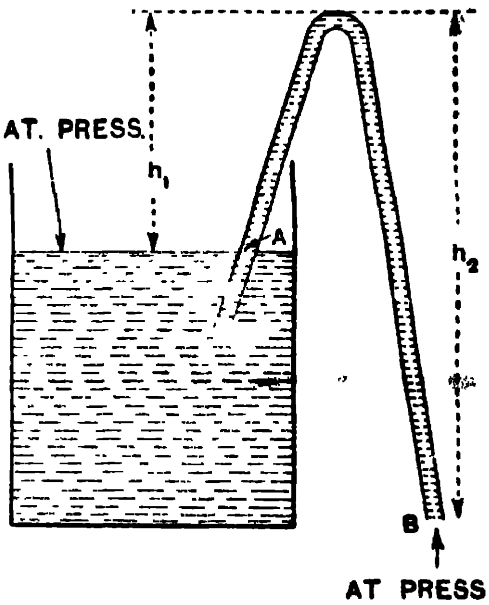

Fig. 106. Siphon.

According to the above theory, for the siphon to work, (*a*) h_1 must be less than the height of the barometric column of the liquid, or the downward pressure would be greater than the upward pressure at A, and the liquid would run back into the vessel; and (*b*) B must be lower than the surface of the liquid in the vessel for the resultant upward pressure at B to be less than that at A. It has been shown that a mercury siphon will work when h_1 exceeds the height of the mercury barometer. Apparently this is due to there being considerable cohesive forces in mercury so that the mercury thread does not easily break but acts more like a very flexible rod and the extra weight on side B pulls it through the tube. However, siphons will work when the A limb is much wider than the B limb and so contains a much greater weight of liquid. Hence the pressure theory gives the most general explanation of siphon action.

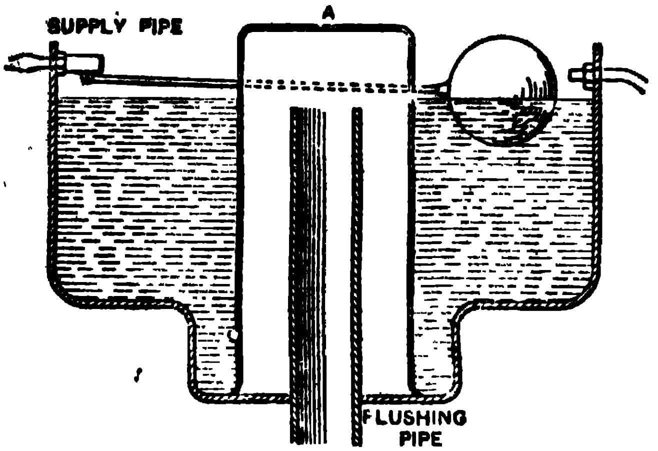

Fig. 107. Flushing Tank.

Soda-water siphons do not work by atmospheric pressure. They contain compressed gas which forces the liquid out when the exit pipe is opened by the action of the lever.

Lavatory flushing tanks have a siphon action. The bell, A (Fig. 107), is raised by a lever, not shown in the figure, when the chain is pulled. This allows the water to rise to the top of the flushing pipe and flow down it. The rush of water keeps the bell from falling right back until the tank is empty. The lever attached to the floating ball closes a valve at the end of the supply pipe when the tank is filled to the proper level again.

Pressure of Gas and Water Supplies

These pressures may be measured by using U-tubes, which are referred to as *manometers* when used for pressure measurements. In the case of the gas supply, water is placed in the U-tube which is connected to the gas tap with rubber tubing. When the gas is turned on the water will be forced away from the tap, and the difference between the two water-levels measures the excess of the gas pressure above that of the atmosphere. Thus, if the height BA (Fig. 108) exceeds the height BC by 6·5 cm., and the barometer is standing at 75 cm.,

Pressure of gas supply = 6·5 cm. of water + 75 cm. of mercury

$= 6{\cdot}5 + (75 \times 13{\cdot}6)$

$= 6{\cdot}5 + 1020$ grm. per sq. cm.

$= 1026{\cdot}5$ grm. per sq. cm.

Water supply pressures are much greater than those of gas supplies, and a mercury

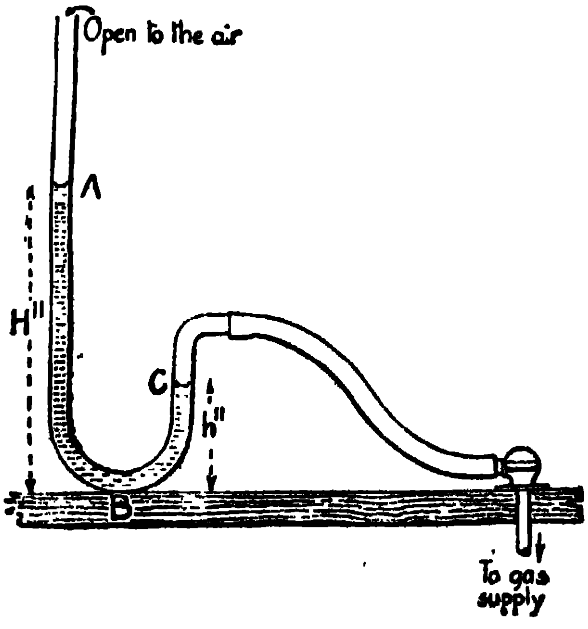

Fig. 108.

manometer with the open limb about 2 yd. long should be used (Fig. 109). In this case the difference in mercury levels can be directly added to the barometric height. The pressure of the water column between the mercury and the tap should be deducted from that sum.

Compressibility of Gases

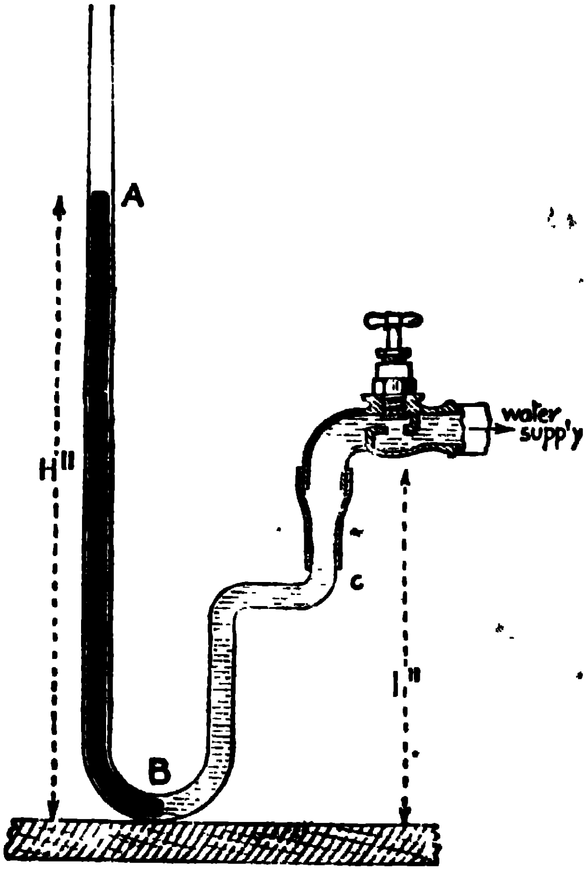

FIG. 109.

For commercial purposes gases such as oxygen and hydrogen may be obtained stored under great pressure in steel cylinders. When the contents of a cylinder are released at atmospheric pressure, they occupy a volume much greater than that of the cylinder.

The alteration in the volume of a mass of gas produced by change of pressure may readily be demonstrated with a good bicycle pump. If the nozzle is closed the piston may be pushed in some distance without any air escaping. When the handle is released it springs back until the enclosed air has its original volume again.

It will be shown in Chapter XVII. that the volume of a mass of gas may also be altered by changing its temperature. Thus the statement that a certain mass of gas has a volume of so many cubic feet has little meaning unless the pressure and temperature of the gas at the time it was measured are mentioned. Also, if we wish to investigate the effect of pressure on a mass of gas, we must take care that all measurements are made at the same temperature.

Boyle's Law

The effect of pressure on a mass of gas was investigated by Robert Boyle (1626-1691). His results may be summarised in the statement, **The volume of a fixed mass of gas at constant temperature is inversely**

proportional to its pressure. This may be verified by a modification of Boyle's method. The apparatus (Fig. 110) consists of two glass tubes connected by a yard or two of stout rubber tubing. A is open and B closed at the top. They are supported vertically by the side of a vertical scale, and A can be moved upwards or downwards. Mercury is poured in until it shows in both tubes, trapping a mass of air in B, which may be graduated to enable the volume of the enclosed air to be read. If not graduated B should be of uniform bore, so that a measurement of the height from the mercury surface to the top of B may be taken as measuring the volume of the air enclosed.

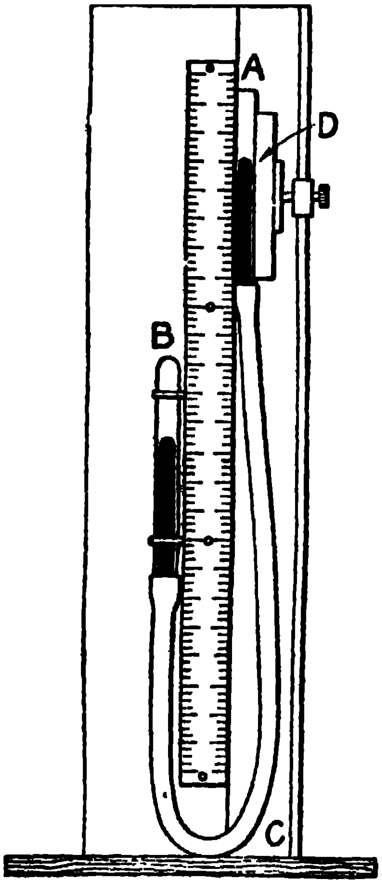

FIG. 110.

A barometer reading is taken, the volume of the enclosed air noted, and the difference between the heights of the two mercury columns is measured. The tube A is then moved to a fresh position and the new readings are taken. This is repeated for a number of different positions of A. For a setting, such as that shown in the figure, where D is higher than the mercury surface in B, the pressure on the air in B would be that of the atmosphere plus the difference between the two mercury columns. If D were lower than the mercury surface in B, the excess mercury column would be balancing part of the atmospheric pressure, so that the pressure on the air in B would be atmospheric pressure minus the difference between the mercury levels. If in the latter case the difference in levels is given a negative sign we may say that in all cases the pressure on the air is the sum of atmospheric pressure and the difference in levels. Results should be tabulated as on page 129.

The product in the last column is approximately constant. This indicates that, if the pressure is doubled, the volume is halved; if the pressure is increased to three times its former value, the volume is reduced to one-third of its previous magnitude, and so on. This is what is meant by saying that the volume is inversely proportional to the pressure.

Height of Barometer = 75·5 cm.

VOLUME (V)	DIFFERENCE OF MERCURY LEVELS	TOTAL PRESSURE P	P × V	
7·5 c.cm.	25·3 cm.	100·8 cm.	755	Corrected to three significant figures.
8 ,,	19·6 ,,	95·1 ,,	761	
9·5 ,,	5·6 ,,	80·1 ,,	760	
10 ,,	0 ,,	75·5 ,,	755	
11 ,,	– 7·0 ,,	68·5 ,,	754	
11·5 ,,	– 9·5 ,,	66·0 ,,	758	
12·5 ,,	– 14·9 ,,	60·6 ,,	759	
13 ,,	– 17·4 ,,	58·1 ,,	755	

It should be noted that the measurements were all made at laboratory temperature which would not alter much during the experiment, so they may be said to have been made at constant temperature. Also, if samples of other gases are substituted for the air in B, similar results are obtained, so that Boyle's Law applies to all gases.

Boyle's Law may be expressed mathematically as—

$$\text{Pressure} \times \text{Volume} = \text{A constant.}$$

Hence, if the pressure changes,

$$\text{New vol.} \times \text{New pressure} = \text{Old vol.} \times \text{Old pressure};$$

$$\text{New vol.} = \text{Old vol.} \times \frac{\text{Old pressure}}{\text{New pressure}}$$

$$\text{or New pressure} = \text{Old pressure} \times \frac{\text{Old vol.}}{\text{New vol.}}.$$

In working examples it is better not to rely on remembering these formulae. A higher pressure will reduce the volume, and that fact will indicate which way up to put the multiplying ratio.

EXAMPLES.—(1) *A mass of gas had a volume of* 200 *c.cm. when its pressure was* 75 *cm. What volume will it have at* 80 *cm. pressure, the temperature remaining unaltered?*

(Note pressure to be increased, so volume will be less than 200 c.cm.)

$$\text{New volume} = 200 \times \tfrac{75}{80} = \tfrac{375}{2} = 187{\cdot}5 \text{ c.cm.}$$

(2) *A mass of gas which had a volume of* 500 *cub. in. when under a pressure of* 15 *lb. per sq. in. is compressed without charge of temperature until it occupies* 300 *cub. in. What is its new pressure?*

$$\text{New pressure} = 15 \times \tfrac{500}{300} = \tfrac{75}{3} = 25 \text{ lb. per sq. in.}$$

Sounding Tubes

The *sounding tube* makes application of Boyle's Law in determining depths of water. It consists of a glass tube of uniform bore closed at its upper end and fixed in a heavy metal case also open at the bottom. The inner surface of the tube is coated with a substance which changes in appearance when in contact with water. It is lowered vertically into the water. The pressure on the enclosed air increases so that it is compressed into a smaller volume and the water rises some distance up the tube. The height to which the water entered is indicated by the sensitive coating when the tube is raised again.

EXAMPLE.—*A sounding tube* 12 *in. long was lowered into sea water of specific gravity* 1·15. *The water rose* 4·5 *in. in the tube. The barometer stood at* 30 *in. What was the depth to which it was lowered?*

Taking each inch of the tube as a unit of volume,

Vol. of air at atmos. pressure (30 in. mercury) = 12 units.

Vol. when compressed = (12 − 4·5) = 7·5 units;

$$\therefore \text{ New pressure} = \frac{30 \times 12}{7{\cdot}5} = 48 \text{ in. of mercury;}$$

$$\therefore \text{ Increase of pressure} = 18 \text{ in. of mercury.}$$

$$18 \text{ in. of mercury would balance } \frac{18 \times 13{\cdot}6}{1{\cdot}15} = 213 \text{ in. of sea water;}$$

$$\therefore \text{ Depth of water} = 213 \text{ in.} = \underline{17 \text{ ft. } 9 \text{ in.}}$$

Air Pumps

These are for pumping air out of vessels, and work in a way very similar to lift pumps. Fig. 111 illustrates the essential principles. When the piston makes an upstroke, the valve A is closed by pressure of air above it, so that air is swept out of the barrel and the pressure below the piston is reduced. This causes air from X to expand into the barrel through B. The downstroke of the piston compresses the air below it, closing B and opening A. These actions are repeated with each stroke, so that more air is removed from X with each upward stroke of the piston.

Suppose X contains 500 c.cm. of air at 76 cm. pressure. Let the volume of the barrel be 200 c.cm. When the piston is pulled up the enclosed air expands to fill 700 c.cm.;

$$\therefore \text{ New pressure} = 76 \times \tfrac{500}{700} = 54{\cdot}3 \text{ cm.}$$

The second stroke causes 500 c.cm. of the expanded air to become 700 c.cm., so that the pressure becomes $\frac{500}{700}$ of its former value;

$$i.e.\ 76 \times \frac{500}{700} \times \frac{500}{700} = 76 \times \left(\frac{500}{700}\right)^2 = 38{\cdot}8 \text{ cm.}$$

Similarly, after the third stroke the pressure becomes

$$76 \times \left(\frac{500}{700}\right)^3 = 27{\cdot}7 \text{ c.m.},$$

and so on.

It will be seen that the pressure in X decreases very rapidly but will never become zero, so that perfect evacuation is not possible by this means.

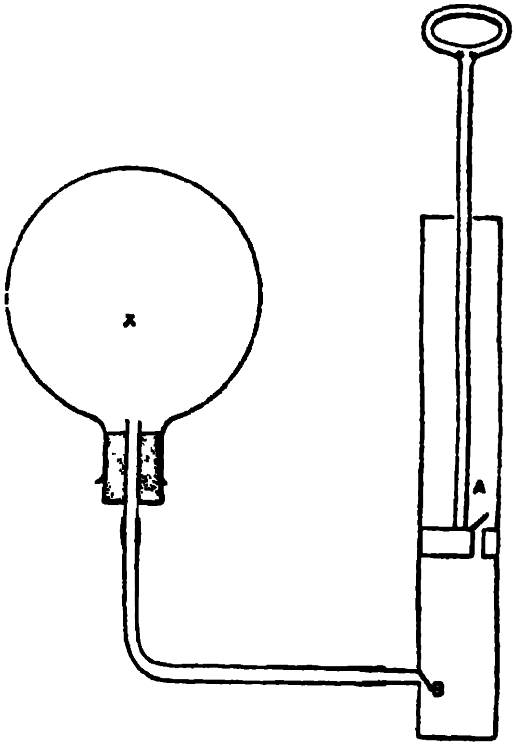

FIG. 111. AIR PUMP.

The Atmosphere. Troposphere and Stratosphere

The atmosphere is an envelope of gas surrounding the earth. By volume about one-fifth of it consists of oxygen and four-fifths of nitrogen. There are also present smaller quantities of water vapour and carbon dioxide and a very small proportion of rare gases such as argon and helium.

It has been shown earlier in the chapter that the atmosphere exerts a pressure of about 15 lb. per sq. in. on the surface of the earth. This is of course due to the weight of the air above it, so with increasing altitude the pressure of the air decreases. This decrease in pressure means that at high altitudes the air is much less dense and much more rarefied than at the surface of the earth. An upward limit to the height of the atmosphere cannot be stated, but it can be shown that at a height of about fifty miles the air is so rarefied that it does not exert any measurable pressure and that about half of the atmosphere is contained in a layer about 3½ miles high. The height at which meteors begin to glow indicates the presence of traces of air up to heights of about 150 miles.

Two distinct regions of the atmosphere are recognised, a lower region known as the troposphere and a higher region known as the stratosphere. They are sharply divided by a layer known as the tropopause which is about 11 miles high over the tropics and 3 to 4 miles high at the poles. In the troposphere temperature falls with increasing height at the rate of about 0·56° C. per 100 metres, so that above a height of 2 kilometres the average temperature is below 0° C. and at a height of about 6 miles it is below − 50° C. In the stratosphere, however, there is very little variation of temperature with altitude but there are temperature variations horizontally, the temperature of the stratosphere being greatest above the poles and least above the equator. In the stratosphere winds may have considerable velocity but they are steadier than those in the troposphere and turbulent convection currents are absent. Almost all the water vapour and dust in the atmosphere are in the troposphere so clouds do not form in the stratosphere from which the sky appears to be black, the blue colour of the sky as seen from the troposphere being due to the scattering of light by dust particles in the atmosphere.

QUESTIONS ON CHAPTER XI

1. Describe three simple experiments to illustrate that the atmosphere exerts a pressure. How can you show that the pressure is exerted in all directions?

2. Describe the setting up of a simple barometer and also describe an experiment to show that the mercury column in the barometer is supported by the pressure of the atmosphere.

3. How would the readings of a barometer be affected by (*a*) unevenness in the bore of the tube, (*b*) the tube being tilted, (*c*) a few air bubbles having been left in the tube when the barometer was constructed? Give reasons for your answers.

4. When the barometric height is 75 cm., what is the value of the pressure of the atmosphere in dynes per sq. cm.?

What would be the corresponding height of a glycerine barometer? The density of glycerine is 1·3 grm. per c.cm.

5. Fig. 112 shows Hare's apparatus for finding the specific gravities of liquids. With one limb dipping into water and the other into the liquid whose specific gravity is to be found, some of the air is sucked out of the tube through the rubber tube at the top which is then closed by a clip. This results in columns of liquid rising in the tubes as shown.

Justify the statement that the column of water of height h_1 and the column of the other liquid of height h_2 have equal pressures.

If h_1 is 26·5 cm. and h_2 is 24·2 cm., what is the density of the liquid?

6. Fig. 113 shows an automatic flushing tank. Describe and explain how it acts.

7. Describe simple experiments to show that the volume of a mass of gas can be changed by changing the pressure on it.

8. State Boyle's Law, and describe a method of verifying it.

If a barometer were not available, how could you determine the barometric height by means of a Boyle's Law apparatus?

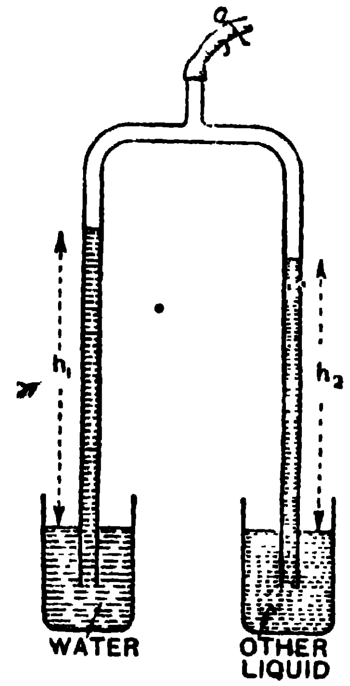

FIG. 112. HARE'S APPARATUS.

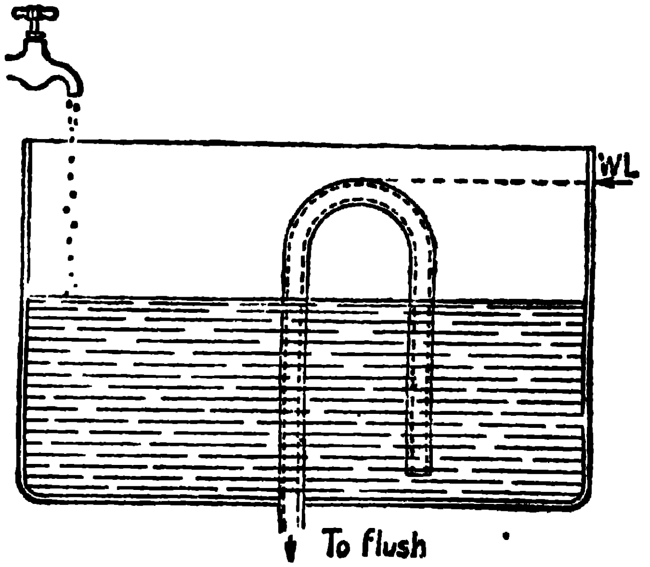

FIG. 113. WHEN WATER REACHES WL, SIPHON IS COMPLETED AND THE WHOLE TANK EMPTIES.

9. Calculate the numbers to fill the spaces in the following table, assuming that there is no change of temperature in any case:—

OLD PRESSURE	OLD VOLUME	NEW PRESSURE	NEW VOLUME
76 cm. of mercury	250 c.cm.	80 cm. of mercury	c.cm.
15 lb. per sq. in.	12 cub. ft.	lb. per sq. in.	30 cub. ft.
80 cm. of mercury	325 c.cm.	cm. of mercury	275 c.cm.
18 lb. per sq. in.	x c.cm.	lb. per sq. in.	$2x$ c.cm.
75 cm. of mercury	700 c.cm.	76 cm. of mercury	c.cm.

10. Instructions are given that the tyres on a certain motor car should be inflated until the pressure inside them is 24 lb. per sq. in. If the capacity of the tyre is 700 cub. in., what volume of air at atmospheric pressure, 15 lb. per sq. in., must be pumped into it when it is empty?

11. A mass of oxygen which will occupy a volume of 10 cub. ft. at atmospheric pressure, 15 lb. per sq. in., is stored under pressure in a cylinder with a capacity of 180 cub. in. What is the pressure inside the cylinder?

12. A quantity of gas, collected in a tube which is inverted over a bowl of mercury, measures 25 c.cm. The mercury stands 8 cm. higher in the tube than in the bowl, and the barometer stands at 76 cm. What will be the volume of the gas if the tube is pushed down into the bowl until the mercury inside it is at the same level as that in the bowl?

13. Describe with the aid of a diagram the principle of the construction of an aneroid barometer.

Explain why it can be used to measure altitude. [L.U.

14. Describe and explain the action of a pressure gauge suitable for the measurement of the pressure of (*a*) the domestic gas supply, (*b*) the steam in a boiler. [L.U.

15. Describe the action of the suction pump and of the force pump. Indicate whether there is any limit to the height to which water can be raised by these pumps, giving reasons. [L.U.

16. State Boyle's Law.

Describe any one experiment you have seen, which shows that the pressure of a liquid increases with depth.

An empty gas-jar is inverted and pushed mouth downwards into water. Explain what would happen if the jar were immersed to a depth of 34 ft. below the surface. [L.U.

17. Describe a method of measuring the pressure of the domestic gas supply, and give a diagram of the apparatus used.

If the gas pressure is read as 4 in. of water, what is the pressure *exerted by the gas* in lb. per sq. in., if the atmospheric pressure is 14.83 lb. per sq. in.? [1 cub. ft. of water weighs 62·5 lb.] [J.M.B.

18. One arm of a U-tube, which is half-filled with a liquid A, is connected to a partially exhausted vessel so that there is a difference between the levels of A in the two arms. On what does this difference depend?

When a similar tube containing a liquid B is also connected to the vessel, the difference between the levels is 30 cm. for A and 36 cm. for B. What is the specific gravity of B if the liquid A is water?

How would you confirm the result of your calculation? [J.M.B.

19. Describe a mercury barometer, illustrating your answer with a clear diagram. Give briefly the principles on which its action depends.

Show that there is a difference of a little more than 2 cm. in the barometric height at sea-level and at an altitude of 250 m. What would be the difference in the levels of water barometers under similar conditions? [Take weight of 1 litre of air = 1·2 grm., and weight of 1 c.cm. of mercury = 13·6 grm.] [J.M.B.

20. Briefly describe and explain the action of (*a*) a simple mercury barometer, (*b*) an aneroid barometer. Give a diagram of each.

If the height of a barometer is 75·0 cm. of mercury, what is the pressure of the air in grm.-wt. per sq. cm.? Take the density of mercury as 13·6 grm. per c.cm. What would be the corresponding height of a water barometer? [J.M.B.

21. State Boyle's Law and describe in detail how you would verify it for air.

A diving bell of volume 9 cub. m. is lowered into water until the water level inside is 17 ft. below the external level. If no extra air has been pumped into the bell, what is the approximate volume of the air inside? [Approximate height of "water barometer" = 34 ft.] [J.M.B.

22. In an experiment to verify Boyle's Law two sets of results were:

Volume of air when compressed (in c.cm.)	17·6	12·1
Mercury pressure in excess of the atmospheric pressure, compressing the air (in cm.)	10·0	50·0

Calculate the atmospheric pressure at the time of the experiment. [J.M.B.

23. Make a sketch of the lower part of a Fortin barometer and write brief notes in explanation of it.

Will a Fortin barometer give correct readings if the tube is not upright? Give reasons for your answer.

24. Explain, illustrating with a drawing, the action of a simple siphon used for drawing liquid off from a vessel. Discuss the conditions necessary for its working.

25. Describe, giving a sketch, the construction and working of a simple form of pump for withdrawing air from closed vessels.

If the barrel of such a pump has a volume $\frac{1}{10}$ that of the vessel being evacuated, what will be the pressure inside the vessel after the fourth stroke of the pump, the original pressure being 1 atmosphere?

26. Air is pumped into a flask of volume 2 litres by means of a bicycle pump. If the cross-section of the pump is 5 sq. cm. and the length of stroke of the piston 20 cm., what is the pressure inside the flask after 40 strokes of the pump? The original pressure in the flask was equal to that of the atmosphere and the barometer stood at 75 cm.

27. If 6000 litres of gas at atmospheric pressure are pumped into a cylinder having a capacity of 50 litres, what will be the pressure inside the cylinder?

What will be the pressure inside the cylinder when 10 litres of the compressed gas has been withdrawn and what will be the mass of the remaining gas if its density at atmospheric pressure is 0·09 gm. per litre?

28. A simple barometer is set up with the tube standing vertically with its mouth in a deep jar of mercury. The mercury column is 74 cm. high, but when the tube is pushed down into the jar until the space above the mercury in the tube is half its original volume, the column is only 72 cm. high. What is the reason for this and what is the true barometric height?

29. A thread of mercury 5 cm. long encloses a quantity of air in a narrow tube closed at one end. When the tube is placed horizontally, the enclosed air fills 40 cm. of the tube; but when the tube is vertical and open end upwards the enclosed air only fills 37·5 cm. What is the height of the barometer and what length of the tube will the enclosed air fill if the tube is placed vertically with the open end downwards?

30. A sounding tube of length 24 in. is lowered into sea water of specific gravity 1·03 and the water rises 8 in. within it. To what pressure was the air in the tube raised and to what depth did the mouth of the tube sink?

(Barometric height = 76 cm.; specific gravity of mercury = 13·6.)

31. At high tide the sea rises to a height of 20 ft. above the mouth of a cave in which it traps a quantity of air. What would be the barometer reading in the trapped air when the reading in the open was 30 in.? If the volume of the trapped air is 5000 cu. ft., what does it become when the tide falls and releases it?

(Specific gravity of sea water = 1·03; specific gravity of mercury = 13·6.)

CHAPTER XII

ARCHIMEDES' LAW. FLOTATION

Apparent Loss of Weight in Liquid

If you try to lift a large block of stone out of water you will notice that, although it may be easily lifted while it is in the water, it seems to become much heavier as it comes out into the air.

The apparent change of weight may be followed if a large piece of stone is hung from a spring balance and slowly lowered into water. As it enters the water, the reading of the balance will decrease, and this decrease will continue as more and more of the stone is submerged. Once the stone is completely covered by the water the balance reading will remain constant as the stone is lowered more deeply. On raising the stone again reverse changes of the balance readings take place until the stone is completely out of the water, when the original reading will be given once more.

Archimedes' Law states that the apparent loss of weight of a body when immersed in liquid is equal to the weight of the liquid displaced by the body. This may be verified by lowering the piece of stone as above into water in the displacement vessel described on page 15. Note the reading of the spring balance when the stone is in the air and again when the stone is completely submerged, and measure the volume of water displaced by it. Results such as the following will be obtained:—

Wt. of stone in air = 498 grm. ....(1)
Apparent wt. of stone in water = 299 grm. ....(2)
Volume of water displaced = 198 c.cm. ....(3)

From (1) and (2)—

Apparent loss of wt. of stone = 199 grm.

From (3):—Wt. of water displaced = 198 grm.

This gives approximate verification in the case of immersion in water.

If other liquids of known densities are used in the displacement vessel, the weight of liquid displaced may be calculated from the product of its volume and density, or the displaced liquid may be caught and weighed in a previously weighed beaker. Again approximate agreement with the law will be found.

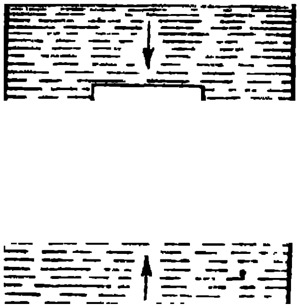

Fig. 114.

The following reasoning shows that this result might be expected. Owing to the difference in depth, the upward pressure on the lower surface of the body B (Fig. 114) will be greater than the downward pressure on its upper surface. Thus there will be a resultant upthrust on the body balancing part of the downward pull of its weight and so reducing the pull of the string on the balance. Now imagine the body to be removed and the space occupied by it to be filled with the liquid. There will still be the same upward and downward pressures on this space, and hence the same resultant upthrust on it as before. But the liquid in the space will not be in motion, so the upthrust on it must be balanced by its weight. Therefore the upthrust experienced by the body B must be equal to the weight of liquid which would fill the same space.

Relative Densities by Archimedes' Law

(1). Solids which Sink.—Weigh a piece of the solid in air and then weigh it again completely submerged in water. For bodies with volumes of 100 c.cm. and upwards spring balance weighing will be sufficiently accurate. Smaller bodies may be weighed suspended from the hook above the pan of a physical balance. A bridge may be placed over the pan and a beaker of water stood on it

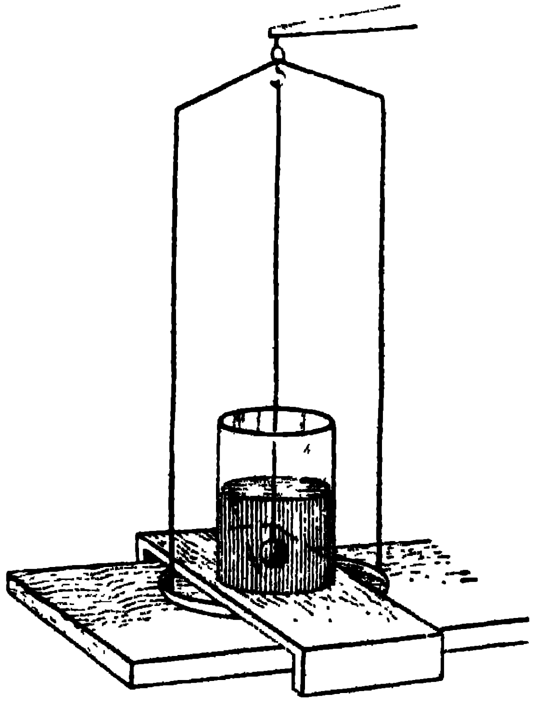

Fig. 115.

for weighing the body in water, as in Fig. 115. The results are dealt with as follows:—

Wt. of piece of iron in air = 99·2 grm.
Apparent wt. in water = 85·8 grm.;
∴ Apparent loss of wt. in water = 13·4 grm.;
∴ Wt. of water displaced = 13·4 grm.

But the water displaced has the same volume as the piece of iron;

$$\therefore \text{ Relative density of iron} = \frac{\text{Wt. of piece of iron}}{\text{Wt. of equal vol. of water}}$$

$$= \frac{\text{Wt. of iron}}{\text{Wt. of water displaced}}$$

$$= \frac{99{\cdot}2}{13{\cdot}4} = \underline{7{\cdot}4.}$$

(2) Liquids.—Weigh a piece of solid in air and water as above, and then in the liquid whose relative density is to be found.

Example of results—

Wt. of solid in air = 99·2 grm.
Apparent wt. in water = 85·8 grm.
Apparent wt. in turpentine = 87·4 grm.;
∴ Apparent loss of wt. in water = 13·4 grm.,
and Apparent loss of wt. in turpentine = 11·8 grm.,
or Wt. of water displaced = 13·4 grm.,
and Wt. of turpentine displaced = 11·8 grm.

But the volumes of water and turpentine displaced are equal;

$$\therefore \text{ Relative density of turpentine} = \frac{\text{Wt. of turpentine displaced}}{\text{Wt. of water displaced}}$$

$$= \frac{11{\cdot}8}{13{\cdot}4} = \underline{{\cdot}88.}$$

Flotation

The experiment on page 137 shows that a certain upthrust is exerted on a body as soon as any part of it has been immersed, and that this increases as more of the body enters the liquid. If, before the whole of the body is submerged, the upthrust is equal to its weight, it will be completely supported, and its own weight will not sink it any further *i.e.* it will float.

At all stages of the immersion the upthrust will be equal to the weight of liquid which has been displaced. Thus the condition for a body to float in a given liquid is that it can displace its own weight of liquid before it is completely submerged. It follows from this that **the weight of liquid displaced by a floating body is equal to the weight of the body.**

The last statement may be verified by using the displacement vessel again. Load a fairly large tin, such as a cocoa tin, with lead shot or sand so that it will float upright. Weigh the tin and its contents and gently lower it into the water in the displacement vessel until it floats. Measure the displaced water and calculate its weight. Compare this with the weight of the floating body. Repeat a number of times, varying the weight of the body.

From the law of flotation given above it will follow that, for a solid to float in a given liquid, it must be less dense than the liquid, for the weight of liquid displaced equals the weight of the solid, but its volume is less than the volume of the solid. If such a body is thrust right under the liquid it will shoot upwards when released for the weight of liquid displaced, and therefore the upthrust on the body will be greater than the weight of the body. When it breaks the surface, the amount of liquid displaced becomes less, so the upthrust decreases. When sufficient of the body has risen above the water surface for the weight of water displaced to be just equal to that of the body, the latter will be in equilibrium.

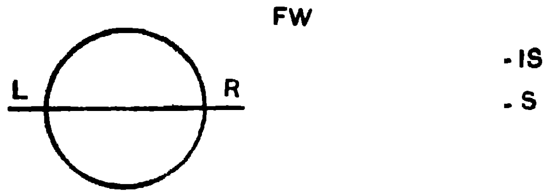

FIG. 116.

Ships

Although iron and steel in solid lumps will not float in water, an iron ship will float. This is because the large space inside it enables it to displace a volume of water much greater than the actual volume of iron used in its construction, and so to displace its own weight of water before it is completely submerged. The "Queen Mary" is said to have a displacement of 73,000 tons. This means that, when loaded until she has sunk as far in the water as it is considered safe for her to do, she displaces 73,000 tons of water, and so the total weight of the ship and her permissible load is 73,000 tons.

The Plimsoll Line

You may have noticed a mark like that shown in Fig. 116 on the side of a ship in harbour. It is known as the Plimsoll line. Years ago ships were frequently overloaded so that they were in great danger of being swamped if they met with rough weather. Owing to the agitation of Captain Samuel Plimsoll, a law was passed requiring each ship to bear a mark showing the depth to which it might be loaded. The line marked LR across the circle gives this depth for sea-water of normal density.

Ships are often loaded while floating in fresh water in river ports. Sea water is more dense than fresh water, hence when the ship passes from the latter to the former, it will not need to displace so large a volume of water in order to displace its own weight, and so will float less deeply. Hence in fresh water it may be loaded to the FW mark, and it will rise till only submerged to the LR line when it passes into the sea. This is the loading generally allowed in summer, but the ship may be loaded down to IS in Indian seas in summer and only to W for most seas in the winter. WNA refers to winter in the North Atlantic.

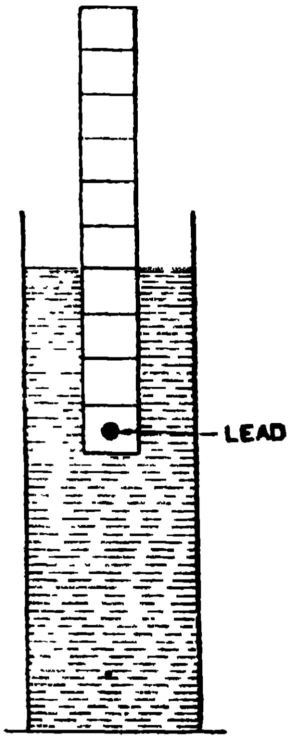

FIG. 117.

Relative Densities by Flotation

Relative densities of liquids may readily be determined by the use of a uniform wooden rod, loaded at one end with lead so that it will float upright, and marked off in equal lengths as indicated in Fig. 117. These lines mark off equal units of volume along the rod. The rod is floated, first in water and then in the liquid whose relative density is required, the mark to which it sinks being noted in each case.

Suppose it sinks to the sixth mark in water and to the fourth mark in the liquid. Then it displaces 6 volumes of water or 4 volumes of the liquid when it floats. But, when floating, it always displaces its own weight of liquid;

$\therefore$ 4 vols. of liquid weigh as much as 6 vols. of water;

$\because$ 1 vol. of liquid weighs as much as $\frac{6}{4}$ vols of water,

i.e. $\frac{6}{4}$ times as much as 1 vol. of water;

$\therefore$ Relative density of liquid $= \frac{5}{4} = 1{\cdot}25$.

Hydrometers

Instruments, such as the rod mentioned above, which are designed for measuring specific gravities by flotation are called **hydrometers.**

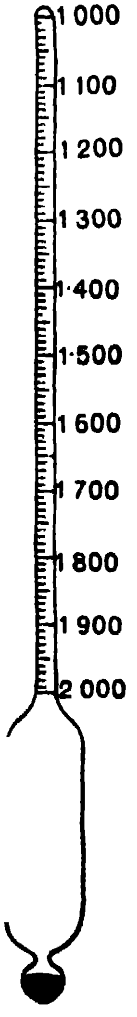

FIG. 118.

The usual commercial form of hydrometer is a sealed glass vessel of the form shown in Fig. 118. The small bulb at the bottom is loaded with mercury or lead shot to ensure its floating upright. The narrow stem carries a scale from which the specific gravity of a liquid in which it is floating may be read directly. The narrowness of the stem allows these graduations to be widely spaced since a considerable length of the stem must be immersed to make much difference to the volume of liquid displaced.

To avoid inconveniently long instruments, hydrometers are usually made in pairs. The one illustrated has been loaded so that it sinks nearly to the top of the stem in water. Thus it would float at a mark on the scale in a liquid denser than water but would sink beyond the scale in liquids less dense than water. The companion instrument, for liquids of the latter type, would be less heavily loaded, so that in water it only sinks to a point just above the bulb where the 1·000 graduation would be placed. In less dense liquids it would sink more deeply, so the graduations would read upwards 0·900, 0·800, etc. Such hydrometers are frequently used for testing the specific gravity of acids, petrol, oils, etc.

For testing milk, which should have a specific gravity between 1·029 and 1·033, a special small hydrometer, called a *lactometer*, is used. The graduations are marked as *"degrees" from 15° to 45°, and indicate specific gravities ranging from 1·015 to 1·045.*

Commercial hydrometers do not give results as accurate as the *experimental* determinations but they are convenient in practice.

Submarines, Balloons, and Airships

A submarine, like any other ship, displaces its own weight of water when floating on the surface. To make it dive, its weight must be increased to more than the weight of the water it will displace when totally covered. This is accomplished by allowing water to run into tanks inside it. When it is to rise, the water is pumped out of the tanks, and when the total weight is less than that of the water displaced, the upthrust on it will be sufficient to bring it to the surface. The same principle has been used in raising sunken ships. Divers make parts of them watertight. Pipes are then connected to these compartments into which air is pumped, displacing the water they contain.

Because gases, as well as liquids, exert pressure, Archimedes' Law will also apply to bodies in the air. Thus a body suspended in air has an apparent weight less than its true weight in vacuum by the weight of air it displaces, for it experiences an up-thrust equal to the weight of this air displaced. Hence, if a body weighs less than the air it displaces, it will float in air. Balloons and airships are filled with hydrogen, which has only about one-fourteenth the density of air, or with helium, which is about twice as dense as hydrogen, in order to secure this. Clearly, helium has less "lifting power" than hydrogen, but it has the advantage of not being inflammable, whilst hydrogen burns very readily.

An interesting illustration of floating on gas can be carried out by filling a trough with carbon dioxide, a dense invisible gas. Soap bubbles are blown and allowed to fall into the trough. On reaching the carbon dioxide they bounce off from its invisible surface, since the air with which they are filled is less dense than the carbon dioxide, and so they cannot sink into that gas.

QUESTIONS ON CHAPTER XII

1. State Archimedes' Law, and describe an experiment to verify it.

2. Describe how, using a spring balance and no measuring instrument, you could find (*a*) the volume, (*b*) the density of a brick.

3. A piece of iron weighed 155 grm. in air and 133 grm. in water. What is its volume and the specific gravity of the iron? What would the same piece of iron appear to weigh in alcohol of density 0·8 grm. per c.cm.?

4. Briefly explain why a dense body appears to weigh less in liquid than in air, and why the apparent loss of weight is equal to the weight of liquid displaced.

A thread which will break when the pull on it exceeds 20 lb.-wt., is attached to a piece of iron weighing 21 lb. which is under water. What fraction of the volume of the piece of iron must remain under water in order that the thread may just be able to support it? [Specific gravity of iron = 7·2.]

5. State the Law of Flotation and explain how it follows from Archimedes' Law. Also describe an experiment to verify it.

6. Explain (*a*) why a lump of iron sinks in water, but a ship made of iron plates will float; (*b*) why an egg will sink in pure water but will float in a strong solution of salt; (*c*) why toy balloons, if filled with hydrogen, will rise to the ceiling, but if filled with carbon dioxide will sink to the floor; (*d*) why in the Plimsoll mark there are different lines for loading in sea water and in fresh water.

7. Describe, giving a diagram, a common hydrometer for measuring specific gravities of liquids less dense than water, and explain the principles on which it is based.

8. A uniform rod of wood, 2 cm. square and 20 cm. long, has 1 c.cm. of lead of density 11·4 grm. per c.cm. attached to one end. When floated in water, 7·4 cm. of the rod remain above the surface. What is the density of the wood?

If, when floated in another liquid, 11·5 cm. of the rod remain above the surface, what is the density of that liquid?

9. A piece of wood of specific gravity 0·5 has a volume of 200 c.cm. What weight of copper of specific gravity 9 must be attached to it so that it will just be pulled below the surface of the water?

10. A piece of zinc, when weighed on a sensitive balance, appears to weigh less than its true weight if brass weights are used, but more than its true weight if aluminium weights are used. Why is this? [Specific gravities: zinc = 7·2, brass = 8·5, aluminium = 2·6.]

11. The balloon and attachments of an airship weigh 2500 kilog. The balloon will hold 11,000 cub. m. of gas. What load can it carry when filled with hydrogen? Density of hydrogen, 0·09 grm. per litre. Air is 14·3 times as dense as hydrogen.

12. A rectangular log 4 ft. long, 2 ft. wide, and 18 in. high weighs 600 lb. Show that it will float in sea water of density 64 lb. per cub. ft., and find the least weight which must be placed on the top of the log in order to sink it in the sea water. [L.U.

13. State the Principle of Archimedes and show how it is applied to a floating body. Illustrate this by reference to (*a*) a submarine, (*b*) an airship. [L.U.

14. Define the *specific gravity* of a substance.

A cube of wood whose edge is 4 cm. weighs 48 grm. What is its specific gravity? When this cube is floated on glycerine it is found that the upper horizontal face is 1·6 cm. above the surface. Show that the specific gravity of the glycerine is 1·25. [L.U.

15. Define *density* and *specific gravity.*

Sketch and describe a common hydrometer graduated to read specific gravities from 0·8 to 1·0.

Why is the stem of the hydrometer of small diameter? [L.U.

16. A submarine, sailing in fresh water with the top of its tower on a level with the surface of the water, passes into the open sea. Describe, giving reasons for your answer, what will happen to the submarine.

If the volume of the whole submarine is 7000 cub. ft., find what change must be made in the weight of the water in the water compartments in order that it may leave the tower just on a level with the open sea surface. [Weight of 1 cub. ft. of fresh water = 62·5 lb. Specific gravity of sea water = 1·024.] [J.M.B.

17. Explain how the principle of Archimedes may be applied in finding the specific gravity of a liquid.

A piece of steel of specific gravity 7·80 weighs 0·50 lb. in air. What is the pull in the suspending cord when it is immersed in methylated spirit of specific gravity 0·83? [J.M.B.

CHAPTER XIII

SURFACE TENSION. VISCOSITY. DIFFUSION. OSMOSIS

The Skin Effect in Liquids

You have probably noticed that drops of rain on a window pane have curved, almost spherical, free surfaces. If drops of mercury on a clean sheet of glass are observed, very small ones are seen to be spherical, while larger ones tend to have the more flattened shapes shown in Fig. 119. It is instructive to form a large drop of mercury by allowing it to run out slowly from a tube drawn to a fine point on to a sheet of glass. The various stages shown in Fig. 119 will be seen as the drop grows.

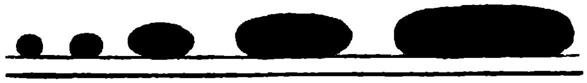

FIG. 119.

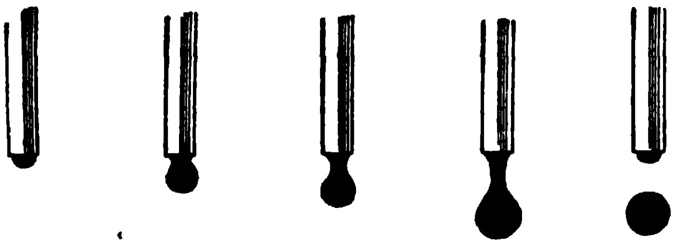

FIG. 120.

If a water tap fitted with a fine jet is turned on slowly so that a drop of water forms slowly on the jet, the stages shown in Fig. 120 will be observed. In the formation of both the water drops and the mercury drops, the stages remind one of the effects which would be observed if the liquid were being run into an elastic skin.

The fact that water behaves as though it had an elastic skin may also be shown by filling a tumbler to the brim and then carefully dropping small nails into it one at a time. It will be found that a considerable number can be dropped in without the water overflowing, its surface becoming heaped up above the brim of the tumbler, as shown in Fig. 121. The same fact may also be shown by floating a needle on the surface of some water. This may be accomplished by resting the needle on a piece of blotting paper floating on the water. As the paper becomes soaked it will sink and leave the needle resting on the water surface. That it is not floating in the ordinary sense, but is supported by the surface, is seen if one end is pushed just below the surface, for then the needle quickly sinks.

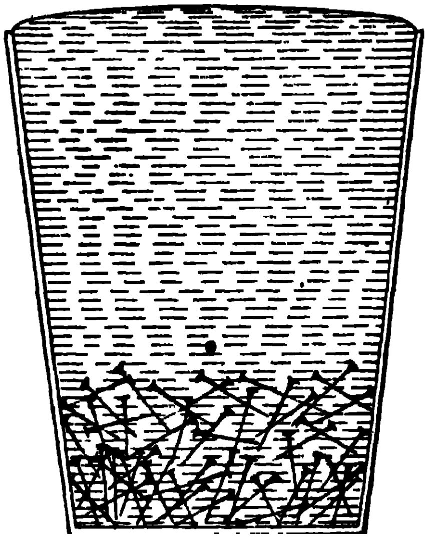

FIG. 121.

Surface Tension

The preceding paragraph has shown that the free surface of a liquid behaves as if it were enclosed in an elastic skin. If we think of a stretched piece of rubber from a toy balloon, we shall realise that there are forces acting along its surface. Thus, if there is a weak place in it a hole will probably be formed, and the hole tends to become circular since its edges are pulled outwards in all directions. Similar forces act along the surface of a liquid, as may be shown by placing a thin layer of water in a flat-bottomed vessel and carefully running a little coloured methylated spirit from a fine tube into the middle of it. The spirit is seen to form a circular patch as shown in Fig. 122. There are also forces along the surface of the spirit, but these are weaker than those of the water surface, so that the spirit is pulled out like the hole in the india-rubber.

The existence of these forces is also shown by those small toy boats which will move about on a saucer of water when small pieces of camphor are fixed to the back of them. The camphor gradually dissolves, and the forces along the surface of the camphor solution are smaller than those of the water surface. Thus the front of the boat experiences a stronger pull than the back, and so it moves forward.

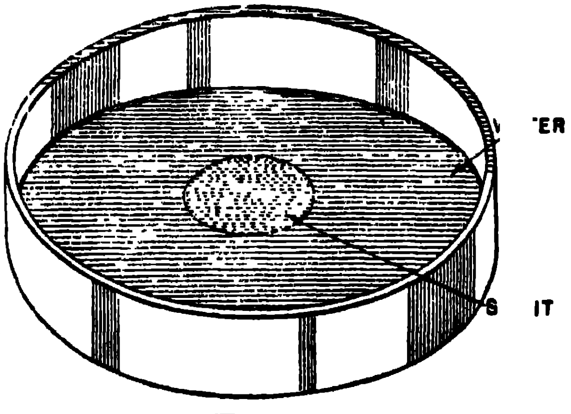

FIG. 122.

The forces acting along the surface of a liquid set up a state of strain in the surface layer which is referred to as **surface tension.** They arise from the attraction which the small particles—molecules—of the liquid have for one another. The particles in the surface layer experience an inward pull from the attraction of the particles below them which tends to make the surface shrink and so results in a state of strain in the surface layer.

Soap bubbles and films may be used to demonstrate the existence of surface tension. If a bubble is blown and left on the pipe when you cease blowing, it will be seen to contract, while a flame at which the stem is pointed will be blown aside.

If a wire ring is dipped into soap solution a film of the solution may be formed across it. When a fine thread is dropped across the film and the film is broken on one side of it, the thread is pulled into the form of an arc of a circle by the tension of the other part of the film (Fig. 123). If a loop is made in the thread and the film broken inside it, the loop will be pulled into a circle (Fig. 124).

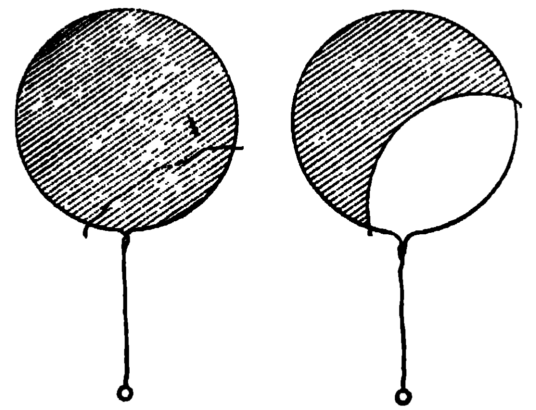

FIG. 123.

All these experiments show that there are forces acting along the surfaces of the films.

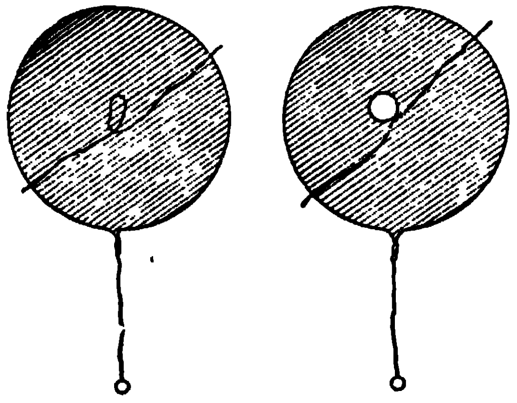

FIG. 124.

The **coefficient of surface tension** of a liquid is defined as the force per centimetre acting across any line in the surface.

The surface tension of a soap film may be measured by means of a wire frame, as shown in Fig. 125. AB is made so that it will slide easily on the sides of the frame. By dipping in soap solution a film is formed over ACDB. The frame is hung vertically, and half-broken match-sticks are hung from AB, as shown, until sufficient weight has been added just to break the film. AB and the match-sticks hung on it are then weighed and the length of AB is measured.

Suppose AB measures 3 cm., and the weight required to break the film is ·42 grm.

Then the film exerts a force of $\frac{\cdot 42}{3}$ grm.-wt. on each centimetre of AB.

But there are two surfaces to the film, so each surface exerts a force of $\frac{\cdot 42}{3 \times 2}$ grm.-wt. on each centimetre of AB;

$\therefore$ The surface tension of the soap solution = ·07 grm.-wt. per cm.

The Wetting of Surfaces.—When water is dropped on to a sheet of glass which has been well cleaned with methylated spirit to remove all grease, it spreads considerably and much of the water remains sticking to the glass when the drops are run off. If a little grease is rubbed on the glass the drops tend to do less spreading and to be almost spherical and may be almost completely run off.

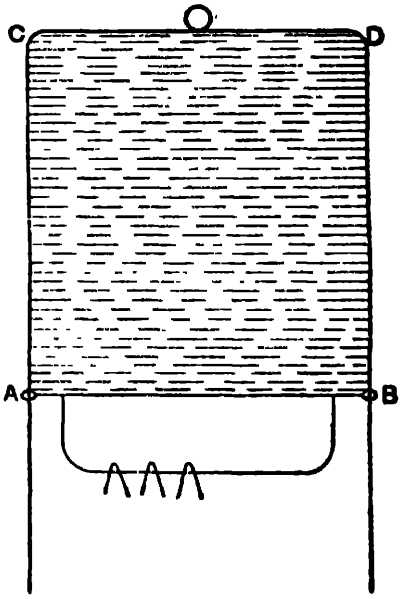

Fig. 125.

The explanation of this is that there is attraction between the particles of water and those of the glass. Thus, while the surface tension of the water tends to make the surface contract and so heap up the water, the attraction of the glass particles pulls the water particles downwards and tends to spread them. Between the water particles and the grease particles there is very little attraction, so that the surface tension of the water is opposed by the weight of the water only, and tends to heap the water into drops. Soapy water will be found to spread much more than pure water since the presence of the soap tends to lower the surface tension of the water. Mercury does not wet glass at all, since there is practically no attraction between glass and mercury particles.

One of the reasons for using soap in washing is that it lowers the surface tension of water. It enables the water to make much closer contact with greasy surfaces so that it can remove the grease and clean those surfaces. For the same reason soap is much used in gardeners' solutions for spraying the leaves of plants. To have the required effects, the solutions should spread thinly over the whole surface of the leaves. The addition of soap lowers the surface tension of the solution and so gives it increased "wetting power."

Materials for rain-proof coats and tents owe their "water-proof" properties to surface tension. The threads have been treated with substances which have little attraction for water particles, so that the threads themselves do not become wet and the surface tension " skins " on the rain drops prevent the water from running through the spaces between the threads. Thus a fabric is obtained which will shoot off the rain but through which air for ventilation can pass.

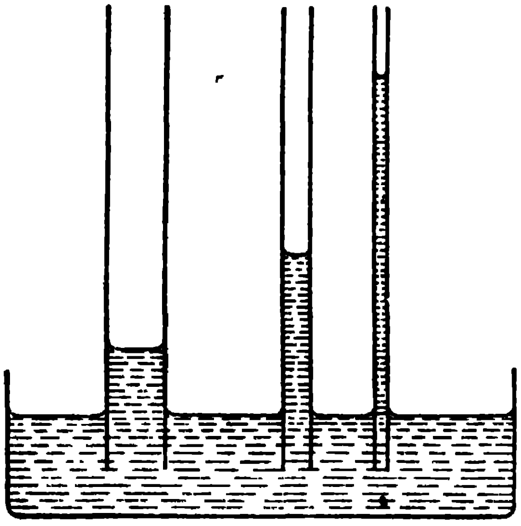
FIG. 126.

If you touch the inside of the fabric of a tent during a rainstorm you will find drops of water coming through the spot you have touched. Your finger is wetted by water and so breaks the " skins " of the drops and allows them to wrap round on the inside of the threads. The water will then soak through between the threads which are already covered with a water film.

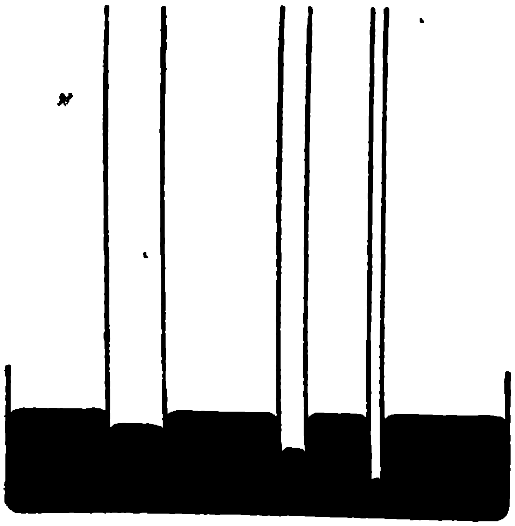
FIG. 127.

CAPILLARITY.—You have probably observed that, when a tube is dipped into a liquid which wets it, the liquid rises some distance in it, and the surface of the liquid is curved, bending downwards towards the middle and forming a meniscus. The height to which the liquid rises is greater in narrow tubes than in wide ones—see Fig. 126 which illustrates glass tubes in water—and, since it is most easily observed in capillary tubes, the phenomenon is called capillarity.

If the liquid does not wet the tube, as in the case of a glass tube in mercury, the liquid is depressed inside the tube and the meniscus bends upwards to the middle, as in Fig. 127.

Capillarity depends on surface tension. If the liquid wets the tube, a film of liquid will form on its inner surface. This will tend to contract owing to its surface tension, and so the surface of the liquid will be drawn up the tube. The film will then creep still further up the inner surface, and by further contraction draw the liquid surface still higher. This will continue until the weight of liquid in the column balances the force with which the surface tension pulls it upwards when the rise will cease. Evidently a longer column will be required in order to have the necessary weight in a narrow tube than in a wide one and so, the narrower the tube, the higher the liquid rises.

In the case of mercury and glass, the mercury particles attract one another more than the glass particles attract them, and so they are pulled away from the glass forming a convex surface. The tendency of this surface to shrink will then depress the liquid. This will continue until a depth is reached where the pressure is such that the upward force it exerts on the surface is equal to the downward force due to the surface tension.

The surface tensions of liquids may be compared by measuring the heights to which they rise in the same capillary tube. When the tube is to be used for a fresh liquid, some of it should be drawn up the tube and allowed to run out several times in order to wash out the film of the previous liquid. The densities of the liquids to be compared must be known. The weight of liquid supported by the surface tension in each case will be equal to height of column × area of cross-section of tube × density.

Thus, in a certain capillary tube water rose to a height of 4·5 cm. A solution of specific gravity 1·1 rose in the same tube to a height of 3·6 cm.

$$\text{Wt. of water supported} = 4{\cdot}5 \times \text{Area of cross-section} \times 1 \text{ grm.}$$

$$\text{Wt. of liquid supported} = 3{\cdot}6 \times \text{Area of cross-section} \times 1{\cdot}1 \text{ grm.};$$

$$\therefore \frac{\text{Wt. of liquid}}{\text{Wt. of water}} = \frac{3{\cdot}6 \times 1{\cdot}1}{4{\cdot}5} \text{ (Since cross-section the same in both cases);}$$

$$\therefore \frac{\text{Force supporting liquid}}{\text{Force supporting water}} = \frac{3{\cdot}6 \times 1{\cdot}1}{4{\cdot}5}.$$

The supporting forces are exerted where the liquid surface meets the glass, *i.e.* around the inner circumference of the tube;

$$\therefore \frac{\text{Surface tension of liquid} \times \text{Inner circumference}}{\text{Surface tension of water} \times \text{Inner circumference}} = \frac{3{\cdot}6 \times 1{\cdot}1}{4{\cdot}5};$$

$$\therefore \frac{\text{Surface tension of liquid}}{\text{Surface tension of water}} = \frac{3{\cdot}6 \times 1{\cdot}1}{4{\cdot}5} = \frac{3{\cdot}96}{4{\cdot}5} = {\cdot}88;$$

i.e. the surface tension of the liquid is ·88 times as great as the surface tension of water.

The rise of oil in lamp wicks and the absorption of ink by blotting paper is due to capillarity which also partly accounts for the rise of sap in plants. It also plays a very important part in causing the movement of water in the soil. When the soil is firm innumerable small capillaries are formed between the soil particles in which the water from the lower soil layers rises to the surface. The object of keeping gardens well hoed in very dry weather is to break up these capillaries and to give a coarser texture to the surface layer, so that the water around the roots of the plants will not be brought up to the surface where it would evaporate into the air.

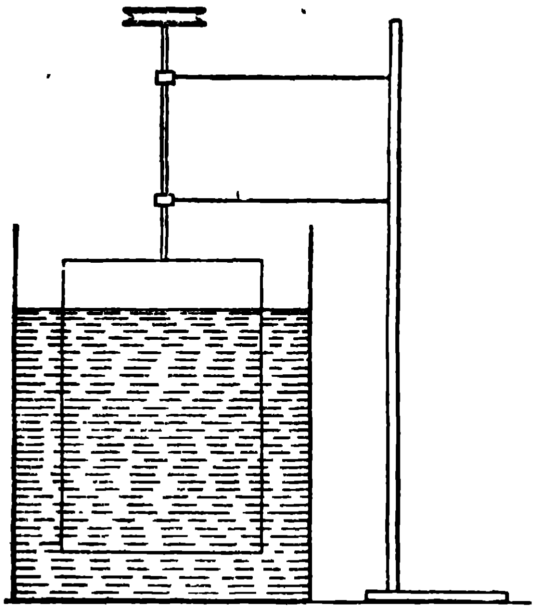

Fig. 128.

Viscosity

It is well known that, while many liquids, such as water, flow very readily, others, such as treacle, flow much more slowly under similar conditions. Liquids of the type that flow readily are said to be **mobile**; those of the treacle type are said to be **viscous.** Viscosity is due to friction in the interior of the liquid. Just as there is friction opposing movement between two solid surfaces when one slides over the other, so there is friction between two liquid surfaces even when they consist of the same liquid, and this internal friction opposes the movement of one layer of liquid over another, and so, when it is great, makes the flow of the liquid very slow.

Even mobile liquids possess a certain amount of viscosity. This may be shown in the case of water by rotating a wire frame, such as that shown in Fig. 128, in a large beaker of water. If it is rotated fairly rapidly, in a short time the whole of the water will be swirling in the

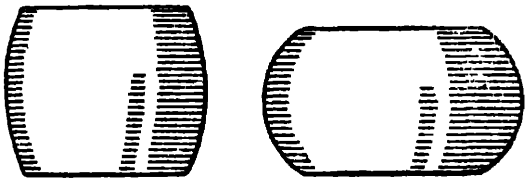

Fig. 129.

same direction as the stirrer. The layer in front of the wire is carried round with it and, owing to viscosity, this drags the next layer with it, and so on, until the motion has been transmitted to the whole of the water.

The viscosity of a liquid rapidly decreases as its temperature rises. Treacle will run off a spoon much more rapidly when it is hot than when it is cold. Similarly, when tar (which is very viscous) is to be run into cracks in the roadway, it is first heated so that it will flow readily.

Some liquids have such high viscosity that they almost have the properties of solids. Thus, the pitch which is frequently used in road-making has a solid appearance, but if a lump of it is left in one position for a long time, it will be found to become flattened as indicated in Fig. 129, showing that it does flow very slowly and so should be regarded as a liquid with very high viscosity.

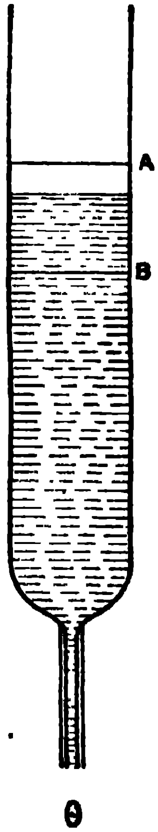

FIG. 130.

Comparison of Viscosities

The viscosities of two liquids may be approximately compared by comparing the rates at which they flow from a tube with a capillary jet (Fig. 130). Two marks, A and B, are made on the tube which is fixed vertically and filled to a point above A with one liquid. The time taken for the liquid surface to fall from A to B is noted. This is repeated with the other liquid. The times refer to the driving of equal volumes of the liquids through the capillary, so it may be taken that time × driving force is proportional to the viscosity, *i.e.*

$$\frac{\text{Viscosity of 1st liquid}}{\text{Viscosity of 2nd liquid}} = \frac{\text{1st time} \times \text{1st driving force}}{\text{2nd time} \times \text{2nd driving force}}.$$

The driving forces are provided by the pressures of the liquid columns. These depend on the heights of the columns and the densities of the liquids. The height varies during each experiment, but for the times noted the average height of the column is the same in both cases. Hence the driving forces are proportional to the densities of the liquids, and

$$\frac{\text{Viscosity of 1st liquid}}{\text{Viscosity of 2nd liquid}} = \frac{\text{1st time} \times \text{Density of 1st liquid}}{\text{2nd time} \times \text{Density of 2nd liquid}}.$$

The above method is not suitable for very viscous oils which would flow too slowly through the capillary. In such cases a simple method of comparison is afforded by the apparatus illustrated in Fig. 131. It consists of a hemispherical steel cup and a steel ball of slightly smaller diameter. Slight projections inside the cup prevent the two surfaces from coming into contact when the ball is in the cup. A drop or [illegible] of oil is placed in the cup and then the ball is pushed in and the w[illegible] apparatus is hung vertically. The oil causes the ball to adhere to the cup for a time and the time which elapses before the ball falls off is proportional to the viscosity of the oil.

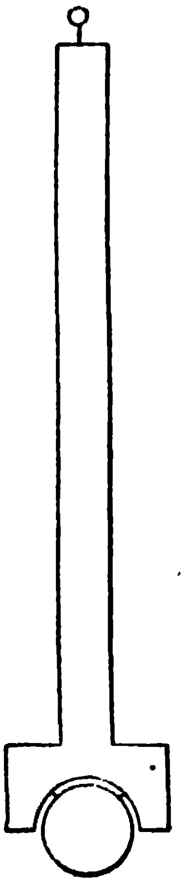

FIG. 131.

Viscosity opposes movement, but the viscosity of a liquid is usually much less than the friction between two solid surfaces, so that an oil with considerable viscosity may be used as a lubricant. Where there is considerable pressure in a bearing it is an advantage to have a viscous lubricant, as otherwise it would be too easily squeezed out of the bearing. Hence, oils for light machinery, such as sewing machines, usually have little viscosity, while those for heavy machinery have considerable viscosity. Since viscosity is greatly reduced by rise of temperature, a more viscous oil is used in motor car engines in summer than in winter.

Diffusion

If some strong copper sulphate solution, which has a deep blue colour, is poured into the bottom of a tall jar and water is then gently run into the jar from a fine jet, the water will float on the solution so that distinct colourless and blue layers will be seen (Fig. 132). If the level to which the blue solution reaches is marked and the jar is left undisturbed for some days it will be found that the blue colour can then be seen at a considerably higher level than before. Clearly, although the liquid as a whole has been at rest, the small particles of copper sulphate dissolved in it have been moving, and this has resulted in some of them moving into the water layer. If the jar is left long enough the copper sulphate will become evenly spread throughout the whole of the liquid. Similarly the particles of a lump of sugar or salt placed at the bottom of a glass of water gradually spread out until they are evenly spread through the water.

A similar experiment may be carried out by floating a layer of methylated spirit on water. In time a uniform mixture of spirit and water will be formed, showing that the particles of liquids, as well as those of substances dissolved in them, are in motion.

The same process may be observed in gases. If two or three drops of bromine are placed at the bottom of a gas jar, they will form a dense brown vapour which will form a distinct layer at the bottom of the jar. If the jar is covered and left for a time the brown colour will be seen to rise until a uniform mixture of bromine and air results.

From these illustrations it is evident that there are movements of the particles of gases, liquids, and dissolved substances even when the substances in bulk are at rest. The process of the mixing of substances arising from these movements is known as **diffusion.**

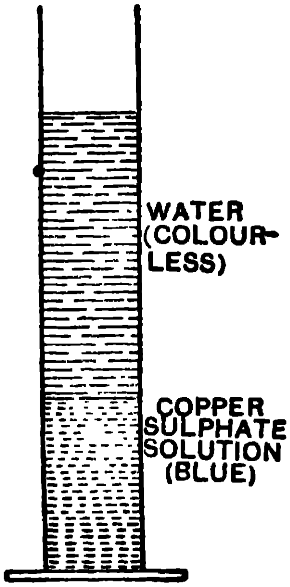

Fig. 132.

Dialysis

A sheet of parchment is tied tightly over the end of an open cylinder and soaked in water for some time (Fig. 133). Into the vessel thus formed a solution containing both salt and starch is poured, and it is hung half submerged in a trough of water. In a short time it will be possible to detect salt in the water in the trough, but no starch will be detected there. To show this, drops of liquid from the inner and outer vessels may be tested with iodine solution which turns starch blue.

The water in the two vessels is continuous since the pores of the soaked parchment are full of water; and the salt, diffusing in the water, passes from one vessel to the other. The starch, however, will not pass through the parchment membrane. Substances which behave like salt in this experiment are called **crystalloids**; those which behave like the starch are called **colloids.** Actually whether a substance in solution behaves like a crystalloid or a colloid depends very largely on

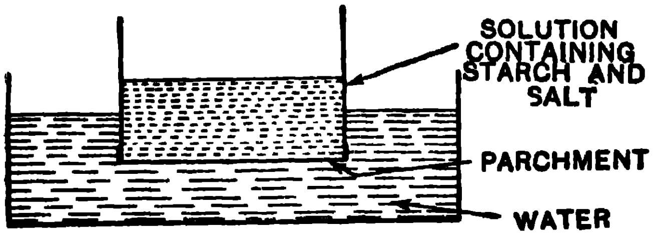

Fig. 133.

the way in which the solution has been made. The difference is due to a difference in the size of the particles in the solution. When dissolved in the ordinary way salt breaks up into much smaller particles than starch, so that the salt particles can diffuse through the pores in the parchment, but the starch particles are too large to do this.

As salt accumulates in the trough some of it will diffuse back into the inner vessel. When the salt concentrations on the two sides of the parchment are equal, equal quantities will diffuse into and out of the inner vessel in a given time, so that the concentration does not undergo further change. If, however, the water in the trough is frequently changed, salt will continue to diffuse out into it and the starch solution may be almost entirely freed from salt. This process is frequently used in chemistry to separate substances when one is in the crystalloid state and the other in the colloidal state, and it is called **dialysis**.

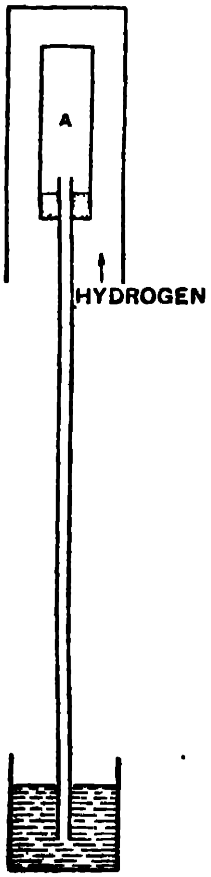

FIG. 134.

Diffusion of Gases

Just as liquids and dissolved solids can diffuse through a parchment membrane, so gases can diffuse through a dry porous plate. If there is gas on both sides of the plate, diffusion will take place in both directions. By placing different gases on the two sides, their rates of diffusion may be compared.

Fig. 134 shows a tube corked into a porous pot and dipping into water in a beaker. A large beaker is inverted over the porous pot. If hydrogen, which is less dense than air, is allowed to float up into the inverted beaker, bubbles of air will be seen to rise through the water from the end of the tube, showing that hydrogen has diffused into A faster than air has diffused out of it, and so set up sufficient pressure to drive air out of the tube.

If the tube is bent so that A dips into a beaker placed mouth upwards, carbon dioxide, which is denser than air, may be poured into the beaker. Water will then be seen to rise in the tube. Air diffuses from A more rapidly than carbon dioxide diffuses into it, and so the pressure in A is reduced below that of the atmosphere.

Experiments such as the above lead to the general conclusion that the less dense a gas is, the more rapidly it diffuses. **Graham's Law.**

due to Thomas Graham who made measurements on the rate of diffusion, states that **the rate at which a gas diffuses is inversely proportional to the square root of its density.** Thus, oxygen is 16 times as dense as hydrogen, so hydrogen will diffuse $\frac{\sqrt{16}}{\sqrt{1}} = 4$ times as fast as oxygen.

Osmosis

When dried raisins or prunes are soaked in water, they swell until their skins, which were originally wrinkled, are quite tight. If they are then placed in a strong solution of salt they shrink and the skins become wrinkled again. The swollen condition can be restored by transferring them to fresh water once more. It is clear that there is a tendency for water to pass through the skins until it sets up a considerable pressure inside the fruit, but that it can be withdrawn by the presence of strong solutions outside the skins. The fruits contain a certain amount of sugar which will form a solution when water enters, so the above observations suggest that the diffusion of water through the skin depends on there being solutions of different concentrations on the two sides of it. The passage of a liquid through a membrane owing to this cause is termed **osmosis.** For osmosis to be noticeable the membrane must be one which allows the solvent but not the solute to diffuse through it. Such a membrane is said to be *semi-permeable* to the solution.

Osmosis may be further studied by means of artificial semi-permeable membranes. Such a membrane can be made by filling a wide test-tube with a thick solution of collodion in ether. Allow it to stand until all air bubbles have risen to the surface. Pour out as much of the solution as possible, leaving a thin layer sticking to the sides of the tube. Blow dry air through the tube to evaporate the ether, and a skin of collodion will be left lining the tube. Fill up with distilled water and place the tube in a vessel of distilled water. The skin may then be removed from the tube.

Fit into the skin a tight-fitting rubber bung carrying a short bent tube and a long straight narrow tube. Pour a strong sugar solution into the skin through the straight tube. The bent tube allows air to escape as the solution runs in, and is closed by means of a clip on a piece of rubber tubing when the skin is quite full. Support the apparatus so prepared vertically with the skin dipping into a beaker of water, as in Fig. 135.

Liquid will be found to rise in the tube and may continue to rise for some days until it forms a column of some 30 or 40 cm. in height, but eventually its height will become constant. It is clear that a considerable pressure, tending to drive water into the solution, exists and that the flow of water ceases when the pressure of the column in the tube is sufficient to counteract the pressure driving the water.

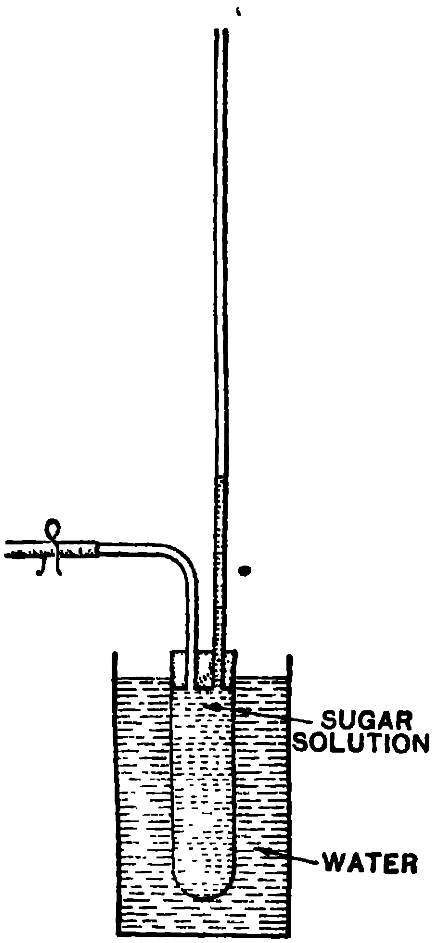

FIG. 135.

If a number of experiments are made, using sugar solutions of different strengths, it will be found that the stronger the solution, the higher the liquid will rise in the tube. If sugar solutions are put into the beaker as well as into the skin, it will be found that water enters the skin, causing the column to rise when the stronger solution is in the skin, but that water leaves the skin and the column falls when the stronger solution is outside the skin.

The results of the above experiments show that, when two solutions of different concentrations are in contact, there is a difference of pressure between them, and if they are separated by a semipermeable membrane, the pressure difference tends to drive solvent from the weaker to the stronger solution. The pressure which can be exerted by a solution in contact with its solvent is called the **osmotic pressure** of the solution, and this pressure may be measured by the height to which the liquid column will rise in experiments such as those mentioned above.

Diffusion and osmosis play very important parts in both plant and animal life. The bodies of plants are built up of cells which have a lining of living matter called *protoplasm*. The interior of the cell is often occupied by *sap*, which is a watery solution containing food substances for the plant. The protoplasm acts as a semi-permeable membrane allowing water to pass through it but preventing the passage of the foodstuffs. Thus osmosis assists the flow of water from one part of a plant to another for, if adjacent cells contain sap at different concentrations, water will pass from the one with lower concentration to that with higher concentration. Soft plant stems are maintained stiff and upright by the pressure of water in their cells causing them to swell out and become rigid. In hot, dry weather much of this water evaporates from the plants, and they tend to droop. When water is supplied to their roots it diffuses into the plant cells owing to the high osmotic pressure of the remaining sap, and makes them rigid once more.

When foodstuffs are to be stored in plants they are usually converted into colloidal, non-diffusible substances such as starch. When they are to be used by the plant chemical changes take place which convert them into diffusible substances, so that they can pass from cell to cell until they reach the part of the plant where they are required.

In the animal body, food is changed by the digestive processes into diffusible substances. At certain places blood vessels with very fine membranes are in close contact with the intestines and the digested substances diffuse into the blood to be carried to various parts of the body by the blood stream.

The air spaces in the lungs are also surrounded by blood vessels with very fine membranes, so that the oxygen from the air breathed in may diffuse into the blood.

QUESTIONS ON CHAPTER XIII

1. What is meant by surface tension? Describe experiments to illustrate its existence.

2. Describe experiments, one in each case, to illustrate the following statements: (*a*) water behaves as if it had a skin; (*b*) the surface tension of water is lowered by dissolving soap in it; (*c*) alcohol has a lower surface tension than water.

3. Explain: (*a*) why small drops of mercury are globular but large drops have a more flattened shape; (*b*) why material through which air can pass may make a rainproof tent; (*c*) why a light sieve will "float" if its wires are waxed.

4. What is meant by capillarity? Briefly explain why water rises in a tube dipped into it and why it rises higher in a narrow tube than in a wide one.

5. How does mercury differ from water with regard to capillarity? Briefly explain the difference. Why should a mercury barometer not have a very narrow tube?

6. When plants in pots have to be left unattended during summer holidays they may be kept watered by placing beside them a pail of water and running threads of wool from the water to the soil. Explain this.

7. What is meant by the viscosity of a liquid? How would you compare the viscosities of water and glycerine?

8. How does the viscosity of a liquid vary with temperature? How would you demonstrate this in the case of a lubricating oil?

9. Describe simple experiments to illustrate the process of diffusion (*a*) of dissolved substances, (*b*) of gases.

10. Describe an experiment to show that carbon dioxide diffuses more slowly than air. Why will carbon dioxide, though considerably denser than air, slowly escape from an uncovered jar with its mouth upwards?

11. State Graham's Law of the diffusion of gases.

If 100 c.cm. of hydrogen escapes from a porous pot in 10 sec., how long would it take 200 c.cm. of carbon dioxide (22 times as dense as hydrogen) to escape from the same pot?

12. Explain, by means of an example, the process known as dialysis.

13. Describe experiments to illustrate what is meant by the osmotic pressure of a solution. How can such a pressure be measured?

14. What connexion is there between the concentration of a solution and its osmotic pressure? How would you demonstrate this connexion? Give some account of the importance of this in connexion with the movement of water in plants.

SECTION II—HEAT

CHAPTER XIV

TEMPERATURE, THERMOMETERS

In the early days of the study of heat it was regarded as a *fluid* which flowed from one body to another. To-day, as will be shown in Chapter XXII., we know that heat is not a material fluid but **a form of energy**. We may, however, still speak of the flow of heat, since energy in the form of heat is frequently transferred from one body to another.

When heat enters a body, several results may follow. (i) Usually the **temperature** of the body rises, that is the body gets hotter, though this is not always the case. (ii) The body may **change its state.** If a solid it may be converted to liquid, and if it is a liquid it may be vapourised. (iii) **Chemical changes** may also result, but in this book we are only concerned with cases where there is no chemical change. (iv) A less obvious, but very important, effect in most cases is that the body **expands**, that is increases in size.

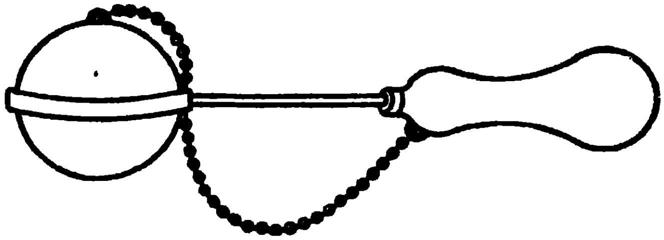

FIG. 136. THE BALL AND RING.

(v) Change of [illegible] properties

Expansion of Solids

The increase in the size of a solid body when it is heated is so small a fraction of its original size that it is not obvious to the eye. Various simple forms of apparatus, however, readily demonstrate that an increase does occur.

Fig. 136 illustrates a metal ring with a metal ball which will just pass through it when both are cold. If the ball is heated in a Bunsen flame and then placed on the ring, it will be found to have become too big to pass through. When it is cold again it will pass through the ring once more.

Fig. 137 shows a metal rod and a metal gauge. The rod will just fit lengthwise in the slot in the side of the gauge, and its ends will just fit into the holes in the gauge. When hot it will not go into the slot nor into the holes, showing that it expands in both length and thickness.

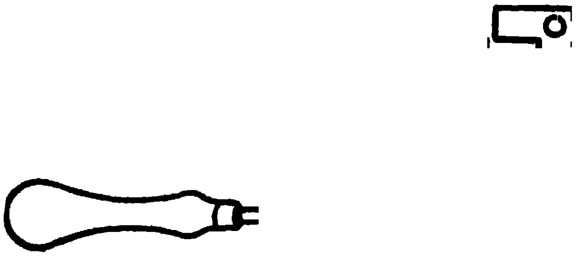

FIG. 137. THE BAR AND GAUGE.

Solids may exert very great forces when they undergo expansion and contraction due to heating and cooling. This is demonstrated by the apparatus shown in Fig. 138. An iron bar is threaded at one end and an internally threaded wheel is screwed on to it. The other end is pierced by a hole through which a pencil of cast-iron is passed. This is fitted into a strong iron frame as shown. By screwing the wheel outwards it is made to press against the projections at A, while the cast-iron pencil presses against those at B. A long gas burner fitted into the base of the frame is lighted to heat the bar. The resulting expansion sets up sufficient force in the bar to break the cast-iron pencil.

While the bar is hot another pencil may be passed through it, and the wheel screwed up until it is pressing against the projections at C and the pencil presses against those at D. On cooling its contraction will snap the pencil.

Equal lengths of different solids undergo unequal expansions when equally heated. This is shown by a compound bar. As shown in Fig. 139, equal strips of copper and iron are rivetted together to form the bar. When it is heated it bends with the copper strip on the outside of the curve. As the outside of a curve is longer than the inside, this shows that the copper strip has expanded more than the iron one.

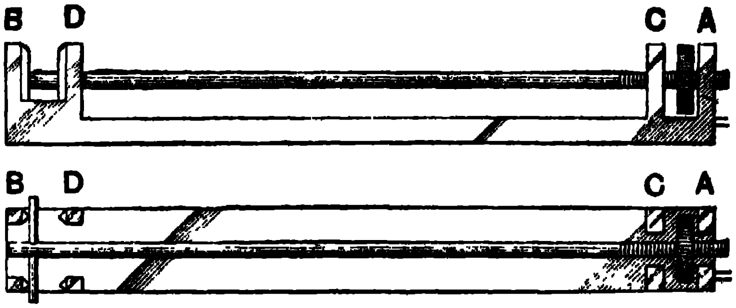

FIG. 138. UPPER FIG.—*Side view:* LOWER—*Plan.*

Expansion of Liquids

Fit a flask with a cork through which a long tube is passed. Fill the flask to the brim with water coloured with ink (Fig. 140). When the cork is replaced some of the water will be forced up the tube. Slip a paper scale on the tube to assist observations of movements of the surface of the water.

Heat the flask by means of a Bunsen burner. At first the water will fall a little in the tube for the flask gets hot and expands, making more room for the water, before the heat reaches the liquid. Soon the liquid will be seen to rise in the tube showing that it does expand when it gets hot, and it will rise beyond the mark from which it first fell. This shows that *the expansion of the water is greater than that of the flask.*

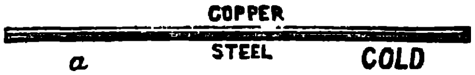

FIG. 139.

If the flame is removed the liquid will continue to rise for a time. The flask cools more quickly than the liquid, and its contraction squeezes more liquid into the tube. Eventually the liquid will fall again, indicating that it does contract as it cools.

As in the case of solids, equal volumes of different liquids expand unequally when equally heated. This can be shown by fitting up two flasks as in the last experiment. Choose equal flasks and equal tubes, so that when filled they will contain equal volumes of liquid. Fill one with water and the other with some other liquid, such as methylated spirit. Fill a large bowl with warm water and stand the flasks side by side in it. Thus both liquids will be equally heated. The rise of the spirit in its tube will be greater than that of the water, showing that the spirit expands more than the water.

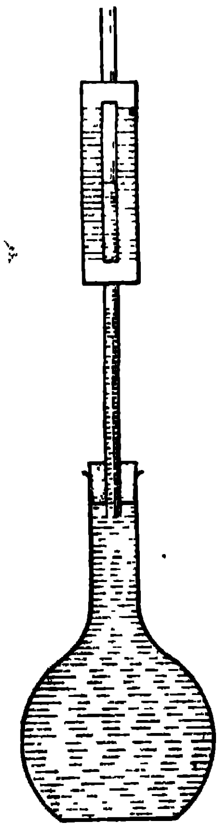

FIG. 140.

Expansion of Gases

Fit a flask with a cork and tube, arranging the tube so that it reaches nearly to the bottom of the flask, and place in it a little coloured liquid (Fig. 141). The upper part of the flask will be full of air. Warm this air by placing the hands round the upper part of the flask. Liquid will be seen to rise in the tube, showing that the air has expanded and displaced some of the liquid from the flask.

Fit up two equal flasks as above, putting equal volumes of liquid in them, so that they have equal gas spaces. Leave the air in one but fill the other with a gas such as hydrogen. Place them together in a bowl of warm water, and it will be found that the liquid rises to the same height in both cases. Thus *for different gases, unlike liquids and solids, equal volumes equally heated undergo equal expansions.*

Expansion in Everyday Life

In some instances the effect of the expansion of bodies when heated may be troublesome, and precautions have to be taken against these effects. In other cases use may be made of expansion. A few cases of each type are noted below. Other examples are given in the chapters where expansion is dealt with more fully.

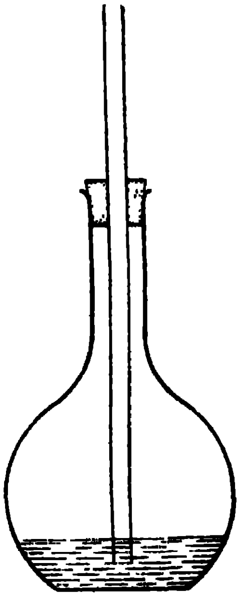

Fig. 141.

(*a*) Railway lines are subject to changes of temperature as the air temperature changes. Although the expansion of a single rail in changing from winter to summer temperatures would be very small, the total expansion over miles of rails might amount to several feet, and if the rails were already tightly end to end, they would buckle under the force set up by the expansion. Hence they are laid with small spaces between consecutive rails to allow for expansion.

(*b*) In a similar way, if long iron bridges were firmly fixed into masonry at both ends, they would tend to buckle or damage the masonry when the temperature rose in summer. To avoid this one end is usually rested on rollers and a space left in the masonry to allow for expansion.

(*c*) Long wires, such as telephone wires or wire fences, will contract considerably when the temperature falls in winter. If they were already tightly stretched between their supports the strain might be sufficient to snap them. Hence, if they are put up in hot weather, they should be allowed to sag a little.

(*d*) When a stone gets amongst the coal on the fire, pieces tend to fly off from it as it gets hot. This is due to the outer layers expanding before the inner part becomes hot, so that they break away. The same process occurs in Nature, particularly in regions where there are great differences between day and night temperature, and tends to break up surface rocks, so assisting in the formation of soil.

In a similar way thick glass tumblers often crack when very hot liquids are poured in them. The inner layers tend to expand before the outer layers, and the latter are burst by the resulting pressure. Laboratory vessels which have to be heated are usually made of thin glass because of this.

In the case of both the stone and the glass the effects are partly due to the fact that they are *bad conductors* (see Chapter XVIII.), that is, heat does not readily pass through them.

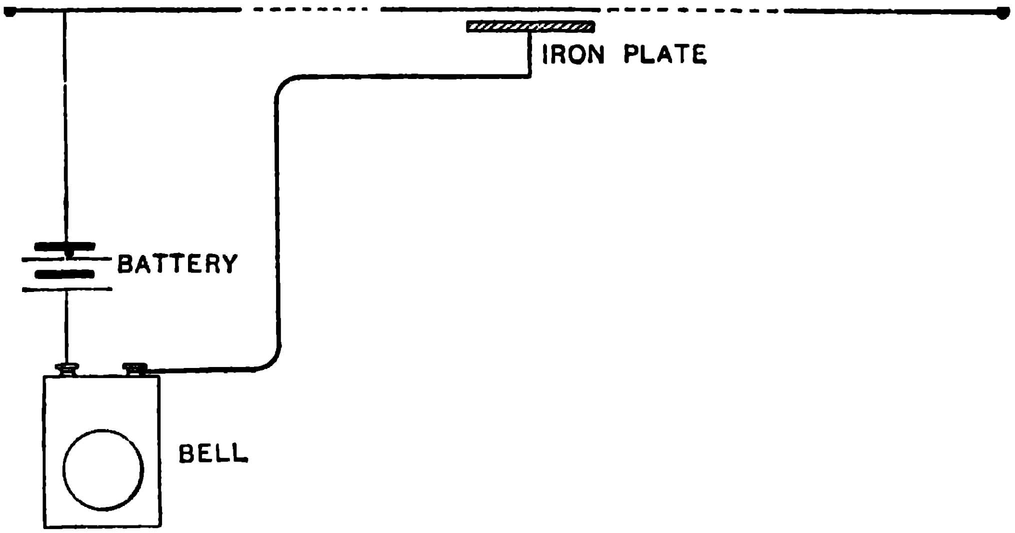

Fig. 142

Use may be made of this effect when a glass stopper has become tightly fixed in a bottle neck. If a cloth which has been dipped in hot water is wrapped round the neck, the stopper is loosened by the expansion of the neck.

(*e*) When making a cart-wheel, the wheelwright makes the iron rim slightly too small to slip on the wheel. It is expanded by heating to get it on, and on cooling it contracts and presses all the parts of the wheel firmly together.

(*f*) Expansion may be utilised in the construction of automatic fire alarms. In one system a long wire is stretched across the room and a metal plate is supported just below it. Electrical connexions are made as shown in Fig. 142. The gap between the wire and the plate is so adjusted that, at a temperature somewhat higher than that which may reasonably be expected in the room, the sagging of the wire due to its expansion will bring it into contact with the plate, completing the electrical circuit and setting the bell ringing.

Alternatively, a compound bar may be used as shown in Fig. 143. Note that the more expansible metal of the compound bar must be on the outside to cause bending in the right direction.

Temperature and Heat

Temperature has been spoken of as rising or falling when bodies become hotter or cooler, so it may be said that *the temperature of a body is its "degree of hotness."* This must not be confused with the quantity of heat in a body. A cupful of boiling water has the same temperature as a kettleful of boiling water, but more heat would have to be given to the latter quantity than to the former to bring it to the point of boiling. A definite experiment may be made on this point by making a small iron nail red hot and an iron pound weight so warm that it can only just be held in the hand. The nail will then be at a considerably higher temperature than the pound weight. They are then dropped separately into equal quantities of cold water. That into which the weight is dropped will be warmed much more than that which receives the nail. Thus the weight contained more heat than the nail although it was at the lower temperature.

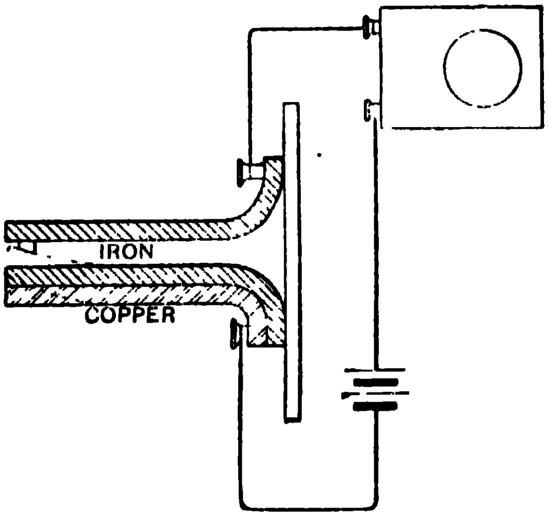

Fig. 143.

A further important idea concerning temperature may be obtained by considering what would happen if the red hot nail were placed on the warm iron weight. The nail would become cooler and the weight a little warmer, that is, heat would flow from the body at high temperature to the one at low temperature in spite of the fact that the latter already contained the greater quantity of heat. Because of facts such as these, temperature is often defined as **the condition which determines in which direction heat will flow if two bodies are placed in contact.**

Measurement of Temperature. Thermometers

Rough ideas of differences in temperature may be obtained from our sense of touch, but it is evident that accurate comparisons when temperature differences are small could not be made in that way. Instruments for measuring temperatures are called thermometers. Common thermometers use the expansion of a liquid to show changes of temperatures, and are based on the principle of the apparatus described on page 163. If a thermometer is examined it will be found to consist of a small bulb on the end of a capillary tube. The bulb and part of the tube are filled with liquid, and the upper end of the tube is sealed so that the liquid will neither spill nor evaporate from the tube. In order that the liquid may expand freely into the upper part of the tube, air has been removed from it so that it is a vacuum. A scale of degrees is marked on the stem to indicate temperatures corresponding to various levels of the liquid in the tube.

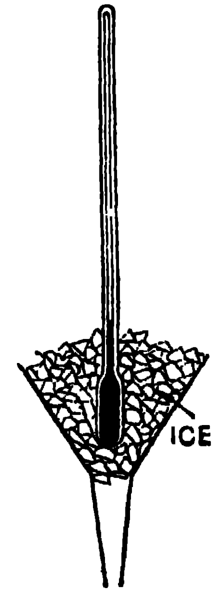

Fig. 144.

(1) Thermometer Scales.—To obtain a standard scale on a thermometer two " fixed points " must first be marked on it. By a fixed point is meant a temperature that will always be the same under given conditions so that it may be readily reproduced to ensure that the " fixed point " mark means the same temperature on all thermometers.

Convenient " fixed points " are *the temperature at which ice melts* and *the temperature of steam from boiling water.* These do change a little under changing pressures (see Chapter XX.), but vary by only very small amounts under ordinary atmospheric conditions.

(*a*) *The lower fixed point.* The unmarked thermometer is stood with its bulb inside a funnel into which small chips of ice are packed (Fig. 144). Using a funnel allows the water formed from the melting ice to drain away, and ensures that ice is in contact with the bulb all the time. Ice should be packed round the stem until the liquid thread is only just visible above it. This ensures that all the liquid is brought to the temperature of the ice. The mark should not be made until the liquid surface becomes stationary, showing that it has fallen to the temperature of the ice.

(*b*) *The upper fixed point.* The apparatus shown in Fig. 145 and known as a **hypsometer** may be used for marking this. It is made of metal, and consists of a boiler, in which the water is placed, surmounted by a double cylinder. The steam from the water must rise up the inner cylinder and pass down the outer one to escape. The outer layer of steam protects the inner layer, in which the thermometer is placed, from the cooling effect of the surrounding air. As the boiling point of water is more affected by pressure changes than the melting point of ice, a manometer is connected with the inner cylinder. The rate at which steam is produced should be adjusted until the mercury stands at the same level in both arms of the manometer, thus ensuring that the pressure of the steam is equal to that of the atmosphere. The bulb should not touch the water but should be completely surrounded by steam. At the same time the thermometer should be inserted far enough into the hypsometer for the liquid to be only just visible above the cork. The mark should not be made until the liquid column is stationary.

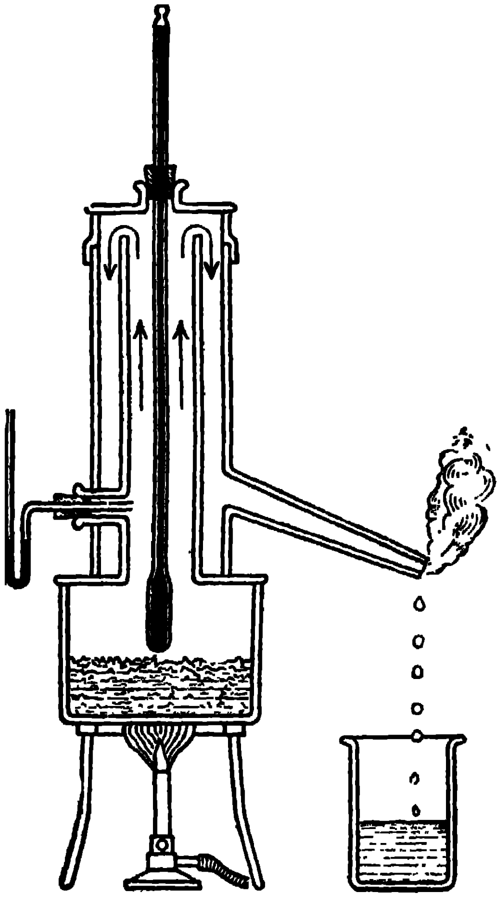

FIG. 145.

(2) FAHRENHEIT AND CENTIGRADE SCALES.—In order that different thermometers shall give the same readings when at the same temperature, the space between the *melting point* and *boiling point* should be divided into the same number of degrees on each, and the fixed points themselves should be similarly numbered on each. There are two standard ways of numbering thermometer scales. For domestic and

industrial purposes in England thermometers marked with the *Fahrenheit scale* are frequently used. On the continent of Europe the *Centigrade scale* is generally used, as it also is by all scientists.

For the Fahrenheit scale the melting point is marked 32° and the boiling point 212°, the space between being divided into 180 parts for single degrees. The corresponding numbers on the Centigrade scale are 0°, 100°, and 100 parts. In each case the scale may be extended above or below the fixed points by continuing to mark degrees equal to those between the points.

Conversion of Temperatures

After taking a reading with one type of thermometer it is sometimes necessary to calculate the temperature that would be registered by the other type. Fig. 146 shows a comparison between the two scales. It will be seen that, if the space between boiling point and freezing point is divided into 20 equal parts each of those parts will contain 9 Fahrenheit degrees and 5 Centigrade degrees;

$$\therefore\ \textbf{9 Fahr. degs.} = \textbf{5 Cent. degs.}$$

$$\text{or } 1 \text{ Fahr. deg.} = \tfrac{5}{9} \text{ Cent. deg.}$$

$$\text{or } \tfrac{9}{5} \text{ Fahr. degs.} = 1 \text{ Cent. deg.}$$

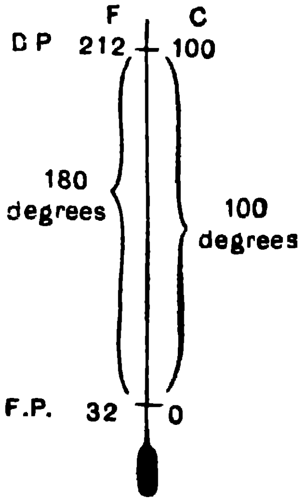

FIG. 146.

Conversion calculations are complicated by the fact that the zero is not at the same level on both scales, but are easy if they are always based on ***distance above or below Freezing Point.***

EXAMPLES.—(1) *Convert* 40° *C. to the Fahrenheit scale.*

$$\text{Distance above F.P.} = 40 \text{ Cent. degs.}$$

$$= \frac{40 \times 9}{5} \text{ Fahr. degs.} = 72 \text{ Fahr. degs.};$$

$$\therefore \text{ Required temperature} = (32 + 72)^\circ \text{ F.} = 104^\circ \text{ F.}$$

(2) *Convert* − 10° *C. to the Fahrenheit scale.*

$$\text{Distance } \textit{below} \text{ F.P.} = 10 \text{ Cent. degs.}$$

$$= \frac{10 \times 9}{5} \text{ Fahr. degs.} = 18 \text{ Fahr. degs.};$$

$$\therefore \textbf{ Required temperature} = \textbf{(32} - \textbf{18)}^\circ \textbf{ F.} = \textbf{14}^\circ \textbf{ F.}$$

(3) *Convert 68° F. to the Centigrade scale.*

Distance above F.P. = (68 − 32) Fahr. degs.

$$= \frac{36 \times 5}{9} \text{ Cent. degs.} = 20 \text{ Cent. degs.};$$

$$\therefore \text{ Required temperature} = 20^\circ \text{ C.}$$

(4) *Convert − 4° F. to the Centigrade scale.*

Distance below F.P. = (32 + 4) Fahr. degs.

$$= \frac{36 \times 5}{9} \text{ Cent. degs} = 20 \text{ Cent. degs.};$$

$$\therefore \text{ Required temperature} = -20^\circ \text{ C.}$$

RÉAUMUR SCALE.—In some parts of Eastern Europe a thermometer scale known as the Réaumur scale is used. It has freezing point marked 0° and boiling point marked 80°. If compared with the other scales by diagrams similar to Fig. 146 it will be found that 4 R. degs. = 5 Cent. degs. or 9 Fahr. degs.

Filling a Thermometer

The very fine bore of a thermometer tube prevents liquid being poured into it as there is not room for the liquid to pass the air in the tube. A piece of wider tube is attached to the top of the thermometer and the liquid is poured into it (Fig. 147). The bulb is then warmed. This expands the air inside so that some of it is forced to bubble out through the liquid. On cooling the bulb again, the remaining air contracts and the pressure of the atmosphere forces some of the liquid down the tube and into the bulb. The process may be repeated so that more air is driven out and more liquid enters, but the whole of the air cannot be removed in this way. When the bulb is about half-full, the liquid in it is boiled so that some turns into vapour which fills the whole tube, sweeping out the remaining air. On cooling again, this vapour will condense, and since no air remains, liquid will flow in to fill the tube completely. The thermometer is then placed in a vessel of liquid with a high boiling point which is heated until it is a little above the highest temperature the thermometer is likely to be used to measure. While the thermometer is at this temperature, a flame is directed against the upper part of the tube to seal it. The liquid which remains in the tube will contract, leaving a vacuum above it, on cooling.

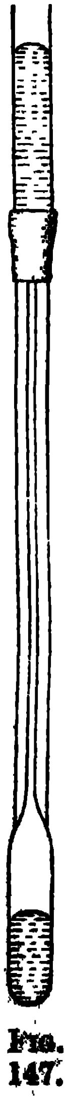

FIG. 147.

Thermometer Liquids

Very Important

A liquid to be suitable for use in a thermometer* should have the following properties:—

(1) It should be opaque so as to be readily seen.

(2) It should be a good conductor of heat, so that all the liquid in the thermometer rapidly comes to the same temperature.

(3) It should have a high coefficient of expansion so that a small change of temperature will cause a considerable change in its volume.

(4) Its expansion should be regular, that is its expansion per degree should be the same at different points on the temperature scale.

(5) It should have a low specific heat and low density, so that the thermal capacity of the volume used will be small, and it will not appreciably cool the body whose temperature is to be taken.

(6) It should have a high boiling point and a low melting point, so that both high and low temperatures can be measured by it.

No one liquid has all these properties. The following table compares the two liquids most commonly used:—

MERCURY	ALCOHOL
1. Opaque.	1. Transparent, but may be coloured by dyes.
2. Good conductor.	2. Poor conductor.
3. Coef. of expansion, 0·00018.	3. Coef. of expansion, 0·00104.
4. Expands regularly.	4. Expansion somewhat irregular.
5. Specific heat, 0·033. Specific gravity, 13·6.	5. Specific heat, 0·6. Specific gravity, 0·8.
6. Freezing point, — 39° C. Boiling point, 357° C.	6. Freezing point, — 130° C. Boiling point, 78° C.

Sensitive and Quick-Acting Thermometers

A thermometer should be both sensitive and quick acting, that is, there should be a visible movement of the liquid thread for a small change in temperature, and it should quickly register temperature changes.

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* Some of the points mentioned in this section will be more fully understood when later chapters have been read.

Sensitivity can be increased by increasing the size of the bulb as a bigger volume of liquid will give a bigger expansion per degree. It can also be increased by reducing the bore of the tube as a given increase in volume will then fill a greater length of tube.

Quick action can be secured by making the glass of the bulb very thin, having a small bulb, and using a liquid of good conductivity, so that the heat quickly passes to all parts of the liquid.

Special Thermometers

(1) Maximum and Minimum Thermometers.—These are used for recording respectively the highest and lowest temperatures during a given period. Fig. 148 represents Sixe's thermometer which gives both maximum and minimum readings. Bulb A, and the tube down to B, contain alcohol. From B to C there is a thread of mercury, and from C to D more alcohol. Above D is a space from which air has been removed. The left-hand side is graduated according to the expansion of the alcohol in AB. The right-hand side is graduated so that, as the expansion of the alcohol in AB drives the mercury round towards D, both mercury surfaces will always point to the same number. If the alcohol in AB contracts, the pressure of that in CD will drive the mercury back.

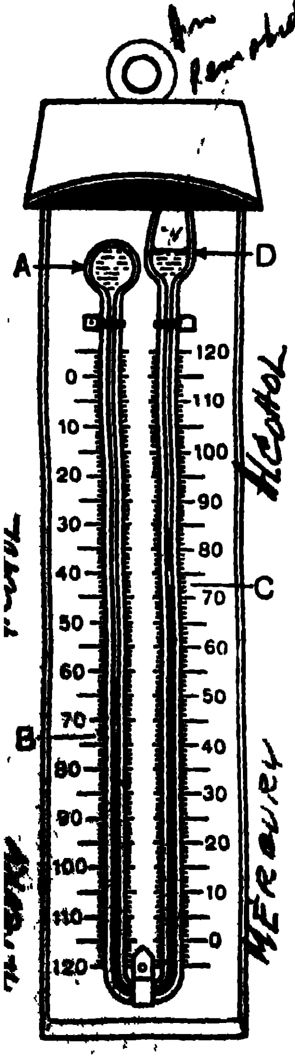


Fig. 148.

Just above the mercury in each limb is a small iron index. Since alcohol, but not mercury, will wet these, surface tension will prevent them from breaking the mercury surfaces. To set the thermometer, each index is brought into contact with the mercury by drawing a small magnet down the tubes. If the temperature rises the mercury will push the right-hand index upwards, but the alcohol will just flow round the left-hand one without moving it. A fall of temperature would raise the left-hand index but leave the right-hand one unmoved. Thus, however long the thermometer is left, the right-hand index will indicate the highest temperature there has been since it was set, and the left-hand one the lowest temperature.

(2) CLINICAL THERMOMETERS.—Clinical thermometers are a special type of maximum thermometers used for taking the temperature of the human body by placing the bulb under the tongue or in the arm pit. Their construction is shown in Fig. 149. At *c* there is a very narrow constriction of the bore. When the mercury contracts after warming the thread breaks at *c*, so that some mercury is left in the stem registering the temperature that had been reached. This thread is shaken back into the bulb before using the thermometer again.

The *normal temperature of the body is about* 98·2° F., *and it only varies by about a few degrees above or below this.* Hence a scale going below 90° F. would have no value. For this reason in the bore below *c* there is a loop which takes up the expansion of the mercury until a temperature of about 90° F. is reached. This allows the thermometer to be kept conveniently short.

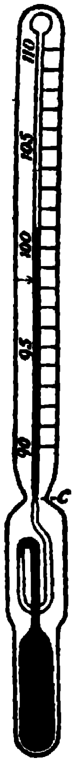


FIG. 149.

QUESTIONS ON CHAPTER XIV

1. Describe simple experiments to show that solids, liquids, and gases expand when heated.

What evidence do your experiments give to show that gases expand more than liquids and liquids expand more than solids?

2. How would you show (*a*) that equal lengths of different solids equally heated have unequal expansions, (*b*) equal volumes of different gases equally heated have equal expansions?

3. Describe three cases in which the expansion of bodies on heating have troublesome effects, and explain the precautions that may be taken to prevent those effects. Also describe three cases in which use is made of expansion.

4. Describe an experiment to illustrate the difference between the temperature of a body and the quantity of heat it contains.

It is sometimes said that temperature bears the same relation to quantity of heat as level bears to volume of water. Illustrate that statement.

5. Briefly describe a thermometer and explain how it is filled.

6. Describe the marking of the "fixed points" on a thermometer. State how, when the fixed points had been marked, you would proceed to mark (*a*) a Centigrade scale, (*b*) a Fahrenheit scale on the thermometer.

7. (*a*) Complete the following table of corresponding Centigrade and Fahrenheit temperatures.

C.	− 50°			10°		45°	75°	
F.		− 13°	23°		59°			203°

(*b*) From the completed table plot a graph to show the relation between the two scales, and use the graph to determine the temperature which is represented by the same number on both scales.

8. Enumerate the steps in the construction and graduation of a mercury thermometer whose range is from − 10° C. to 110° C. What is (*a*) the advantage, (*b*) the disadvantage, of providing the thermometer with a large bulb for a specified cross-sectional area of the stem? [L.U.

9. Describe and explain the action of a thermometer that may be used to indicate (*a*) maximum, (*b*) minimum temperatures.

The maximum and minimum temperatures recorded on a particular day were 75° F. and 44° F. respectively. Express the temperatures in degrees Centigrade. [L.U.

10. Make a careful drawing of a clinical thermometer (actual size), including the usual graduations and the mark for the normal temperature of the body.

What special features in its construction determine—

(*a*) That it records the maximum temperature;

(*b*) Whether this maximum temperature is reached in half a minute or in one minute;

(*c*) The ease with which the instrument can be read? [L.U.

11. Explain why mercury and alcohol are chosen as suitable liquids to be used in thermometers.

Describe how you would test the accuracy of the fixed points of a mercury thermometer. [L.U.

12. Describe the method of graduation of a mercury in glass thermometer, such as would be suitable for the determination of the melting point of naphthalene (which is between 70° C. and 80° C.).

Also describe the method of making this determination.

What kind of thermometer could be used for measuring a low temperature, such as the freezing point of mercury? [J.M.B.

CHAPTER XV

EXPANSION OF SOLIDS

Measuring Expansion of a Rod

It has been shown that the expansion of a solid is very small compared with its original size. It follows that, to measure the increase in length of a rod with accuracy the rod should be long, its temperature should be raised considerably, and an instrument for measuring very small lengths should be used.

Fig. 150 illustrates a form of apparatus which may be used. A rod of metal at least 50 cm. long is passed through corks in the ends of a long glass tube, a thermometer having been tied to it with its bulb at about the middle of the rod. Bent tubes A and B are also passed through the corks. This apparatus is fixed vertically in a wooden stand with the lower end of the rod resting on a block. Across the top of the stand is a glass plate with a hole through its centre. A spherometer stands on this plate.

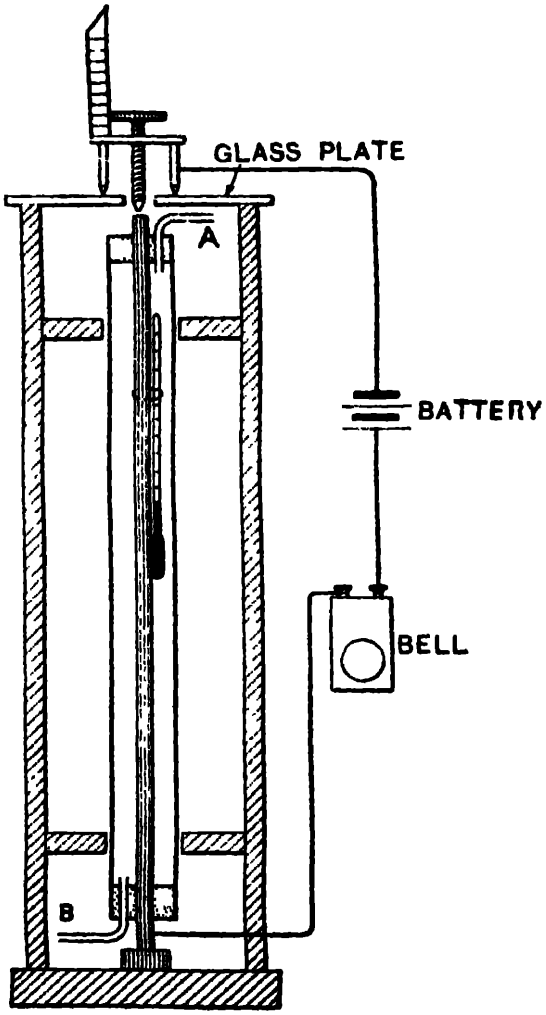


Fig. 150.

Cold water is run into the tube at A and out at B for a few minutes to bring the rod to the temperature of the water. The temperature is then noted from the thermometer. The spherometer screw is screwed down until it makes contact with the top of the rod. The electrical circuit shown will indicate when contact is just made, as the bell will ring when the gap between the screw and the rod is closed. The spherometer is then read.

The screw is then screwed up to give space for the expansion of the rod. The flow of water is stopped, and A is connected by rubber tubing to a flask in which steam is being generated. The steam is allowed to pass through the tube until the thermometer has registered a steady temperature for a minute or two so that the rod may be brought to that temperature. The reading of the thermometer is then taken again, the screw screwed down to make contact with the rod once more, and a second reading of the spherometer is taken. The difference between the two spherometer readings is equal to the expansion of the rod.

The following points concerning the apparatus should be noted:—(1) The temperature may not be quite the same at the two ends of the rod. By placing the thermometer bulb half-way along it, the *average* temperature along the rod is obtained. (2) The parts of the rod outside the corks will not be directly heated by the steam and are subject to cooling by the air, so they should be as short as possible. (3) The rod should be well pressed down on the block at the base, so that the upper end must rise by the full amount that the rod expands.

In such an experiment with an iron rod the following results were obtained:—

$$\text{Length of rod} = 51 \text{ cm.}$$

$$\text{1st spherometer reading} = 0{\cdot}25 \text{ mm.};$$

$$\text{2nd spherometer reading} = 0{\cdot}74 \text{ mm.};$$

$$\therefore \text{ Expansion} = 0{\cdot}49 \text{ mm.} = 0{\cdot}049 \text{ cm.}$$

$$\text{1st temperature} = 16^\circ \text{ C.}$$

$$\text{2nd temperature} = 99^\circ \text{ C.};$$

$$\therefore \text{ Increase in temperature} = 83^\circ \text{ C.}$$

$$\text{Expansion expressed as fraction of original length} = \frac{{\cdot}049}{51};$$

$$\therefore \text{ Fractional expansion per degree} = \frac{{\cdot}049}{51 \times 83} = {\cdot}000012.$$

This number expresses the fact that an iron rod expands by ·000012 of its original length for each degree Centigrade that its temperature rises; which will apply to linear dimensions of iron articles in general, and so is called the "coefficient of linear expansion of iron." The coefficient of linear expansion of a substance is the fractional increase in length of a piece of that substance per degree rise of temperature, *i.e.* :—

$$\text{Coefficient (linear)} = \frac{\text{Increase in length}}{\text{Original length} \times \text{Rise in temperature}}.$$

The above example shows how a coefficient of expansion may be calculated from experimental results. Note that the final result would not be affected by making the measurements in other units than centimetres, so long as both original length and expansion are expressed in the same units, since the ratio between expansion and original length would not be altered. The result does depend on the temperature scale used. One Fahrenheit degree is only $\frac{5}{9}$ of a Centigrade degree, so that the expansion per degree Fahrenheit would be $\frac{5}{9}$ of that per degree Centigrade.

Coefficients of Linear Expansion per Degree Centigrade

Aluminium ...	·000026	Iron ...	·000012
Brass ...	·000019	Lead ...	·000028
Copper ...	·000017	Platinum ...	·000009
Glass ...	·0000085	Quartz ...	·00000042
Invar ...	·000001	Zinc ...	·000028

Examples.—(1) *A copper aerial wire is* 50 *ft. long when it is put up, the temperature being* 10° *C. What will its length be when the temperature becomes* 25° *C.?*

The fractional increase per degree C. = ·000017;

∴ Expansion of 50 ft. for 15° C. rise in temperature

$$= \cdot 000017 \times 50 \times 15 = \cdot 01275 \text{ ft.}$$

∴ Length at 25° C. = 50·01275 ft.

(2) *If iron rails each* 30 *ft. long are laid close up end to end when the temperature is* 90° F., *what space will there be between consecutive rails when it is just freezing?*

$$\text{Fractional increase per degree F.} = \cdot 000012 \times \tfrac{5}{9}.$$

$$\text{Fall in temperature} = (90 - 32)^\circ \text{ F.} = 58^\circ \text{ F.};$$

$$\therefore \text{Contraction of 30 ft.} = \cdot 000012 \times \tfrac{5}{9} \times 30 \times 58 \text{ ft.}$$

$$= \cdot 0116 \text{ ft.} = \cdot 14 \text{ in. (approx.)}.$$

Simple calculations on expansion may be made as above, but coefficient calculations so frequently occur in Physics that it is useful, particularly in theoretical work, to be ready to apply the general coefficient formula obtained below.

Let l_0 be the length of a bar at 0° C., and l_t be its length at t° C. Also let its coefficient of linear expansion per degree C. be a.

Then $l_t - l_0 =$ the expansion of the rod between 0° C. and t° C. But this expansion $= l_0 \times a \times t$;

$$l_t - l_0 = l_0 \times a \times t;$$
$$\therefore\ l_t = l_0 + l_0 at;$$
$$\therefore\ l_t = l_0 (1 + at).$$

This formula may be modified when the length at 0° C. is not known to the form

$$l_{t_2} = l_{t_1} \{1 + a(t_2 - t_1)\}$$

in which l_{t_1} represents the length at t_1° C. and l_{t_2} that at t_2° C. In this form it is not exactly correct but the error for moderate differences of temperature is negligible.

Thus Example (1) above may be worked as follows:—

$$l_{25} = l_{10} \{1 + a\,(25 - 10)\}$$
$$= 50\,\{1 + \cdot000019 \times 15\} = 50 \times 1{\cdot}000285 = 50{\cdot}01425;$$
$$\therefore\ \text{Length at } 25^\circ\text{ C.} = 50{\cdot}01425 \text{ ft.}$$

Superficial and Cubical Expansion

When a solid body is heated its surface area and volume as well as its linear dimensions will increase. Increase of area is spoken of as **superficial expansion**; that of volume as **cubical expansion.**

The coefficient of $\left\{\begin{matrix}\text{superficial}\\ \text{cubical}\end{matrix}\right\}$ expansion of a substance is the fractional increase of the $\left\{\begin{matrix}\text{area}\\ \text{volume}\end{matrix}\right\}$ of a piece of that substance per degree rise in temperature. Using such coefficients, calculations on increases of area and volume may be made as in the case of linear expansion.

Imagine a one-inch square plate of material of coefficient of linear expansion a (Fig. 151). Let its temperature be increased by 1° C. Then each side will expand a in. and become $(1 + a)$ in.;

$\therefore$ Its area becomes $(1 + a)^2 = 1 + 2a + a^2$ sq. in.

a is a very small fraction, so a^2 is extremely small, and may be neglected.

(Note in Fig. 151, where the expansion is very much exaggerated, it is the small square in the corner which represents a^2. In an actual case each side of this square would be about $\frac{1}{10000}$ in. long.)

$\therefore$ The increase of area is approximately $= 2a$ sq. in.;

$\therefore$ Fractional increase per degree, that is, coefficient of superficial

$$\text{expansion} = \frac{2a}{1} = 2a.$$

Hence the coefficient of superficial expansion of a solid is equal to twice its coefficient of linear expansion.

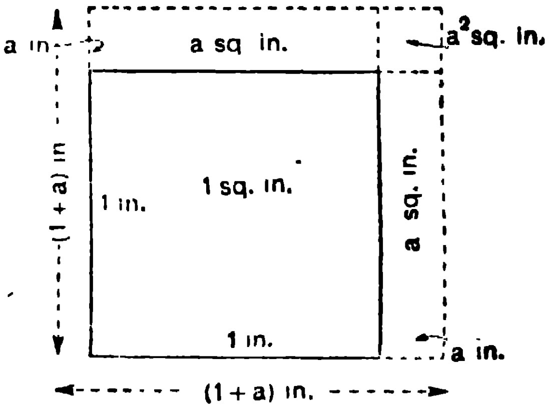


FIG. 151.

In a similar way, by considering a cube with each edge 1 in. long, it can be shown that the coefficient of cubical expansion of a solid is equal to three times its coefficient of linear expansion, for if the temperature rose 1° C., each side would become $(1 + a)$ in. long;

$\therefore$ The volume would become $(1 + a)^3 = 1 + 3a + 3a^2 + a^3$ cub. in.

But a^3 is still smaller than a^2;

$\therefore$ The new volume is approximately $1 + 3a$ cub. in.;

$\therefore$ The expansion is $3a$ cub. in., and the fractional increase per degree C.

$$= \frac{3a}{1} = 3a.$$

Formulae of the same type as that proved on p. 178 may be obtained for superficial and cubical expansion. Thus for superficial expansion we may write

$$A_t = A_0 (1 + st),$$

where A_t is the area at $t°$ C. of a surface whose area is A_0 at 0° C., the coefficient of superficial expansion of the substance being s.

Similarly,

$$V_t = V_0 (1 + ct)$$

when V_t is the volume at $t°$ C. of a mass of substance which has a volume V_0 at 0° C. and whose coefficient of cubical expansion is c.

Example.—*A brass cube has each edge* 10 *cm. long at* 15° *C. What will be the area of each face and its volume at* 60° *C.?*

$$\text{Area of face at } 15^\circ \text{ C.} = 10 \times 10 = 100 \text{ sq. cm.}$$

$$\text{Coef. of superficial expansion} = \cdot 000019 \times 2 = \cdot 000038.$$

Using formula

$$\begin{aligned} A_{60} &= A_{15}\{1 + s\,(60 - 15)\} \\ &= 100\{1 + \cdot 000038 \times 45\} \\ &= 100 \times 1\cdot 00171 = 100\cdot 171; \\ \therefore\ \text{Area at } 60^\circ \text{ C.} &= \underline{100\cdot 171 \text{ sq. cm.}} \end{aligned}$$

$$\text{Vol. at } 15^\circ \text{ C.} = 10 \times 10 \times 10 = 1000 \text{ c.cm.}$$

$$\text{Coef. of cubical expansion} = \cdot 000019 \times 3 = \cdot 000057.$$

Using formula

$$\begin{aligned} V_{60} &= V_{15}\{1 + c\,(60 - 15)\} \\ &= 1000\{1 + \cdot 000057 \times 45\} \\ &= 1000 \times 1\cdot 002565 = 1002\cdot 565; \\ \therefore\ \text{Vol. at } 60^\circ \text{ C.} &= \underline{1002\cdot 565 \text{ c.cm.}} \end{aligned}$$

Applications

The table on page 177 shows that Invar, which is an alloy of nickel and steel, has a very small coefficient of expansion. For this reason it is often used in the construction of instruments where expansion and contraction must be kept to a minimum.

Platinum and glass have approximately equal coefficients. For that reason platinum wire is generally used when wires have to be fused through the walls of glass vessels. If you insert an iron wire into the drawn-out end of a glass tube and then try to seal it into the glass you will find that it breaks away on cooling because the wire contracts more than the glass around it. A platinum wire would not do this, as the contractions would be about equal.

Quartz has a very low coefficient of expansion. Since it can be made into vessels similar to glass vessels it is frequently used in making laboratory ware which can then be made thick and strong without being liable to crack when heated.

A gas regulator, to maintain an oven at a steady temperature, is illustrated in Fig. 152. The arrows indicate the path of the gas through the regulator on its way to the burners. The tubular part is inside the oven. If the temperature inside the oven rises, the brass tube expands, but the Invar rod remains practically constant in length. Thus the cone B is pulled towards the left and checks the flow of gas through the opening at A. If the temperature in the oven falls, the contraction of the tube pushes the cone to the right which allows gas to flow more freely to the burners.

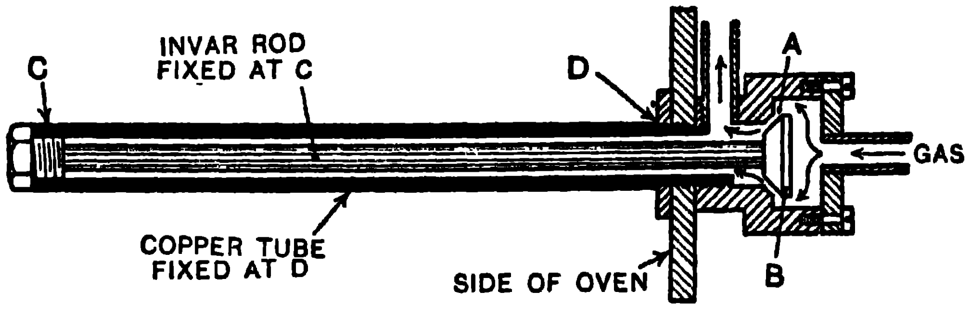


FIG. 152.

Measuring rods and chains will vary slightly in length as their temperature changes. A standard measure of approximately constant length can be made by using two metals with different coefficients of expansion. Reference to the table will show that the coefficient for brass is approximately 1½ times that for iron. Therefore 1 unit length of brass will expand as much as 1½ unit lengths of iron, or 2 unit lengths of brass as much as 3 unit lengths of iron. Thus (Fig. 153), if a 3-ft. rod of iron and a 2-ft. rod of brass are placed side by side and joined at one end, the distance between the free ends will always be 1 ft. whatever the temperature may be.

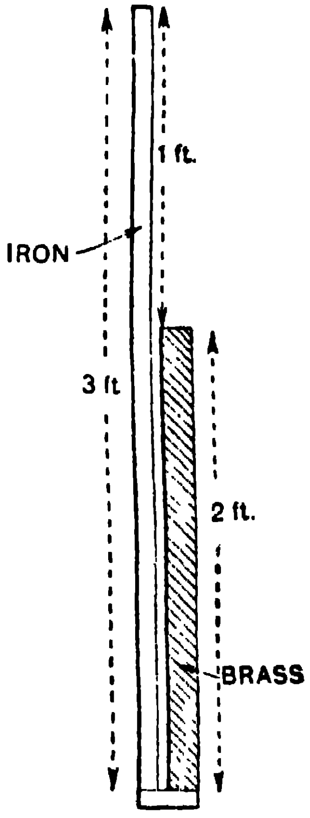


FIG. 153.

The principle of the last paragraph is utilised in the compensation of clock pendulums. The number of swings a pendulum makes in a given time depends on its length. The longer the pendulum, the greater is its time of swing. Since the time of swing of the pendulum regulates the rate at which the clock goes; clocks with uncompensated pendulums tend to lose time when the temperature rises owing to increase in length of the pendulum through its expansion.

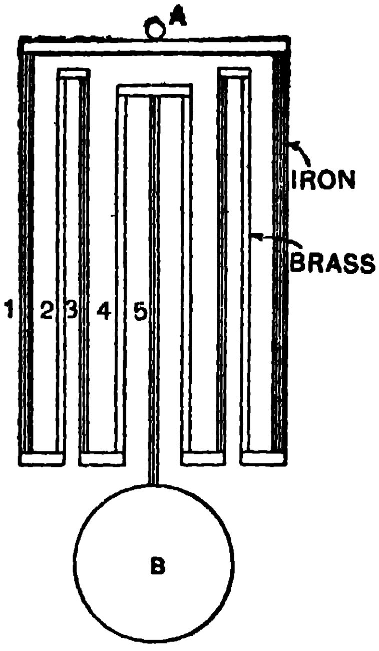


FIG. 154.

A pendulum usually has a light rod and a heavy bob, so that its centre of gravity is approximately at the centre of the bob and its effective length is the distance from its point of suspension to the centre of the bob.

In order to keep this effective length constant, *Harrison's gridiron pendulum* is built up of a system of brass and iron rods, as indicated in Fig. 154. Any expansion of the iron bars will tend to lower the bob, increasing the effective length AB. But expansion of the brass rods will raise the bob and shorten AB. If the total lengths of rods 1, 3, and 5 and the total length of rods 2 and 4 are in the ratio 3 : 2, these two opposing effects will balance each other and the effective length AB of the pendulum will remain constant.

The rate at which a watch goes is regulated by a balance wheel which is a wheel with a heavy rim whose movement is checked by a hair spring so that it oscillates. As most of its weight is in the rim its time of oscillation depends on its radius, the longer the radius the longer being this time. Hence, if the temperature of the watch rises the expansion of the wheel increases its radius and so slows its oscillation and the watch loses time. To compensate for this the rim of the balance wheel may be made in separate sections as shown in Fig. 154 (*a*) A, each section being a compound bar with the more expansible metal on the outside. Then as the spokes lengthen through expansion the free ends of these sections will curl inwards as in Fig. 154 (*a*) B, in which the effect is exaggerated. This keeps the average distance of the particles of the rim from the pivot constant and so maintains a constant time of oscillation.

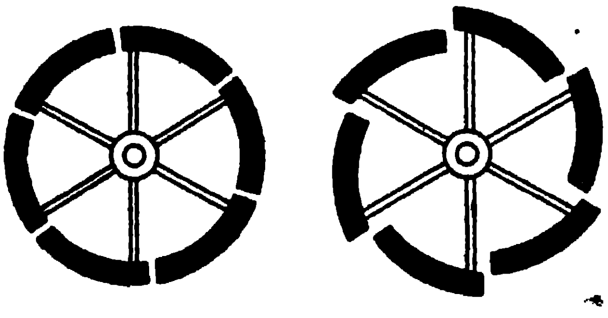

(A) (B)

FIG. 154 (*a*).

Expansion of Hollow Vessels

A hollow vessel will expand by the same amount as a solid piece of the same substance of the same size as the vessel. This will be seen from considering a rectangular block of metal and a box, with dimensions equal to those of the block, made of thin sheets of that metal. If the two are equally heated, the increases in length, breadth, and height will be the same for the two bodies, and therefore they will remain equal to one another in volume. Thus, calculations on the expansion of a hollow vessel may be made in the same way as for a solid body, using the coefficient of cubical expansion of its substance and the formula proved for cubical expansion may be used in such calculations.

Change of Temperature and Density

When a body is heated its volume increases but its mass remains constant. It follows that the density of the matter in the body decreases as the temperature rises.

$$\text{Now Mass} = \text{Density} \times \text{Volume}.$$

But mass remains constant;

$$\therefore \text{Density at 2nd temperature} \times \text{Volume at 2nd temperature}$$
$$= \text{Density at 1st temperature} \times \text{Volume at 1st temperature};$$

$$\therefore \frac{\text{Density at 2nd temperature}}{\text{Density at 1st temperature}} = \frac{\text{Volume at 1st temperature}}{\text{Volume at 2nd temperature}}.$$

If D_1 and V_1 are written for the first values of density and volume respectively, and D_2 and V_2 for their second values,

$$\frac{D_2}{D_1} = \frac{V_1}{V_2}.$$

Also, if c is the coefficient of cubical expansion of the substance, and t is the difference between the two temperatures,

$$V_2 = V_1 (1 + ct) \text{ (coefficient formula)};$$

$$\therefore \frac{D_2}{D_1} = \frac{V_1}{V_1 (1 + ct)}; \quad \therefore \frac{D_2}{D_1} = \frac{1}{1 + ct}; \quad \therefore D_2 = \frac{D_1}{1 + ct}.$$

As a special case of this we may write

$$D_t = \frac{D_0}{1 + ct},$$

where D_t is the density at t° C., and D_0 the density at 0° C.

Example.—*Taking the density of copper to be* 8·9 *grm. per c.cm. at* 0° *C., what is its density at* 20° *C.?*

$$\text{Coefficient of cubical expansion} = \cdot 000017 \times 3.$$

$$\text{Density at } 20^\circ \text{ C.} = \frac{8\cdot 9}{1 + (\cdot 000017 \times 3 \times 20)} \text{ grm. per c.cm.}$$

$$= \frac{8\cdot 9}{1\cdot 00102} = 8\cdot 89 \text{ grm. per c.cm.}$$

QUESTIONS ON CHAPTER XV

Use the table of coefficients on page 177 where necessary.

1. Calculate the lengths at 75° C. of rods of brass, iron, glass, and zinc, each of which is 1 m. long at 0° C.

2. Calculate the total lengths of the gaps which must be left in a mile of iron rails, which are laid when the temperature is 40° F., to allow for a possible summer temperature of 100° F.

3. Calculate coefficients of linear expansion for copper, iron, and zinc from the following data:—

Metal	Length of Rod	Expansion	1st Temp.	2nd Temp.
Copper	85 cm.	1·16 mm.	20° C.	100° C.
Iron	75 cm.	0·68 mm.	15° C.	90° C.
Zinc	60 cm.	1·38 mm.	16° C.	98° C.

4. A surveyor uses an iron measuring rod which is correct at 15° C. What will be the true length of a distance he measures as one mile when the temperature is 25° C., and what will be the percentage error in his measurement?

5. An iron window frame has spaces 20 in. by 12 in. for the panes of glass which are fitted tightly into the frame when the temperature is 5° C. How much smaller than its frame will a pane be (*a*) in length, (*b*) in width, (*c*) in area, when the temperature is 32° C.?

6. An iron rod and an aluminium rod are to be made so that the difference between their lengths is always to be 1 yd. What length must each be?

7. A glass measuring flask, to measure 1 litre, is correctly marked when it is at 15° C. What number of cubic centimetres of liquid does it hold when filled to the mark at 25° C.?

8. An aluminium cylinder is 8 cm. long and 2 cm. in diameter at 20° C. Find (*a*) its length, (*b*) the area of one end, (*c*) its volume when it is heated to 60° C.

9. Define *coefficient of linear expansion* of a solid.

In carrying out a measurement of this coefficient for brass the number 0·000018 was obtained when centimetres and degrees C. were used. What would have been the result if inches and degrees F. had been employed?

Describe two practical applications of the contraction of metals in cooling. [L.U.

10. Explain what is meant by the coefficient of linear thermal expansion.

Describe and explain the action of two contrivances when advantageous use is made of the difference between the values of this coefficient for two metals. [L.U.

11. Describe how you would measure the coefficient of linear expansion for the material of a metal bar.

Two metal bars, A and B, differ in length by 25 cm. whatever the change in temperature. If their coefficients of linear expansion are 0·0000128 and 0·0000192 per ° C. respectively, calculate the actual lengths of A and B at 0° C. [L.U.

12. A metal rod is exactly 1 metre long at 0° C. If the coefficient of linear expansion of the metal is 0·000020 per degree C., what is the temperature when the length of the rod has increased by 1 mm.?

How would you determine the coefficient of expansion of *either* a brass rod *or* a brass tube?

Explain one instance where the expansion of a solid due to a rise of temperature is a disadvantage, and one instance where use is made of it. [J.M.B.

13. Explain why a piece of hot copper has a lower density than cold copper.

What is the ratio of the densities of copper at 15° C. and 30° C.?

CHAPTER XVI

EXPANSION OF LIQUIDS

In the case of a liquid cubical expansion only can be dealt with because the linear and surface dimensions will change as the liquid is poured from one vessel to another without change of temperature.

Real and Apparent Expansion

In the experiment to show the expansion of a liquid (page 163) the expansion and contraction of the containing vessel had to be considered in order to explain the observations made. This expansion of the containing vessel must always be taken into account when dealing with the expansion of the liquid.

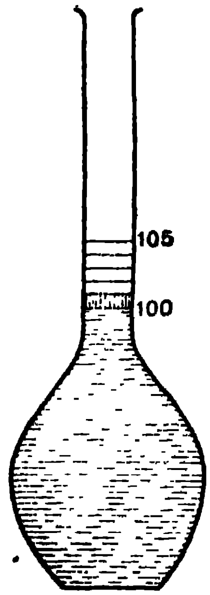


Fig. 155.

Suppose the flask in Fig. 155 is graduated in cubic centimetres along the neck. Let it be filled to the 100 c.cm. mark. If it is heated until the liquid rises to the 101 c.cm. mark, it *appears* to have expanded by 1 c.cm. In reality it has expanded more than that, since the flask will also have expanded and its capacity up to the 101 mark is now more than 101 c.cm. Thus we distinguish between the **apparent expansion** of the liquid, that is, the increase in volume which appears to have taken place if no notice is taken of the expansion of the containing vessel, and its **real expansion**, which is the actual increase in volume of the liquid. The actual volume of the liquid, after heating, in the above case would be:—

101 c.cm. + the amount by which the flask expanded, and its real expansion would therefore be:

1 c.cm. + the amount by which the flask expanded.

hence, we have the following relation:—

Real expansion = Apparent expansion + Expansion of containing vessel.

If we consider the case where the original volume is 1 c.cm. and the temperature is raised 1° C., the expressions in the above equation are, respectively, the coefficient of real expansion of the liquid, its coefficient of apparent expansion, and the coefficient of cubical expansion of the substance of the containing vessel. Therefore:—

Coefficient of real expansion = Coefficient of apparent expansion + Coefficient of cubical expansion of the substance of the vessel.

Coefficients of Apparent Expansion

These are frequently determined by the "weight thermometer" method. A small density bottle forms a convenient vessel for the experiment. Weigh the dry, empty bottle. Fill it from a beaker of the liquid which has been standing for some time in the laboratory so that it will be at air temperature. Take the temperature of the liquid in the beaker. Weigh again to find the weight of liquid in the bottle. Suspend the bottle in a beaker of water so that it is covered to the neck without touching the bottom or side of the beaker. Carefully warm the water, keeping it well stirred. Fix a thermometer in the water with its bulb level with the middle of the bottle. When it shows a suitable rise of temperature, adjust the flame to maintain that temperature until all the liquid is likely to have attained it. Then remove the bottle and allow it to cool. During the heating some of the liquid will have been expelled owing to its expansion. When it is cool weigh the bottle again to find the weight of liquid remaining.

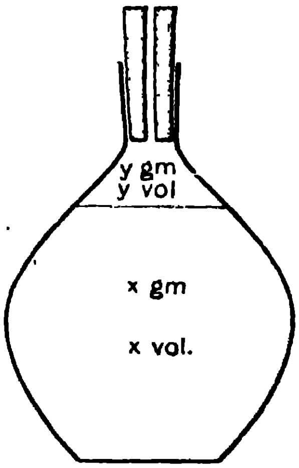


FIG. 156.

Suppose that there were $(x + y)$ grm. of liquid in the bottle originally, x grm. of which remained so that y grm. were expelled (Fig. 156). Regard the space occupied by 1 grm. of the liquid *when cold* as 1 unit of volume. Then there are $(x + y)$ such units of volume in the whole bottle. The x grm. which remained would fill x of these units when it was at the first temperature, but occupied the whole $(x + y)$ units at the higher temperature. Therefore x volumes expanded by y volumes so that its fractional expansion was equal to $\frac{y}{x}$, and the coefficient of expansion was $\dfrac{y}{x \times \text{Rise in temperature}}$. That is,

$$\textbf{Coefficient of expansion} = \frac{\textbf{Weight expelled}}{\textbf{Weight remaining} \times \textbf{Rise in temperature}}.$$

Account has not been taken in the above reasoning of the expansion of the bottle, so it is the coefficient of *apparent* expansion of the liquid in glass which is obtained by this method. The following are the results of an experiment with aniline oil:—

Weight of bottle = 18·67 grm.

Weight of bottle and oil at first temperature = 68·55 grm.

Weight of bottle and oil after heating = 66·50 grm.;

∴ Wt. of oil remaining = 47·83 grm.; Wt. of oil expelled = 2·05 grm.

1st temp. = 31° C.; 2nd temp. = 78° C.; Rise in temp. = 47° C.;

$$\therefore \text{ Coefficient of apparent expansion} = \frac{2{\cdot}05}{47{\cdot}83 \times 47} = 0{\cdot}00092.$$

Determination of Coefficients of Real Expansion. Simple Methods

(1) By Weight Thermometer.—Find the coefficient of apparent expansion using a weight thermometer made of material whose coefficient of cubical expansion is known. Then calculate the coefficient of real expansion from the relation given on page 182.

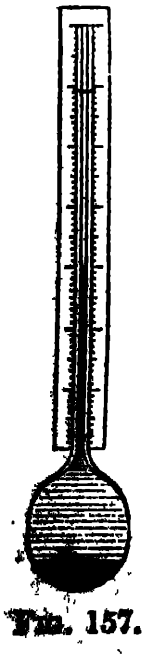


Fig. 157.

(2) By Constant Volume Dilatometer.—The coefficient of cubical expansion of mercury is seven times as big as the coefficient of cubical expansion of glass, so that any volume of mercury will expand as much as seven times its volume of glass when the two are equally heated.

The constant volume dilatometer consists of a glass bulb (Fig. 157) one-seventh of which is filled with mercury, connected to a narrow tube. The part of the bulb above the mercury will have a volume which is constant at all temperatures as the expansion of the mercury will just fill the extra space made by the expansion of the bulb. Along the tube is a scale graduated to give readings of volumes above the mercury. The remainder of the bulb and part of the tube are filled with the liquid whose expansion is to be measured The bulb is immersed in a vessel of water, and readings of the temperature of the bath and the volume of the liquid are taken. The bath is heated, the water in it well stirred, and further temperature and volume readings are taken. Since the expansion of the mercury compensates for the expansion of the bulb, the coefficient of real expansion of the liquid may be calculated directly from the readings, it being equal to

$$\frac{\text{Increase in volume}}{\text{Original volume} \times \text{Rise in temperature}}.$$

Dulong and Petit's Method for Coefficient of Real Expansion

Dulong and Petit's method for the direct determination of the coefficient of real expansion of a liquid is based on the change of density with temperature and the relation between balancing columns of liquids. The liquid is contained in a continuous tube bent into the form shown in Fig. 158. The two vertical portions are surrounded by wider tubes with inlets and outlets. The upper portions of the two limbs are brought near together and backed by a scale so that the levels of the two liquid surfaces may readily be compared.

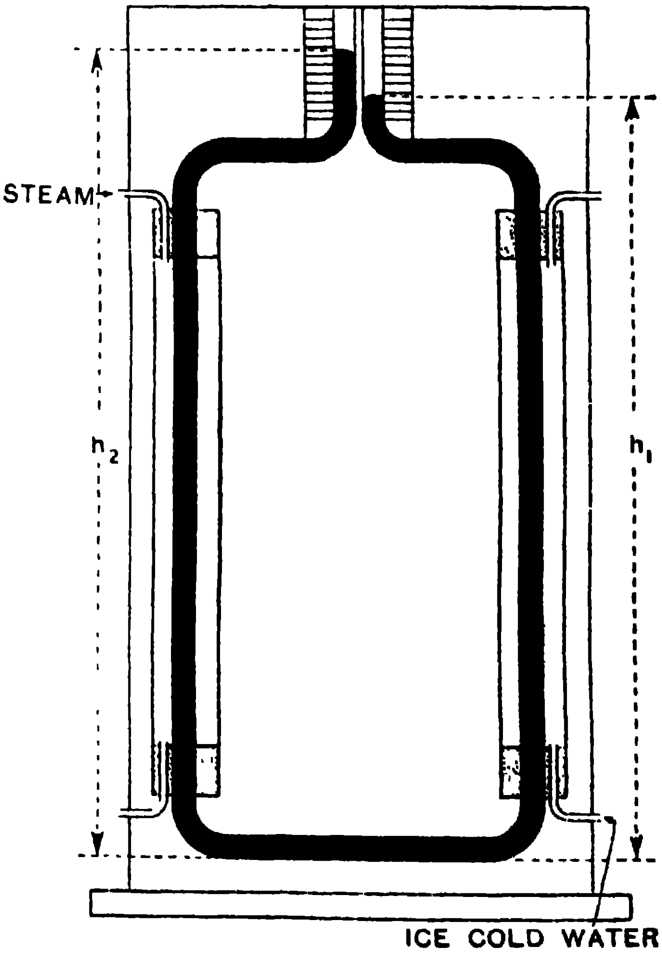


Fig. 158.

Steam is passed into the jacket around the left-hand limb, and water cooled with ice into that round the right-hand limb. Thus, the liquid in the left-hand limb is heated to 100° C. and that on the right cooled to 0° C. Owing to this difference in temperature, the liquid on the left will be less dense than that on the right, and so the left-hand column will have a greater height than the right-hand one to maintain balance. When the two liquid surfaces have attained steady levels, the difference between their levels is read and the height of the cold column, h_1, is measured. Since the columns of liquid balance:—

$$\frac{\text{Height of cold column } (h_1)}{\text{Height of hot column } (h_2)} = \frac{\text{Density of hot column } (D_{100})}{\text{Density of cold column } (D_0)}.$$

As shown on page 183, $D_{100} = \frac{D_0}{1 + 100a}$, where a is the coefficient of expansion of the liquid;

$$\therefore \frac{h_1}{h_2} = \frac{D_0}{1 + 100a} \cdot \frac{1}{D_0} \text{ or } \frac{h_1}{h_2} = \frac{1}{1 + 100a};$$

$$\therefore h_1 + 100ah_1 = h_2, \textit{ i.e. } 100ah_1 = h_2 - h_1;$$

$$\therefore a = \frac{h_2 - h_1}{100h_1}.$$

So the coefficient of expansion may be calculated from the relation:—

$$\text{Coefficient} = \frac{\text{Difference in heights}}{\text{Height of cold column} \times \text{Difference in temperature}}.$$

This expression has been derived entirely from the pressures of the two columns which depend on their heights and densities only, so the result is not affected by the expansion of the tube, and the method gives the coefficient of *real* expansion.

Expansion of Water

It has been assumed in the foregoing that the expansion of a liquid is regular, that is, that the expansion per degree of a given mass of liquid is the same at all points on the temperature scale. This is not quite correct. If the expansion of a mass of liquid between 10° C. and 50° C. is compared with that between 50° C. and 90° C., the two values will probably be found to differ slightly though in many cases the variations are very small indeed, and approximately accurate calculations may be made by using *mean coefficients* which give the average increase per degree over a wide range of temperature. The importance of using a liquid with a regular expansion in a thermometer has already been mentioned.

Water shows very great irregularities in its expansion as its temperature rises. To show this, the behaviour of water when cooled in a constant volume dilatometer may be observed. The dilatometer bulb is placed in a water-bath at about 25° C. The bath is kept well stirred and gradually cools. The volume of the water in the dilatometer at various temperatures is noted. When the bath has cooled to air temperature, it may be further cooled by the addition of ice, so that the observations may be continued until the water is at 0° C.

It will be found that equal falls of temperature cause smaller contractions as the temperature becomes lower, and that after cooling to 4° C., further cooling causes the water to expand instead of continuing to contract, so that at 0° C. the volume is approximately equal to that registered at 10° C. If the results are plotted into a graph the curve will be as shown in Fig. 159.

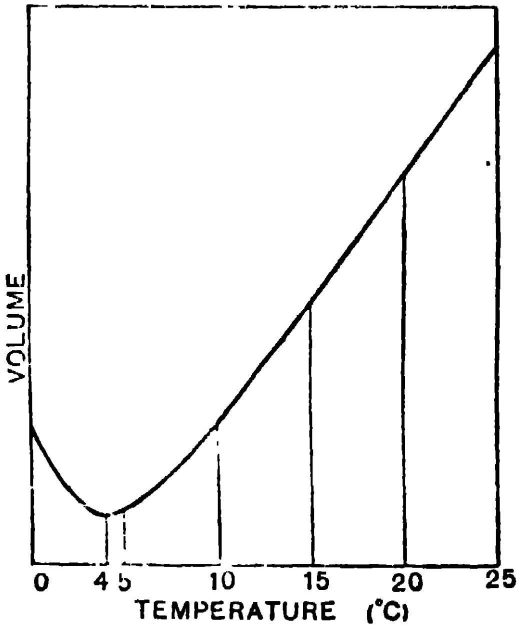


FIG. 159.

From this it follows that a given mass of water has its smallest volume, and therefore its greatest density, at 4° C. Water at this temperature of its maximum density is referred to in defining the gramme.

Changes of density cause important movements in water as it cools to its freezing point. These may be illustrated by means of Hope's apparatus, which is illustrated in Fig. 160. It consists of a tall metal cylinder with a circular trough around it about half-way up its height, and openings for thermometers near the top and bottom. The cylinder is filled with water, ice is packed into the trough, and from time to time simultaneous readings of the two thermometers are taken.

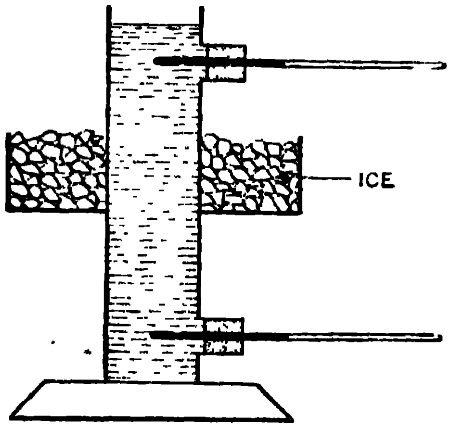


FIG. 160.

At first it will be found that the upper thermometer reading remains steady, while that of the lower thermometer falls rapidly. As the lower reading approaches 4° C. the upper one begins to fall. When the lower one becomes 4° C., it becomes steady, and the upper one continues to fall until it too reads 4° C., showing that the water is at that temperature from top to bottom. During subsequent cooling the upper thermometer will fall slowly to 0° C. while the lower one still gives a reading of 4° C. These observations may be illustrated graphically as in Fig. 161.

Evidently, in the early stages the cooled water, because it has contracted and become more dense, sinks to the bottom. But, when the bottom layer is at 4° C., it has its maximum density, so that neither warmer nor cooler water will displace it. As more water is cooled to 4° C. it sinks until it rests on the first layer, and this continues until all the water is at that temperature. Further cooling of the water at 4° C. in the middle will make it expand and become less dense, so that it floats up through the denser water above it causing the reading of the upper thermometer to fall while that of the lower one remains 4° C.

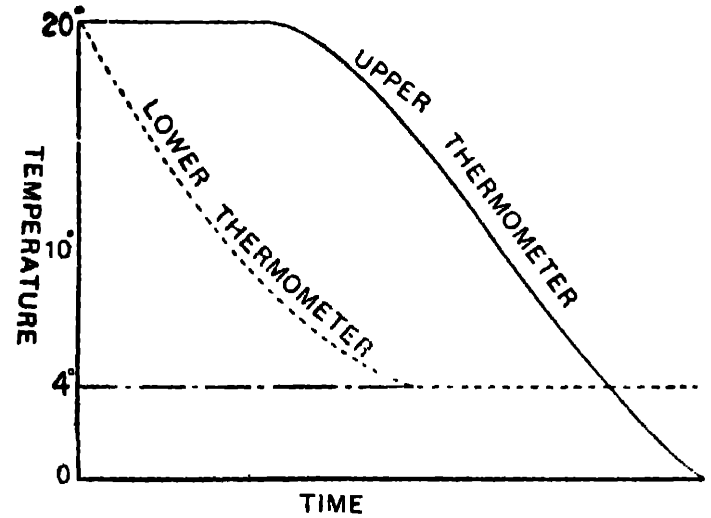


FIG. 161.

These effects are of considerable importance in Nature. During the winter the water of ponds and lakes loses heat from the surface layers. As these cool they will sink, but when a layer at 4° C. has sunk to the bottom, further layers at that temperature will only sink until they rest on it, and gradually the whole mass of water will be cooled to 4° C. As the surface layer cools still more it will float on the water at 4° C. below it. When it reaches 0° C. it will begin to freeze, and as there is a further expansion when water at 0° C. turns to ice, the ice will be less dense than the surface water, and so remain floating. Thus, the water at the bottom of a pond seldom falls below 4° C., and very severe and prolonged frost is required to freeze a pond solid.

If it were not for the expansion of water as it cools from 4° C. and freezes, ponds would freeze from the bottom upwards, and probably all life in them would be destroyed every time they froze. Also the ice would melt much more slowly in the summer, as the heat would have to penetrate the water to get to the ice. This would result in large sheets of water having a very great cooling effect on the climate of surrounding districts.

Thermostats

A thermostat is a device for regulating the heating of an enclosure so that it is kept at a constant temperature. The oven regulator described on page 181 is a form of thermostat. In laboratories thermostats based on the expansion of liquids are frequently used to keep large tanks of water at constant temperature. The gas supplying the burner heating the tank passes through B [Fig. 161 (*a*)], as indicated by the arrows. A is filled with a liquid, such as toluene, with a fairly large coefficient of expansion and is immersed in the tank. As the temperature rises the liquid in A expands and forces mercury up into B and, at a certain temperature, the end of the tube through which gas enters B will be closed by the mercury and the flow of gas will be stopped. A small amount of gas will still flow through the narrow by-pass, C, and keep the burners alight. If the temperature falls, contraction of the liquid will allow the gas to flow freely once more.

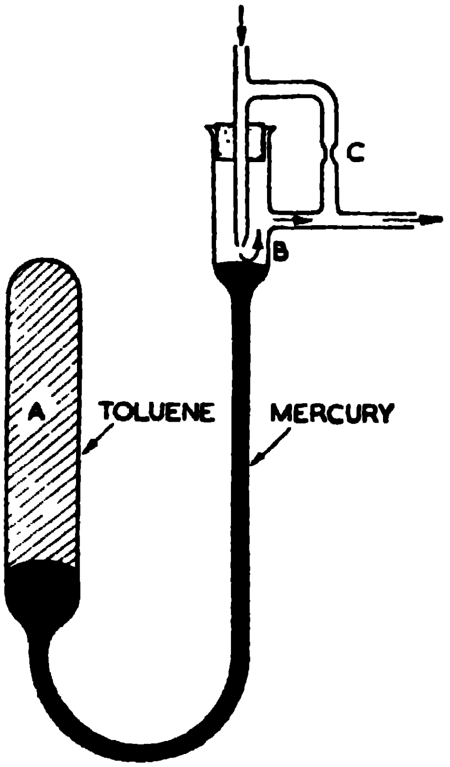


Fig. 161 (*a*).

QUESTIONS ON CHAPTER XVI

Coefficients of Expansion of Liquids (Real).—Alcohol, ·00104. Paraffin, ·00090. Glycerine, ·000485. Turpentine, ·00090. Mercury, ·000182.

1. Distinguish between the real and apparent expansion of a liquid and explain the relation between them.

2. Describe how you would determine the coefficient of apparent expansion of turpentine in glass by experiment.

Calculate that coefficient from the values of the real coefficient of expansion of turpentine and the coefficient of linear expansion of glass.

3. A specific gravity bottle weighed 15·35 grm. when empty and 41·37 grm. when filled with turpentine at 20° C. When it had been heated to 70° C. and cooled again it weighed 40·28 grm. Calculate the coefficient of apparent expansion of turpentine.

From the value you obtain and the coefficient of real expansion of turpentine, calculate the coefficient of linear expansion of the glass of which the bottle is made.

4. Describe a method for determining the coefficient of real expansion of a liquid.

A column of mercury at 100° C. is balanced by a column at 0° C. The respective heights are 76·35 cm. and 75 cm. Calculate the coefficient of real expansion of mercury.

5. Design (*a*) an automatic fire alarm, (*b*) a gas regulator for keeping a water bath at a constant temperature, in both cases utilising the expansion of a liquid.

6. The mean coefficient of expansion of water between 4° C. and 20° C. is 0·00015. Calculate the weight of 1 litre of water at 20° C.

7. The bulb of a thermometer contains 0·45 c.cm. of mercury. What must be the area of cross-section of the bore of the tube in order that the degree graduations may be 2 mm. apart. Coefficient of apparent expansion of mercury in glass is 0·000155.

8. A quartz bulb, with negligible expansion, is loaded until it just sinks in water at its freezing point. Describe what will happen to it as the water is gradually warmed to 20° C.

9. Define the terms *mass*, *volume*, and *density*.

A pipette is marked "25 c.cm., 15 deg. C." What does this mean ? Explain carefully how you would test its accuracy. [L.U.

CHAPTER XVII

EXPANSION OF GASES

The section of Chapter XI which deals with Boyle's Law should be studied or revised before proceeding with this chapter.

Thermal Expansion of a Gas

The simple experiments in Chapter XIV. show that gases expand very much more than either solids or liquids, and that equal volumes of different gases expand equally when equally heated. The work on Boyle's Law shows that the volume of a mass of gas can be altered by changing its pressure as well as by changing its temperature.

From these observations we may make the following deductions:—

(1) Except for purposes of very accurate work we need not distinguish between the real and apparent expansion of a gas. The expansion of a gas will be so much greater than that of the containing vessel that the latter is negligible in comparison with the former.

(2) All gases will have the same coefficient of expansion.

(3) In determining the coefficient of thermal expansion of a gas, all measurements should be made with the gas at a constant pressure.

A further point arises from the great expansion of a gas. A coefficient of cubical expansion has been defined as the ratio

$$\frac{\text{Increase in volume}}{\text{Original volume}} \text{ per degree rise of temperature.}$$

In connexion with solids and liquids we did not state at what temperature the original volume should be measured. The value found for the coefficient will depend on that temperature since, for instance, the volume of a body at 20° C. will be greater than that at 10° C., so that a bigger denominator for the fraction will be obtained if original volume is measured at the former temperature instead of at the latter one. In the case of solids and liquids, the expansion is so small compared with the whole volume that no appreciable difference is made to the results by taking original volumes at different temperatures. In the case of gases, however, the expansion is so great that a coefficient based on an original volume at 20° C. would be appreciably different from one based on a volume at 10° C., and coefficients must always be related to an original volume at some standard temperature. It is convenient to take 0° C. as the standard temperature, so that the coefficient of expansion of a gas is defined as:—

$$\frac{\text{Increase in volume}}{\text{Original volume at } 0^\circ \text{ C.}} \text{ per degree rise in temperature.}$$

Charles' Law

A measurement of the coefficient of expansion of air may be made as follows. A uniform tube, about 50 cm. long and 1 mm. in bore, is sealed at one end after a short pellet of mercury has been drawn to about its mid-point (Fig. 162). Thus a quantity of air is enclosed between the pellet and the closed end. This tube is tied to a half-metre scale and supported with its open end upwards in a deep vessel of water. A thermometer is placed in the water near the tube. Since the tube is uniform, each centimetre of it may be considered to contain one unit of volume, and measurements of the distance from the closed end to the lower end of the mercury pellet may be taken as measurements of the volume of the enclosed air.

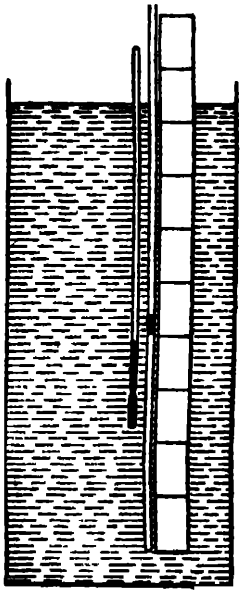


FIG. 162.

The water in the vessel is cooled with ice until the thermometer registers 0° C., and a measurement of the volume of the enclosed air is then taken from the scale. The water is then heated, either by passing in steam or by means of an electrical immersion heater, and kept well stirred until the thermometer registers a steady temperature, and the pellet, which has moved upwards owing to the expansion of the enclosed air, remains stationary. Thermometer and scale readings are then taken again. The measurements have been taken with the enclosed air at constant pressure, as the pressure throughout the experiment has been that of the atmosphere plus that of the pellet of mercury. The results may be dealt with as follows:—

1st temperature, 0° C. 2nd temperature, 99·5° C.

1st volume 26·5 units, 2nd volume, 36·0 units.

Thus a volume of 26·5 units at 0° C. expands by 9·5 units when heated through 99·5° C.;

$$\therefore \text{ Coefficient of expansion} = \frac{9{\cdot}5}{26{\cdot}5 \times 99{\cdot}5} = {\cdot}0036.$$

Accurate measurements give a value equal to $\frac{1}{273}$ (about 0·00366). This value was first announced by a Frenchman named Charles, who stated the law that the volume of a fixed mass of gas at constant pressure increases by $\frac{1}{273}$ of its volume at 0° C. for each degree Centigrade its temperature rises.

Absolute Temperature

Using Charles' Coefficient, calculations on the expansion of gases may be made in the usual way if the volume at 0° C. is given, but as this is not always known, it is convenient to deal with them in another way.

If, in the experiment on page 196, a number of readings at intermediate temperatures are taken, and volumes plotted against temperatures, a straight line will be obtained as in Fig. 163. This shows that the expansion is regular. If the line of the graph is produced to the left, it cuts the temperature axis at — 273° C. It follows from this that the volume is proportional to the number of degrees the temperature is above — 273° C. For example:—

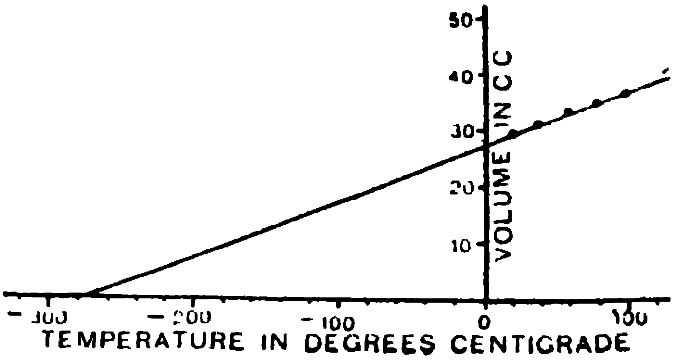


FIG. 163.

$$\frac{\text{Volume at } 20^\circ \text{ C.}}{\text{Volume at } 60^\circ \text{ C.}} = \frac{20 + 273}{60 + 273}.$$

If, then, the temperature axis is numbered from zero at — 273° C. as shown in Fig. 164, the volume will be directly proportional to the temperature on the scale so obtained for—

$$\frac{RS}{PQ} = \frac{OR}{OP} = \frac{373}{233}.$$

From the graph it appears that the volume of the mass of gas would become zero at — 273° C. This does not really happen as the

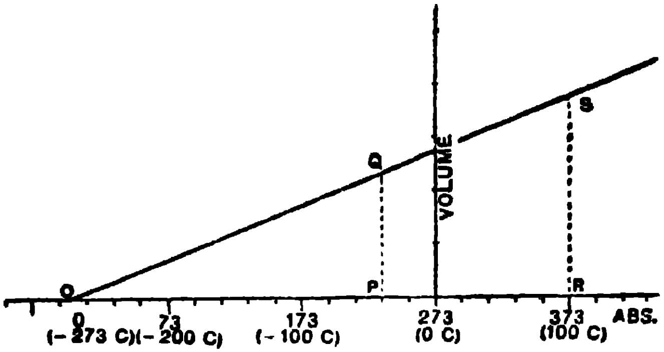


FIG. 164.

gas would liquefy before reaching so low a temperature, and the gas coefficient would then no longer apply to it. But, because theoretically the volume becomes zero at that temperature, − 273° C. is called the **absolute zero** of temperature, and the scale counting from it as zero is called the **absolute scale of temperature.** Using this scale, Charles' Law may be stated in the form that **the volume of a fixed mass of gas at constant pressure is directly proportional to its absolute temperature.** This may be written:—

$$\frac{\text{New volume}}{\text{Old volume}} = \frac{\text{New absolute temperature}}{\text{Old absolute temperature}};$$

$$\therefore \text{ New volume} = \text{Old volume} \times \frac{\text{New absolute temp.}}{\text{Old absolute temp.}}.$$

EXAMPLES.—(1) *A mass of gas measures 200 c.cm. at 17° C. What will be its volume if it is heated to 92° C., its pressure remaining unaltered?*

$$17^\circ \text{ C.} = (17 + 273)^\circ \text{ Abs.} = 290^\circ \text{ Abs.}$$

$$92^\circ \text{ C.} = (92 + 273)^\circ \text{ Abs.} = 365^\circ \text{ Abs.};$$

$$\therefore \text{ New volume} = \frac{200 \times 365}{290} = 252 \text{ c.cm. (approx.).}$$

N.B.—Temperatures must be converted to the absolute scale before working out the volume. As in Boyle's Law calculations, the multiplying ratio is got right way up by considering whether the change makes the new volume greater or less than the old one.

(2) *A mass of gas measures 250 cub. in. when its temperature is 27° C. What is its temperature when its volume is 180 cub. in. at the same pressure as before?*

$$27^\circ \text{ C.} = (27 + 273)^\circ \text{ Abs.} = 300^\circ \text{ Abs.};$$

$$\therefore \text{ New } \textit{absolute} \text{ temperature} = \frac{300 \times 180}{250} = 216 \text{ degs.};$$

$$\therefore \text{ New temperature} = (216 - 273)^\circ \text{ C.} = -57^\circ \text{ C.}$$

Simultaneous Changes of Temperature and Pressure

Both Boyle's Law and Charles' Law are satisfied by the relation—

$$\frac{\text{New volume} \times \text{New pressure}}{\text{New absolute temperature}} = \frac{\text{Old volume} \times \text{Old pressure}}{\text{Old absolute temperature}}$$

which may be expressed in symbols thus:—

$$\frac{V_2P_2}{T_2} = \frac{V_1P_1}{T_1}.$$

If the temperature is constant, that is $T_2 = T_1$, the equation gives $V_2P_2 = V_1P_1$, which is in agreement with Boyle's Law; and if the pressure is constant, that is $P_2 = P_1$, it gives $\frac{V_2}{T_2} = \frac{V_1}{T_1}$, which agrees with Charles' Law. Thus the equation may be taken to express the relation between the initial and final state of a mass of gas after any change of temperature or pressure or of both. Therefore we have as a general relation for a mass of gas,

$$\frac{PV}{T} = R \quad \text{or} \quad PV = RT,$$

where P is the pressure when the volume is V and the *absolute* temperature is T. R is a constant for the particular mass of gas considered. Gram-molecular quantities of all gases give the same value of R, and that particular value is known as the gas constant, and the equation PV = RT is often referred to as the general gas equation. This relation will enable us to solve problems on the change of volume of a mass of gas when pressure and temperature change at the same time. From the equation $V_2P_2/T_2 = V_1P_1/T_1$, we can write—

$$V_2 = \frac{V_1P_1}{T_1} \times \frac{T_2}{P_2} = \frac{V_1P_1T_2}{P_2T_1}.$$

This may be written in words as follows:—

$$\text{New volume} = \frac{\text{Old volume} \times \text{Old pressure} \times \text{New abs. temperature}}{\text{New pressure} \times \text{Old abs. temperature}}.$$

EXAMPLE.—*A mass of gas measures* 250 *c.cm. when at* 62° *C. and* 80 *cm. pressure. What would be its volume when at* 17° *C. and* 75 *cm. pressure?*

$$62° \text{ C.} = (62 + 273)° \text{ abs.} = 335° \text{ abs.}$$

$$17° \text{ C.} = (17 + 273)° \text{ abs.} = 290° \text{ abs.}$$

$$\text{New volume} = \frac{250 \times 80 \times 290}{75 \times 335} = 231 \text{ c.cm. (approx.).}$$

Note.—In writing down the expression consider the pressures first. Change from 80 cm. to 75 cm. would *increase* the volume, so multiply by $\frac{80}{75}$. Then take the temperatures. Change from 62° C. to 17° C. would *decrease* the volume, so multiply by $\frac{290}{335}$.

Chemists frequently wish to compare the gas measurements of one man with those of another. Direct comparison would be useless if the measurements were made under different temperature and pressure conditions, so it is usual for chemists to calculate from their actual measurements the volume that the gas would occupy under standard conditions which are 0° C. and 76 cm. pressure. These conditions are frequently represented by S.T.P. (standard temperature and pressure) or N.T.P. (normal temperature and pressure).

Example.—*A gramme of metal liberated* 953 *c.cm. of hydrogen measured at* 20° *C. and* 80 *cm. pressure. Calculate the volume it would occupy at S.T.P.*

$$20^\circ \text{ C.} = (273 + 20)^\circ \text{ abs.} = 293^\circ \text{ abs.}$$

$$0^\circ \text{ C.} = 273^\circ \text{ abs.}$$

$$\text{New volume} = \frac{953 \times 80 \times 273}{76 \times 293} = 935 \text{ c.cm.}$$

Change of Pressure at Constant Volume

If a gas is heated under conditions which prevent its expansion, for instance in a closed vessel, its tendency to expand will cause it to exert an increased pressure. The general expression for the gas laws will give us the relation between temperature and pressure in that case. From

$$\frac{V_2P_2}{T_2} = \frac{V_1P_1}{T_1},$$

we have, if volume is constant, that is if $V_2 = V_1$:—

$$\frac{P_2}{T_2} = \frac{P_1}{T_1},$$

that is, there is just the same relation between pressure and temperature at constant volume as between volume and temperature at constant pressure, or **the pressure of a fixed mass of gas at constant volume is directly proportional to its absolute temperature.**

The increase in pressure of air and other gases when heated at constant volume was thoroughly investigated, and the law first established, by Regnault: his apparatus worked on the same principle as that dealt with below.

The law may be verified by means of the apparatus shown in Fig. 165. The bulb A contains air and is connected by capillary tubing, E, to a wider tube which in turn is connected by a considerable length of rubber tubing to another vertical tube. The air in A is enclosed by mercury, as shown. A fixed level B is marked on the wider tube. In the diagram it is marked by a point projecting into the tube.

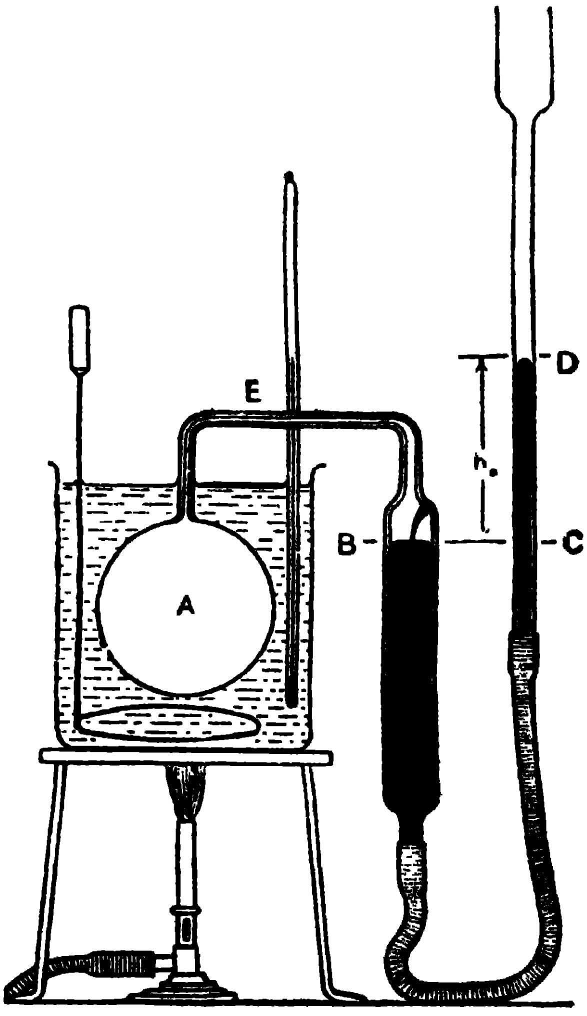


Fig. 165.

A is immersed in water in a large vessel into which a thermometer also dips. The water is well stirred, and when movement of the mercury ceases, showing that the air in A is at the temperature of the water, the right-hand tube is raised or lowered until the left-hand mercury surface is just at the level B. The difference, *h*, between the heights of the two mercury surfaces is then measured and a reading of the thermometer is taken. The water is then heated to a new temperature, the mercury surface again adjusted to B, and readings are taken as before. This is repeated for a number of temperatures. A reading of the atmospheric pressure is also taken from the barometer.

Since the mercury surface was always adjusted to B before readings were taken, they have all been taken with the air in A at a constant volume. Also in each case the pressure supported by the air in A would be equal to atmospheric pressure plus the measured mercury column *h*. Hence the results may be tabulated as follows:—

Height of Barometer (A) = 76·5 cm.

Difference in Mercury Levels (h)	Total Press. ($A + h$)	C. Temp. (t)	Abs. Temp. ($t + 273$)	$\frac{\text{Total Press.}}{\text{Absolute Temp.}}$
2·5 cm.	79 cm.	20°	293°	$\frac{79}{293} = \cdot 270$
4·2 cm.	80·7 cm.	27°	300°	$\frac{80\cdot 7}{300} = \cdot 269$
6·7 cm.	83·2 cm.	35°	308°	$\frac{83\cdot 2}{308} = \cdot 270$
10·5 cm.	87·0 cm.	48°	321°	$\frac{87\cdot 0}{321} = \cdot 271$
14·7 cm.	91·2 cm.	66°	339°	$\frac{91\cdot 2}{339} = \cdot 269$

Approximately constant values are obtained for the ratio in the last column, thus verifying the relation.

Alternatively, if a reading at 0° C. is taken, a coefficient of pressure can be calculated from $\frac{\text{Increase in pressure}}{\text{Original pressure at } 0^\circ \text{ C.} \times \text{Rise in temperature}}$. This should give a value equal to Charles' Coefficient.

Air Thermometers

If the gas laws are assumed either the constant pressure apparatus of Fig. 162 or the constant volume apparatus of Fig. 165, without the mercury thermometers, may be used for measuring temperatures. The processes are as described in the experiments on pages 196 and 201. A fixed scale cannot be marked on these thermometers as their readings at a given temperature depend on atmospheric pressure. Therefore a preliminary set of readings at a known temperature must first be made. This is usually done by immersion in water which has been cooled to 0° C. by adding ice until some remains unmelted after well stirring. A set of readings are then taken at the temperature it is desired to measure.

Examples.—(1) *A constant pressure air thermometer gave a reading of 47·5 units of volume when in ice-cold water, and of 67·0 units when immersed in a boiling liquid. Calculate the boiling point of the liquid.*

$$0^\circ \text{ C.} = 273^\circ \text{ abs.}$$

$$\text{New absolute temperature} = \frac{273 \times 67}{47\cdot 5} = 385 \text{ degrees;}$$

point of liquid 112° C.

(2) *With a constant volume air thermometer in ice-cold water the mercury surface in the open tube was 3·2 cm. below that in the tube connected to the bulb. When the bulb was placed in an oven the mercury in the open tube had to be raised to 92·7 cm. above the other surface to maintain the volume of air constant. The barometer stood at 77·7 cm. What was the temperature of the oven?*

$$\text{1st pressure} = 77{\cdot}7 - 3{\cdot}2 \text{ cm.} = 74{\cdot}5 \text{ cm.}$$

$$\text{2nd pressure} = 77{\cdot}7 + 92{\cdot}7 \text{ cm.} = 170{\cdot}4 \text{ cm.}$$

$$0^\circ \text{ C.} = 273^\circ \text{ Abs.};$$

$$\therefore \text{ New absolute temperature} = \frac{273 \times 170{\cdot}4}{74{\cdot}5} = 624 \text{ degs.};$$

$$\therefore \text{ Temperature of oven} = (624 - 273)^\circ \text{ C.} = 351^\circ \text{ C.}$$

Air thermometers have the following advantages:—

(1) They are very sensitive owing to the large coefficient of expansion of air.

(2) The expansion of air is very regular.

(3) They can be used over a wide range of temperatures.

Their disadvantages are:—

(1) They are large and not suitable for taking temperatures of small enclosures or small volumes of liquids, etc.

(2) They cannot be permanently graduated to give direct readings of temperatures and their use involves making a preliminary experiment as a known temperature each time they are used, and a calculation to give the required temperature.

Owing to these disadvantages they are only used when extremely accurate temperature measurements are required. Ordinary thermometers are often tested for errors in their scales by comparison with air thermometers.

QUESTIONS ON CHAPTER XVII

1. State three important facts relating to the expansion of gases. What are the chief advantages and disadvantages of air as a thermometric substance? [L.U.

2. State Charles' Law and describe an experiment to verify it. Explain how Charles' Law leads to the idea of an absolute zero of temperature.

3. State Boyle's Law and Charles' Law, and show that the equation $\frac{PV}{T}$ = a constant expresses both laws.

4. Calculate the missing numbers in the following table, assuming that pressure is constant in each case.

1st Volume	1st Temp.	2nd Vol.	2nd Temp.
200 c.cm.	20° C.		75° C.
60 cub. in.	− 13° C.		20° C.
1 litre	90° C.		0° C.
250 c.cm.	17° C.	300 c.cm.	
325 c.cm.	72° C.	125 c.cm.	
10 cub. ft.	30° C.	25 cub. ft.	

5. Calculate the missing numbers in the following table, assuming volume to remain constant in each case:—

1st Temp.	1st Pressure	2nd Temp.	2nd Pressure
27° C.	75 cm.	92° C.	
91° C.	25 lb. per sq. in.	0° C.	
26° C.	65 cm.	− 13° C.	
0° C.	76 cm.		95 cm.
100° C.	112 cm.		70 cm.

6. Calculate the missing numbers in the following table:—

1st Vol.	1st Temp.	1st Press.	2nd Vol.	2nd Temp.	2nd Press.
5 litres	13° C.	80 cm.		0° C.	76 cm.
205 cub. in.	0° C.	76 cm.		78° C.	82 cm.
325 c.cm.	26° C.	92 cm.	425 c.cm.	65° C.	
400 c.cm.	98° C.	104 cm.	200 c.cm.		78 cm.

7. A mass of gas is collected over mercury when the barometer reads 82 cm. and air temperature is 18° C. The gas measures 152 c.cm. and the mercury stands 2·5 cm. higher in the tube containing the gas than in the trough in which it is inverted. Calculate the volume which would be occupied by the gas at S.T.P.

8. A motor tyre is inflated to a pressure of 24 lb. per sq. in. when the air temperature is 68° F. What will be the pressure in it if the temperature rises to 95° F.? Assume that the volume of the tyre does not change.

9. 100 litres of oxygen at atmospheric pressure and at 18° C. are compressed into a cylinder whose internal capacity is 10 litres. What will be the pressure inside the cylinder?

The cylinder is guaranteed to withstand a pressure of 200 lb. per sq. in. At what temperature would there be a danger of it bursting?

Take atmospheric pressure as 15 lb. per sq. in.

10. Describe in detail the measurement of the boiling point of a liquid with either a constant volume or a constant pressure air thermometer. Explain how the temperature would be calculated from your measurements.

11. Define the *coefficient of expansion* of a gas when the pressure upon it is kept constant.

Describe how this coefficient can be measured experimentally, showing how the result is calculated from the observations taken. [L.U.

12. Define the *coefficient of increase of pressure of a gas at constant volume*, and describe an experimental method of determining the value of this coefficient. [L.U.

13. Explain fully what is meant by the statement that "the coefficient of expansion of a gas at constant pressure is $\frac{1}{273}$ per degree C. Describe how you would verify this statement experimentally for air.

A mass of air occupies 145 c.cm. at 17° C. and atmospheric pressure. Calculate its volume (*a*) when heated to 100° C., (*b*) when cooled to − 10° C., the pressure remaining the same in both cases. [J.M.B.

CHAPTER XVIII

TRANSFERENCE OF HEAT

Powerful engines burn up large quantities of fuel to provide the heat necessary for their working. To avoid waste of fuel it is important to ensure that as much as possible of the heat is directed to the places where it will be useful. On the other hand in the engine of a motor car, a good deal of heat which cannot be used is generated, and to prevent overheating it must be got rid of as quickly as possible. These examples illustrate the importance of a knowledge of the ways in which heat may be transferred from one place to another.

There are three ways of doing this.

Conduction

Coat a stout piece of copper wire, about a foot long, with paraffin-wax, and place one end in a Bunsen flame. Observe that the wax quickly melts near the flame, and gradually melts further and further along the wire. It appears as though the heat flows along the wire from the heated end. In solids the molecules are very near together so that those which are directly heated by the flame can pass on some of their heat to their neighbours, which in turn can pass some on to the next layer, and so on. Heat transferred in this way is said to be conducted along the body.

Convection

If a hand is held some distance above a lighted Bunsen burner a much greater heating effect is felt than when it is held at an equal distance from the side of the flame. This indicates that the heat is not readily conducted by the air, for in that case we should expect it to be conducted equally in all directions from the flame.

If a wide glass tube is placed around the Bunsen flame and a piece of smouldering paper is held near its lower end, smoke from the paper will be seen to be drawn into the lower end of the tube and rise up it, while an upward flow of hot air will be felt by a hand placed above the tube. This indicates that heat from the burner is transferred through the air by the heated particles of air themselves moving upwards from the flame. When heat is transferred by movements from one place to another of the heated particles, convection is said to take place.

Radiation

When an electric fire is switched on, heat from it can be felt at a considerable distance in front of it. We have seen that heat is not appreciably conducted by air and that it travels upwards by convection, so in this case the heat must be transferred by some other method.

On a cool day, while you may feel warm when sitting in front of such a fire, immediately it is switched off you find that the air around you is cool, so the fire has a heating effect on your body without heating the air in between. The same thing may be noted on a sunny spring day. Your body is warmed by the sun but immediately a cloud obscures the sun you find that it has not warmed the air around you. When a hot body has a heating effect on other bodies without heating the intervening medium radiation is said to take place.

In connexion with the heating of the earth by the sun, it may be noted that the action takes place through millions of miles of vacuum where, since there are no particles, transmission by either conduction or convection is impossible.

Later it will be shown that, in radiation, the source of heat emits energy which is transmitted as a *wave motion* and which is transformed again into heat when the waves fall on other bodies.

Conductivity of Solids

Obtain a number of wires of equal diameters but of different substances. Twist them together at one end and spread them out fanwise, as shown in Fig. 166. Pass the free ends through a cardboard screen and support them on a sheet of heat-sensitive paper. This is paper which has been soaked with a solution of cobalt chloride so that when heated it will turn green.

Place a lighted Bunsen burner below the twisted ends, and after a time examine the paper. Green marks will be found to have been made to various lengths under the different wires, showing that heat is conducted more readily through some substances than through others. Note that the cardboard screen prevents direct heating of the paper by radiation from the flame.

It will be found that the paper is marked for a distance of several inches by metal wires, but if a glass rod is tested, the paper will hardly be marked even when the end of the rod in the flame is melting.

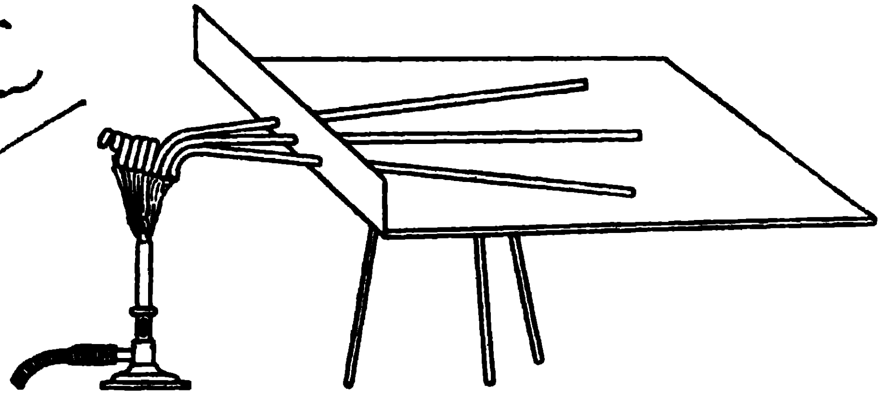


Fig. 166.

Similar experiments will show generally that *metals are good conductors* and *non-metals poor conductors.* Metals vary in their conductivities as shown by different lengths on the paper being marked by different metal wires. The best conductors are silver, copper, and gold, in that order. Other common metals have conductivities in the order aluminium, zinc, platinum, iron, tin, lead.

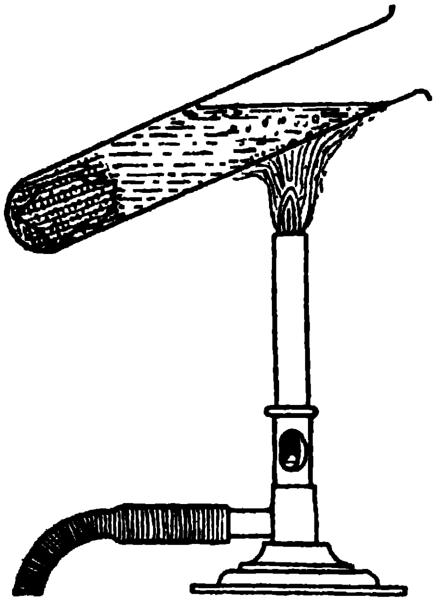


Fig. 167.
Water is a Bad Conductor.

Conductivity of Liquids and Gases

Liquids and gases are usually bad conductors. To test them for conductivity we have to find if heat will travel downwards through them, for it will travel upwards by convection.

A piece of wire gauze is wrapped round a piece of ice to weight it, so that it will sink to the bottom of a test-tube of water. The tube is then heated as shown in Fig. 167. The ice still remains unmelted even when the water at the top of the tube boils. Evidently very little heat is conducted through the water to the ice.

A liquid which does conduct heat well is mercury, which is a liquid metal. This can be shown by melting a little wax in the bottom of a long tube, closed at one end, and allowing it to solidify again. The tube is then filled up with mercury and heated near the top. The wax quickly melts.

That air is a bad conductor may be shown by covering one hand with coarsely broken chalk, which will contain much air between the chalk particles, and the other with finely powdered chalk, which will contain little air. If a hot metal ball is placed on each hand in turn, the heat will be felt much more through the finely powdered chalk than through the coarse particles.

The Process of Convection

Place some water in a flask and drop a few pieces of solid litmus to the bottom of it (Fig. 168). Heat it with a small flame. Streaks of colour will be seen to travel in the direction indicated by the arrows.

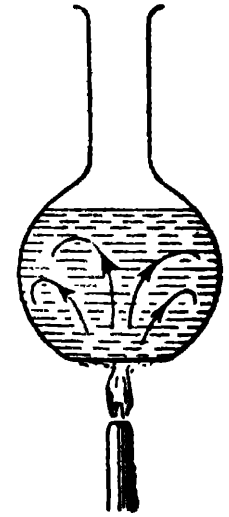


Fig. 168.

Divide a tall, wide jar into two compartments by inserting a strip of cardboard which does not quite reach the bottom. Lower a lighted candle fixed to a piece of wire down one side, and hold a smouldering piece of paper above the other side. Smoke from the paper will be seen to travel through the jar, as indicated in Fig. 169.

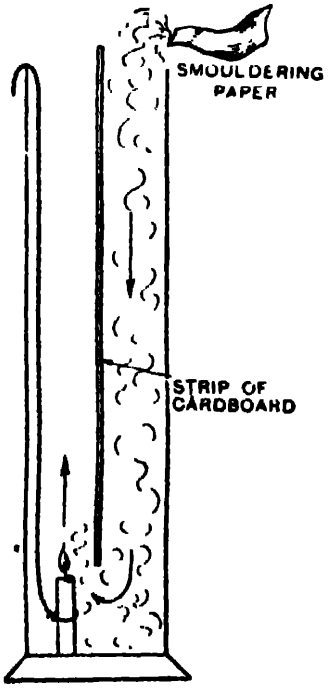


Fig. 169.

The two experiments are similarly explained. The heated portions of the water or air expand and so become less dense. They are therefore displaced by the denser water or air around them and pushed upwards. The cooler water or air flowing to the point of heating in its turn expands and is pushed upwards. Thus, so long as the heating is continued, upward hot currents and downward cooler currents are maintained.

Radiation Apparatus

To detect heat radiation the *differential thermoscope* may be used. This, as illustrated in Fig. 170, consists of two air-filled bulbs connected to a U-tube containing a little liquid. If one bulb is heated more than the other, the air in it will have a greater pressure than that in the other, and the liquid will be driven away from the hotter bulb.

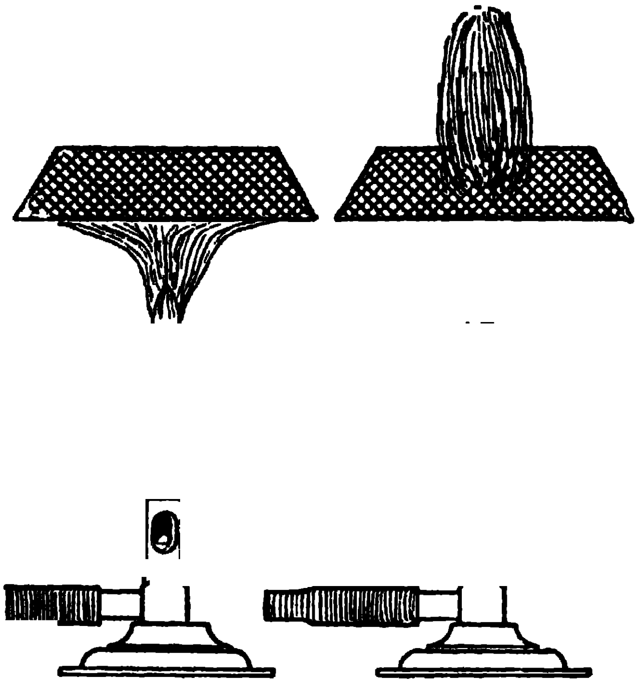


FIG. 174.

Absorbing Powers of Surfaces

Good radiating surfaces readily absorb radiation, while bad radiating surfaces tend to reflect it instead of absorbing it.

To show this, paint a dead black design on a thin sheet of bright tin. Paste a sheet of heat-sensitive paper on the back of the tin. Place the tin with the painted side facing a source of radiation. The design will appear on the paper on the back, those parts of the paper behind the black portions of the design turning green while those behind the bright portions remain white. Many other experiments can be arranged to show the fact.

Applications

(1) WIRE GAUZE.—So that the flame shall not play directly on it, a glass vessel which is to be heated over a Bunsen burner is usually stood on wire gauze, through which the flame does not pass. If a piece of gauze is held an inch or two above a burner, the gas turned on, and a light applied above the gauze, the gas burns above the gauze but the flame does not extend below it to the burner (Fig. 174).

The explanation is that the wires of the gauze are good conductors, so that heat is quickly conducted away from the place where the flame is in contact with it, and as a result, the temperature on the other side of the gauze does not rise sufficiently to ignite the gas.

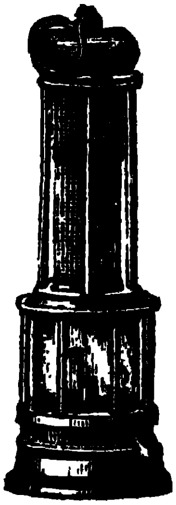


FIG. 175.

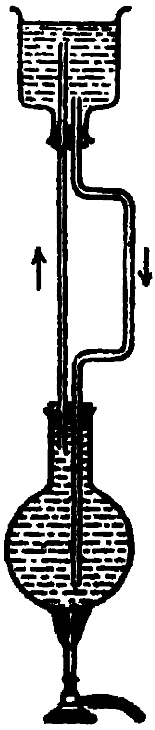


FIG. 176.

This principle is applied in the *Davy Safety Lamp* for miners, in which a small oil lamp has a glass cylinder, surmounted by a cylinder of gauze, completely enclosing the flame (Fig. 175). The gauze allows the necessary air to enter the lamp. If combustible gas enters with the air, it will burn inside the lamp but the temperature outside the gauze will not be raised sufficiently to ignite the gas outside and cause an explosion.

(2) HOT-WATER SYSTEMS.—Fig. 176 shows apparatus which may be fitted up to illustrate the principle of a domestic hot-water system. Note particularly that the bent tube reaches nearly to the bottom of the flask and only just passes into the reservoir at the top, while this arrangement is reversed for the straight tube. If some colouring matter is put in the water in the reservoir the water will be seen to travel as indicated by the arrows when the flask is heated. For the reasons given on page 209, hot convection currents tend to rise and so flow up the straight tube to the reservoir, while water flows back through the other tube to the flask.

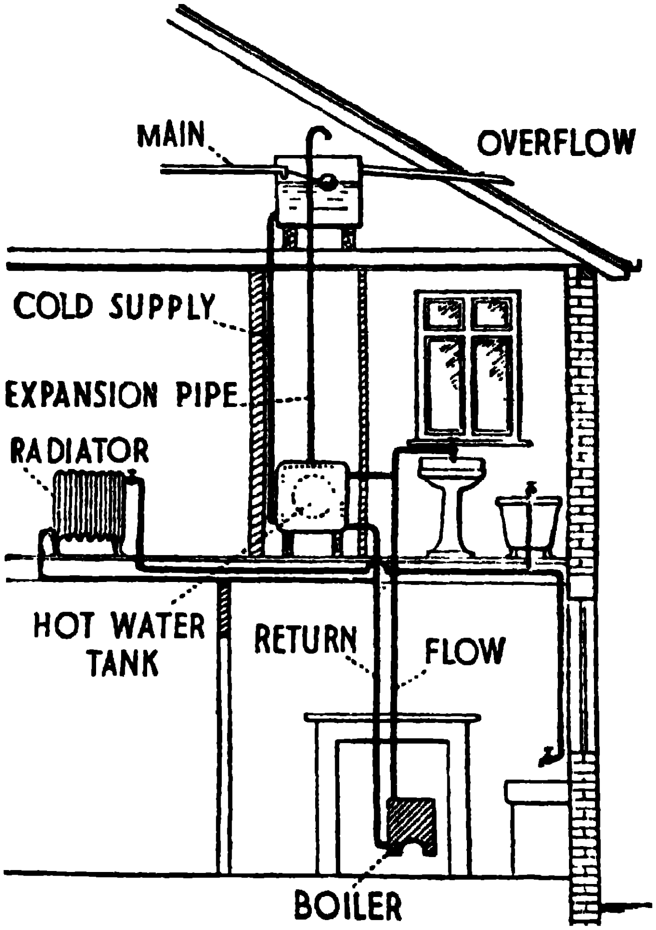


FIG. 177.

Compare this with Fig. 177, which shows a hot-water system in a house, noting that the pipe from the top of the boiler to the top of the tank will correspond with the straight tube and that connecting the bottom of the boiler to the tank corresponds to the bent tube. The radiator is connected to both these pipes, so that water will circulate round it as well as round the tank. The roof tank provides a pressure head to drive the water out of the taps when they are opened.

It should be noted that conduction also plays a part in this system.

The boiler plates should be of good conducting metal so that heat may readily pass from the fire to the water, and, as mentioned in the next section, both boiler and tank should be lagged to prevent loss of heat. The radiator also is made up of good conducting metal, so heat readily passes from the water to its outer surface.

Heat is distributed from the radiator partly by radiation but also by convection. The air in contact with the radiator becomes heated and flows upwards, while cool air from other parts of the room flows to the radiator to be heated in its turn.

(3) Heat Insulation.—Heat which escapes from boilers and steam pipes is wasted, and in addition makes the boiler rooms unpleasantly hot. For this reason, exposed parts of boilers, etc., are often "lagged" with asbestos or some other bad conductor to reduce the loss of heat.

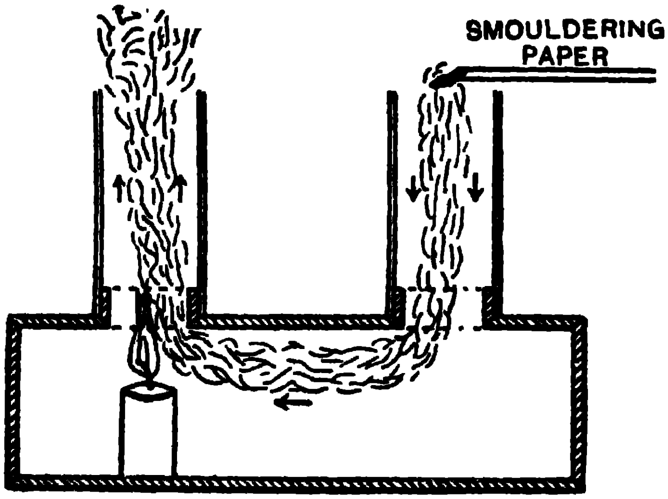


Fig. 178.

Substances of a loose texture, which contain a large amount of trapped air, make good heat insulators. Thus clothing and blankets, which are designed to keep within the body heat produced there, should be of a light fleecy texture. Also domestic hot-water tanks are often lagged by building a casing round them and filling it up with cork shavings. To keep a lump of ice in warm weather it should be wrapped round with loosely-woven material which will prevent the heat outside from getting in to it.

(4) Ventilation.—Convection is the main factor in ventilation. Air which has been warmed by being breathed tends to rise. Hence, if exits are provided for it near the top of the room and inlets for cool air near the bottom, a convection circulation will ensure that fresh air replaces that which has been used.

Fig. 178 shows an apparatus to demonstrate how a mine may be ventilated by heating the air at the bottom of one of a pair of shafts connected with it. The apparatus is conveniently made from a box stood on its side, with a sheet of glass in place of its lid. The explanation of it should be obvious.

(5) WINDS AND OCEAN CURRENTS.—These are convection currents on a large scale. Where the surface of the earth is hot, the air in contact with it becomes heated and is displaced by the cooler air around it which is at a higher pressure. Thus there is an upward flow of air at hot places on the earth's surface and a downward flow at cold places, with a flow along the surface from the cold to the hot places. Ocean currents may be similarly explained.

(6) THE VACUUM OR "THERMOS" FLASK.—This is largely used to-day to keep liquids hot for long periods. It was invented by Sir James Dewar for storing liquid air, which has to be kept very cold.

Fig. 179 shows its construction. A double-walled glass vessel is silvered on the outer surface of the inner wall and the inner surface of the outer wall. Air is withdrawn from the space between the walls which is then sealed. In the domestic form this is protected by a metal case. It rests on a cork at the bottom of the case and is secured at the neck with a pad of felt or a ring of rubber.

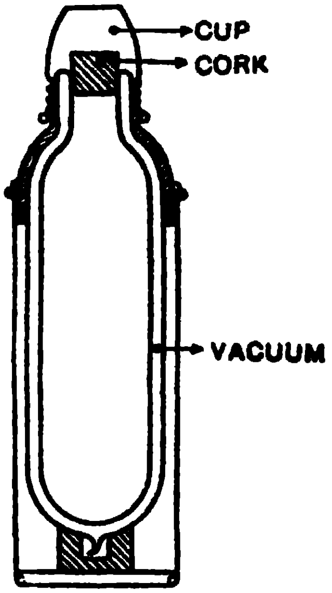


FIG. 179.

When a hot liquid is placed in it, heat can not escape by either conduction or convection across the vacuum. The silvering on the outside of the inner wall makes that wall a bad radiator, while the silver on the outer wall tends to reflect back any heat that is radiated.

The cork, air, and felt between the flask and the container are all bad conductors, so that heat which is conducted along the glass, itself a bad conductor, cannot readily pass to the container. The cork in the neck of the flask, and the cup over that, prevent loss of heat by convection.

Starting from the outside, it is easy to show in a similar way how difficult it is for heat to enter the flask from outside: hence double-walled vacuum vessels of this and similar types not only keep hot drinks hot, but they also keep cold drinks cold.

QUESTIONS ON CHAPTER XVIII

1. Explain the process by which heat is transferred through a solid, and describe an experiment to compare the conducting powers of two metals.

Describe (*a*) an appliance where advantage is taken of the good thermal conductivity of a solid, (*b*) one where the property of poor thermal conductivity is utilised. [L.U.

2. Why does your body keep warm on a cold day and a lump of ice remain cold on a hot day if each is wrapped in a blanket ?

3. Give reasons for each of the following:—

(*a*) Saucepans are usually made of metal.

(*b*) A layer of felt is often placed between the boards and the tiles in a house roof.

(*c*) A metal teapot often has an ebony handle.

(*d*) The bottoms of kettles to be heated on electric hot plates are ground to fit closely to the plates.

4. Why is straw often wrapped round out-of-doors water pipes in the winter ? Is it best to pack it tightly or loosely ?

5. Explain the process of *heat convection.*

Describe how use is made of this process in the heating of a building. [L.U.

6. Describe experiments to show that convection takes place (*a*) in liquids, (*b*) in gases.

Explain the observations which would be made during the experiments.

7. Explain why (*a*) it is hotter in the gallery than on the floor of a crowded theatre, (*b*) night frosts are often more severe in a hollow amongst hills than on the high ground, (*c*) on a hot day the surface water of a pond is warmer than the water below, but on a day when it is nearly freezing, the surface water is the colder.

8. Describe and explain, with the aid of a diagram, the cooling system of a motor car engine. Why is the radiator made in the form of a " honeycomb " of tubes ?

9. Describe experiments to illustrate that good radiating surfaces are good absorbing surfaces. What would you expect to observe if a red-hot metal ball were placed half-way between the bulbs of a differential thermoscope, one of which had been blackened and the other silvered ? Explain your answer.

10. An open copper calorimeter contains a quantity of hot water. Explain the various processes by which it can lose heat, and suggest methods of reducing the loss due to each process. [L.U.

11. Explain the nature of the three chief methods of heat transference. Show how the " thermos flask " is designed to minimise the effects of those methods. [L.U.

12. Describe experiments (one in each case) to show that (*a*) copper is a better conductor of heat than iron, (*b*) water is a bad conductor of heat, and (*c*) a blackened surface is a better absorber of heat than a bright one. [L.U.

13. Briefly distinguish between *heat radiation, heat convection,* and *heat conduction.*

How is it possible to show (*a*) which is the better conductor of heat —iron or fireclay, (*b*) which is the better radiator of heat—a roughened or a smooth surface of a material ?

Explain why the radiants (fireclay pillars) used in a gas fire are (i) roughened, (ii) made in skeleton form with open front, rather than made solid. [J.M.B.

14. How does the density of a gas vary (*a*) when the pressure is changed at constant temperature, (*b*) when the temperature is changed at constant pressure ?

Describe *one* practical application of a variation in density of a gas such as air.

In large modern buildings it is a growing practice to warm the rooms by means of heated panels in the ceiling. Explain how the rooms become heated, and suggest *one* advantage of the method. [J.M.B.

CHAPTER XIX

QUANTITY OF HEAT

Coal bills, gas bills, and electricity bills are, in the main, demands for payment for means of producing heat. So much is this the case that gas bills state the amount of heat for which payment is demanded and electricity bills state the number of units of energy which have been consumed, from which the amount of heat supplied may be calculated. It is clear from this that measurements of quantities of heat may be very important.

Factors Involved in Quantity of Heat

There is clearly a relation between the temperature of a body and the amount of heat it contains. It is assumed that, so long as the state of the body is not changed, its rise of temperature will be proportional to the quantity of heat given to it.

The experiment on page 162 shows that temperature alone is not a measure of the amount of heat in a body. A body at high temperature may contain less heat than one at lower temperature. In the case mentioned the difference was evidently connected with the different masses of the bodies. Just as a fixed volume of water will fill a narrow jar to a greater depth than a wide jar, so a fixed quantity of heat will raise a small mass to a higher temperature than a large mass of the same substance. It is assumed that equal masses of the same substance will require equal quantities of heat to raise their temperatures by equal amounts.

The quantity of heat in a body also depends on its substance. This may be verified as follows. Take two equal beakers. Place a quantity of water in one and then place sand in the other until the two balance, so that equal masses of water and sand are taken. Adjust a Bunsen burner till it gives a small steady flame. Place the beaker of water over it and support a thermometer dipping into it. Stir continuously and note the time taken for the temperature to rise through a given range, say 20° C. to 30° C. Repeat with the beaker of sand, avoiding any alteration in the gas flame and timing over the same temperature range as in the case of the water. The second time will be less than the first, so we can say that, when equal masses of sand and water have their temperatures raised equally, less heat is required by the sand than by the water.

Units of Heat

It follows from the above that, in defining a unit of heat, mass, temperature, and substance will all have to be considered. Because it is so frequently used in heat experiments, water is chosen as the standard substance for this purpose. The following are the heat units mostly used:—

(1) **The calorie,** which is **the quantity of heat required to raise the temperature of 1 grm. of water by 1° C.**

(2) **The British Thermal Unit** (B.Th.U.), which is **the quantity of heat required to raise the temperature of 1 lb. of water by 1° F.**

(3) **The Therm,** which is equal to **100,000 B.Th.U.** This unit is used mainly in connexion with gas supply.

Thus, to raise 20 grm. of water through 15° C. requires $20 \times 15 = 300$ calories of heat; 15 lb. of water cooling from 90° F. to 60° F. lose $15 \times 30 = 450$ B.Th.U. of heat.

Generally, heat gained or lost by a mass of *water* = mass × change of temperature, corresponding units of heat, mass, and temperature being used.

Principle of Mixtures

When cold water is added to hot water in the bath the whole comes to one temperature which is higher than that of the cold water and lower than that of the hot. Thus the cold water gains heat, and the hot loses it. It is reasonable to assume that the heat gained by the cold water is that lost by the hot, so that the two quantities are equal. Note that this applies to *heat* gained and lost, and not to rise and fall of temperature, for if a little cold water is added to a large mass of hot, the final temperature will be very little below that of the hot water.

To test this assumption, take two equal beakers. Measure 200 c.cm. (200 grm.) of cold water into one and 150 c.cm. (150 grm.) into the other. Heat the first beaker until the temperature of the water is just above

70° C. Let it cool, keeping it stirred until the temperature is just 70° C. In the meantime take the temperature of the cold water in the other beaker. Pour the hot water in the cold, stir well, and note from a thermometer dipping into it the temperature to which the whole rises.

Repeat the experiment, but this time pour the cold water into the hot when mixing. Results such as the following will be obtained:—

	Hot into Cold	*Cold into Hot*
Temperature of hot water ...	70° C.	70° C.
Mass of hot water	200 grm.	200 grm.
Temperature of cold water ...	20° C.	20° C.
Mass of cold water	150 grm.	150 grm.
Temperature of mixture ...	47·5° C.	49·5° C.
Fall in temp. of hot water ...	22·5° C.	20·5° C.
Heat lost by hot water	(22·5 × 200) cals. = 4500 cals.	(20·5 × 200) cals. = 4100 cals.
Rise in temp. of cold water ...	27·5° C.	29·5° C.
Heat gained by cold water ...	(27·5 × 150) cals. = 4125 cals.	(29·5 × 150) cals. = 4425 cals.

In neither case is the heat gained by the cold water equal to that lost by the hot. This is readily explained. In the first case, the beaker containing the cold water was part of the cold body and would be warmed up during the mixing, taking a part of the heat lost by the hot water. Thus the cold water gained only a part of that lost by the hot.

In the second case the beaker containing the hot water would help to warm the cold water poured into it. Thus the cold water gained more heat than the hot lost.

To eliminate this unknown quantity of heat take averages for the two experiments.

$$\text{Average quantity of heat lost by hot water} = \frac{4500 + 4100}{} = 4300 \text{ cals.}$$

$$\text{Average quantity gained by cold water} = \frac{4125 + 4425}{2} = 4275 \text{ cals.}$$

This gives approximately equal results. There is still a source of error which has not been dealt with. This is that some heat would escape to the surrounding air during the mixing.

Thermal Capacity

The quantity of heat required to raise the temperature of a body by 1 degree is termed the thermal capacity of the body.

The method described in the last paragraph enables the thermal capacity of a vessel to be determined. Thus, in the first case, the beaker in which the water was mixed, as well as the water in it, would be warmed from 20° C. to 47·5° C., that is its temperature would be raised 27·5° C. This was brought about by the heat which was lost by the hot water and not gained by the cold, that is by (4500 – 4125) cals. = 375 cals. Therefore to raise the temperature of the beaker by 1° C. requires $\frac{375}{27\cdot5}$ cals. = 13·6 cals.

In the second case it may be said that in falling 20·5° C. the beaker lost (4425 – 4100) cals. = 325 cals. Therefore, in falling 1° C. it loses $\frac{325}{20\cdot5}$ cals. = 15·8 cals. So the thermal capacity of the first beaker was 13·6 calories per degree C., and that of the second 15·8 calories per degree C.

Water Equivalent

It is frequently useful to consider the mass of water which has the same thermal capacity as a body. This is known as the **water equivalent** of the body. In the case of the first beaker mentioned above, 13·6 calories will raise its temperature 1° C. But 13·6 calories would raise the temperature of 13·6 grm. of water 1° C. Therefore the water equivalent of the beaker is 13·6 grm. Note that the water equivalent of a body is numerically equal to its thermal capacity.

EXAMPLE.—*200 grm. of water at 18° C. are contained in a vessel with a water equivalent of 20 grm. and 250 grm. of water at 100° C. are poured into it. What will be the temperature of the mixture?*

Let the required temperature be t° C. Then:—

$$\text{Fall in temp. of hot water} = (100 - t)^\circ \text{ C.}$$

$$\text{Rise in temp. of cold water and vessel} = (t - 18)^\circ \text{ C.}$$

The cold water and vessel are together equivalent to (200 + 20) grm. of water.

$$\text{Heat lost by hot water} = 250\,(100 - t) \text{ cals.}$$

$$\text{Heat gained by cold water and vessel} = 220\,(t - 18) \text{ cals.}$$

$$\text{Heat lost} = \text{Heat gained};$$

$$\therefore\ 250\,(100 - t) = 220\,(t - 18);$$

$$\therefore\ 25000 - 250t = 220t - 3960; \quad \therefore\ 470t = 28960;$$

$$\therefore\ t = \frac{28960}{470} = 61{\cdot}6;$$

$$\therefore\ \text{Temperature of mixture} = 61{\cdot}6^\circ\ \text{C.}$$

Thermal

Specific Heat

From the results on page 221 it has been shown that 13·6 cals. were required to raise the temperature of the beaker 1° C. Suppose the beaker had a mass of 100 grm. Then it can be said that each gramme of glass in the beaker requires $\frac{13{\cdot}6}{100}$ cals. = ·136 cals to raise its temperature 1° C. This number may be expected to apply to any one gramme of glass, and so is a physical constant of the substance glass and has not particular reference to any special body. It is called the specific heat of glass. The specific heat of a substance is given numerically by the number of calories required to raise the temperature of 1 grm. of the substance by 1° C.

A more general definition of specific heat can be given as follows. The thermal capacity of the beaker was 13·6 cals. per deg. C. It weighed 100 grm., so the thermal capacity of an equal mass of water would be 100 cals. per deg. C. Thus the specific heat of the substance may be said to equal the ratio

$$\frac{\text{Thermal capacity of a mass of the substance}}{\text{Thermal capacity of an equal mass of water}}.$$

The former definition is the more useful to have in mind when working examples, but the latter is the *exact* definition: it indicates that the value of specific heat does not depend on the heat units used in measuring it so long as corresponding units of heat, mass, and temperature are used throughout a calculation.

TABLE OF SPECIFIC HEATS

Aluminium ...	·21	Silver ...	·056	Alcohol ...	·60
Copper ...	·094	Tin ...	·054	Glycerine ...	·58
Iron ...	·113	Zinc ...	·093	Paraffin ...	·52
Lead ...	·0315	Brass ...	·092	Turpentine ...	·42
Mercury ...	·033	Glass ...	·16	Water ...	1·00
Platinum ...	·0322	Ice ...	·50		

The value 1·0 for the specific heat of water follows from the definition. It will be noted that water has a high specific heat compared with other substances, and specific heats of liquids generally are considerably higher than those of solids.

It should be clear from the foregoing that the thermal capacity, and therefore the water equivalent of a vessel is equal to its mass × the specific heat of its substance. For example, 1 grm. of copper (specific heat ·094) requires ·094 cals. to raise its temperature 1° C. Therefore a copper vessel weighing 50 grm. will require ·094 × 50 cals. to raise its temperature 1° C. Also, for any heating or cooling of a body:—

Heat gained or lost = Mass of body × Specific heat × Change of temp.

Thus, to raise the temperature of the above copper vessel by 35° C. would require ·094 × 50 × 35 cals.

Measuring Specific Heats

The mixture principle is used for this purpose. The vessel in which the mixing is done is called a calorimeter, and is usually made of copper or aluminium. Precautions must be taken to avoid loss of heat to the surrounding air when the hot body is placed in the calorimeter, so the outside of the calorimeter should be polished to reduce radiation from it. It is stood on a piece of cork or other poor conducting material inside a wider vessel (Fig. 180). This cork, and the layer of air between the two vessels, tend to reduce loss of heat by conduction. The outer vessel also shields the calorimeter from radiation from outside and from air currents. A cover with small holes for a stirrer and thermometer will prevent loss of heat by convection from the apparatus.

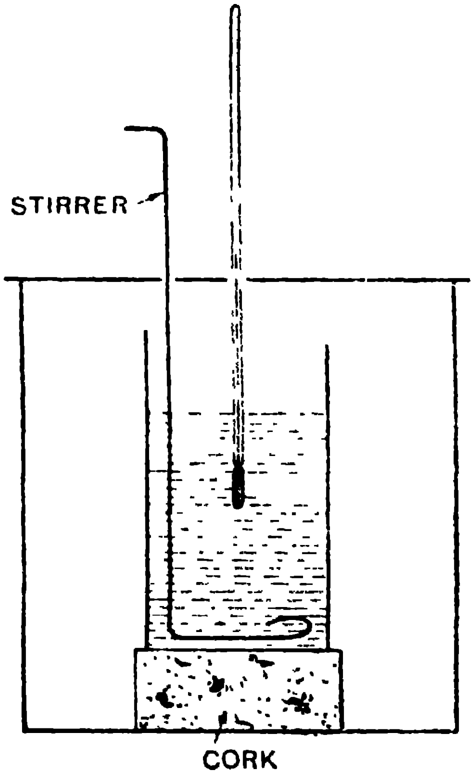


FIG. 180.

The heat gained by the calorimeter and stirrer must also be taken into account, so the stirrer should be of the same material as the calorimeter so that the two together may be taken as one body.

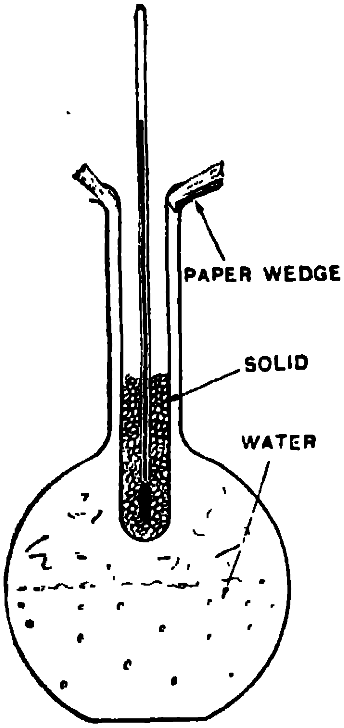


Fig. 181.

(1) Specific Heat of a Solid.—A calorimeter and stirrer are weighed together. The calorimeter is about two-thirds filled with water and weighed again. It is then placed in its shielding vessel. A quantity of the solid is then weighed and heated to about 100° C. Fig. 181 illustrates a convenient way of doing this. The test-tube should fit loosely into the neck of the flask and wedges of paper should be inserted as shown to ensure there being a space between the tube and the neck for the escape of steam. The water is boiled, and when the temperature of the hot solid is steady, the temperature of the water in the calorimeter is taken and the solid quickly poured into it. The " mixture " is well stirred, and the highest temperature recorded by the thermometer in the calorimeter noted. It is best in this case to take account of the heat gained by the calorimeter and stirrer by considering their water equivalent. Since specific heats of copper and aluminium are approximately 0·1 and 0·2 respectively, water equivalents of vessels made of those metals may be taken as one-tenth and one-fifth respectively of their masses.

Results

Mass of calorimeter and stirrer (copper) ... = 60·6 grm.
∴ Their water equivalent ... ... = 6·06 grm.
Mass of calorimeter, water, and stirrer ... = 185·9 grm.
∴ Mass of water ... ... = 125·3 grm.
Mass of solid ... ... = 206·5 grm.
Temperature to which solid heated ... ... = 90° C.
Temperature of cold water ... ... = 16° C.
Temperature of " mixture " ... ... = 18·5° C.
∴ Rise in temperature of water, etc. ... = 2·5° C.
Fall in temperature of solid ... ... = 71·5° C.
Heat gained by water and calorimeter

$$= 2{\cdot}5 \times (125{\cdot}3 + 6{\cdot}06) \text{ cals.}$$

Let s be the specific heat of the solid.

Heat lost by solid = ($s \times 206{\cdot}5 \times 71{\cdot}5$) cals.

But heat lost = Heat gained;

$$\therefore\ s \times 206{\cdot}5 \times 71{\cdot}5 = 2{\cdot}5 \times 131{\cdot}36;$$

$$\therefore\ s = \frac{2{\cdot}5 \times 131{\cdot}36}{206{\cdot}5 \times 71{\cdot}5} = {\cdot}022.$$

(2) Specific Heat of a Liquid.—The process is the same. The liquid is placed in the calorimeter and a solid of known specific heat is used.

Example of Results. 65 grm. of a liquid are placed in an aluminium calorimeter which weighs 10·5 grm. 200 grm. of lead, of known specific heat ·0315, are heated to 100° C. The temperature of the liquid is 15° C. When the lead is dropped into the calorimeter, the temperature rises to 30° C.

Rise in temperature of liquid and calorimeter ... = 15° C.

Fall in temperature of lead ... ... = 70° C.

Let s be the specific heat of the liquid.

Heat gained by liquid ... ... = $s \times 65 \times 15$ cals.

Heat gained by calorimeter ... = $\cdot 21 \times 10{\cdot}5 \times 15$ cals.

Heat lost by lead... ... ... = $\cdot 0315 \times 200 \times 70$ cals.

Heat gained = Heat lost;

$$\therefore\ (s \times 65 \times 15) + (\cdot 21 \times 10{\cdot}5 \times 15) = \cdot 0315 \times 200 \times 70;$$

$$\therefore\ 975s + 33{\cdot}075 = 441{\cdot}0;$$

$$\therefore\ 975s = 407{\cdot}925;\quad \therefore\ s = \frac{407{\cdot}925}{975} = \cdot 418.$$

Calorific Value of Fuels and Foodstuffs

The calorific or heating value of a fuel is the quantity of heat which can be produced by burning a pound of it in the case of a solid or liquid fuel or a cubic foot of it in the case of a gas.

To determine the calorific value of solid or liquid fuels, bomb calorimeters are used (Fig. 182). The bomb consists of a strong iron vessel to which a strong lid can be screwed down, so that a big internal pressure can be withstood. A weighed quantity of the fuel in a crucible is introduced and a thin wire connected to stout insulated leads is dipped into it. Enough oxygen to ensure complete combustion is pumped in under pressure through an opening in the lid, which is then closed with a screw. The bomb is then suspended in a large calorimeter containing water which is shielded in the usual way. The weight of water and the water equivalent of the bomb and calorimeter must be known. The thin wire is heated to redness for a moment by passing an electric current through the leads, and so the fuel is ignited. The rise in temperature of the water as the fuel burns is noted. Then Rise in temperature × (Mass of water + Water equivalent of apparatus) gives the heat produced by burning the weighed amount of fuel.

For gaseous fuels, a calorimeter based on the geyser principle is used. That is, a steady stream of water passes through the apparatus and is heated by enclosed gas burners supplied with gas at a steady rate. The temperature of the water is taken as it flows into the apparatus and again as it flows out. When these two temperatures are steady the water flowing through in a given time is collected and weighed. The amount of gas burned in the same time is noted from a meter connected to the supply pipe. The quantity of heat produced by burning that amount of gas is calculated by multiplying the weight of water collected by its rise in temperature.

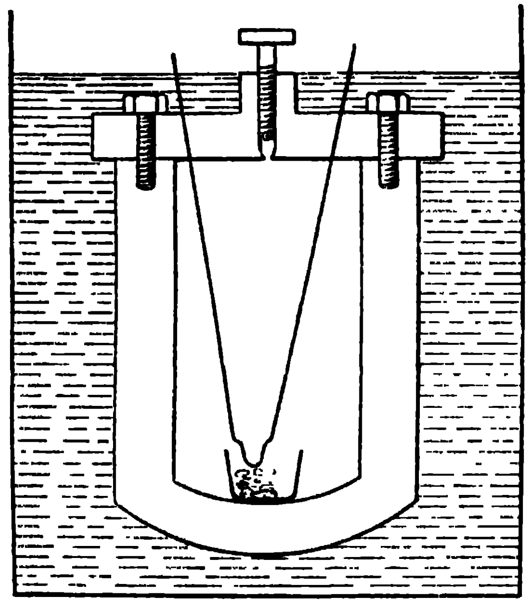


FIG. 182.

Gas companies are obliged to declare the calorific value of their gas which is tested as above from time to time by public officials. The gas meters measure the volume of gas supplied, and from this the number of therms supplied is calculated. The following information is taken from a gas bill.

Calorific value, 500 B.Th.U. per cub. ft.

Gas consumed, 5·2 thousand cub. ft.

Charge, 26 therms at 9¾d. per therm = £1 0s. 10d.

The number of therms was obtained as follows: 5·2 thousand cubic feet of gas are equivalent to:—

$$5{\cdot}2 \times 1000 \times 500 \text{ B.Th.U.}$$

$$= \frac{5{\cdot}2 \times 1000 \times 500}{100000} = 26 \text{ therms.}$$

One of the main purposes of food is to supply the body with energy which is liberated by the oxidation in the blood stream of products of the digestion of the food. It will be shown in Chapter XXII. that heat is a form of energy and so the calorie is in fact a unit of energy. Hence one of the ways of determining the value of a foodstuff is to find its calorific value. The same method may be used as for fuels but certain corrections have to be applied as the oxidations taking place in the body may not be so complete as that in the bomb calorimeter.

In giving the calorific value of foodstuffs a unit known as the large calorie or kilo-calorie is used. This is the quantity of heat required to raise the temperature of one kilogram of water by 1° C. and so is equal to 1000 ordinary calories. Some typical calorific values which have been determined in kilo-calories per gram are: fat, 9·3; carbohydrate, *e.g.* starch, 4·1; protein, *e.g.* lean meat, 4·1. It has been calculated that the energy requirement per day of the human body varies from about 2500 kilo-calories for those doing practically no manual work to 5500 kilo-calories for those doing very strenuous work. From these figures the quantities of various kinds of foodstuffs needed in the diet of an individual can be calculated. It should be noted that a diet which has a sufficient calorific value is not necessarily one which will maintain health. For example, a diet consisting entirely of starch could provide sufficient calories but would not provide the necessary material for body building and replacement.

QUESTIONS ON CHAPTER XIX

(Use the Table of Specific Heats on page 222 when necessary.)

1. On what factors does the quantity of heat in a body depend? Describe experiments to illustrate your answer.

2. Explain what is meant by saying that (*a*) the expansion of water with rise of temperature is anomalous, (*b*) water has a high thermal capacity.

Give examples in Nature where these distinctive thermal properties are advantageous. [L.U.

3. Describe carefully an experiment you have carried out in which a *calorimeter* was used.

Point out the precautions taken to prevent loss of heat during the experiment. [L.U.

4. Name and define the unit in which *quantity of heat* is measured. Show that if 40 grm. of water at 70° C. are mixed with 120 grm. of water at 10° C., the final resulting temperature is 25° C. (neglecting loss of heat). [L.U.

5. Define *calorie*, *British Thermal Unit*, *therm*.

Calculate the number of calories in a B.Th.U., taking 1 lb. as equal to 454 grm.

6. Calculate the heat gained in each of the following cases:—

(i) 75 grm. of water heated from 16° C. to 100° C.
(ii) 36 lb. of water „ „ 60° F. to 212° F.
(iii) 10 gall. of water „ „ 48° F. to 100° F.
(iv) 5 litres of water „ „ 15° C. to 80° C.
(v) 7 grm. of copper „ „ 15° C. to 200° C.
(vi) 8 lb. of iron „ „ 40° F. to 350° F.

7. Find the resulting temperatures in the following mixtures, neglecting heat losses:—

(i) 250 grm. of water at 99° C. into 200 grm. water at 15° C.
(ii) 5 gallons of water at 40° F. into 20 gall. of water at 200° F.
(iii) 300 grm. of lead at 90° C. into 150 grm. of water at 16° C.
(iv) 250 grm. of copper at 100° C. into 100 grm. of turpentine at 20° C.
(v) 10 lb. of iron at 600° F. into 1 gall. of water at 50° F.

8. Calculate the temperatures of the pieces of iron which gave the following results when dropped into water:—

WT. OF IRON	WT. OF WATER	TEMP. OF WATER	FINAL TEMP.
500 grm.	500 grm.	20° C.	40° C.
30 lb.	20 lb.	50° F.	85° F.
400 grm.	1 kilog.	15° C.	50° C.

9. Calculate the specific heats of the metals in the following cases, neglecting heat absorbed by the calorimeter.

(i) 100 grm. of copper at 100° C. dropped into 200 grm. of water at 15° C. raised the temperature to 19° C.
(ii) 300 grm. of lead at 99° C. dropped into 100 grm. of water at 16° C. raised the temperature to 23° C.

(iii) 1 lb. of mercury at 200° F. dropped into 1 pt. of water at 50° F. raised the temperature to 53·5° F.

10. An electric heater dipped into 500 grm. of water in an aluminium calorimeter weighing 10 grm. raised the temperature from 20° C. to 30° C. in 3 min. At what rate did it give out heat?

If, when 600 grm. of oil was substituted for the water, the heater raised the temperature from 20° C. to 30° C. in 2 min., what was the specific heat of the oil?

11. Which is the cheaper fuel for heating purposes, gas at 9d. per therm or coal of calorific value 12,000 B.Th.U. per lb. at £2 per ton?

12. 32 grm. of water at 60° C. are poured into 60 grm. of cold water at 12° C. which is contained in a calorimeter of mass 40 grm. and specific heat 0·1.

Show that, neglecting loss of heat, the final resulting temperature is 28° C. [L.U.

13. Define *specific heat* and *water equivalent.*

A calorimeter contains 85 grm. of water at a temperature of 16° C. and into this is placed a piece of aluminium weighing 80 grm. at a temperature of 100° C. The final temperature of the water is 29·8° C. Calculate the water equivalent of the calorimeter. [Sp. Ht. of aluminium = 0·22.] [L.U.

14. Explain what is meant by the *water equivalent* of a body.

A calorimeter contains 100 grm. of water at 16° C. 50 grm. of water at 45° C. are added, and the resulting temperature is 25·2° C. Calculate the water equivalent of the calorimeter. Enumerate the sources of error which may occur in performing an experiment of this nature and the precautions you would take to eliminate them. [L.U.

15. Explain the meaning of *specific heat* and *water equivalent.*

A calorimeter of water equivalent 6·3 grm. contains 57·6 grm. of a liquid at a temperature of 15° C. Into this is placed a block of copper of mass 123 grm. at a temperature of 100° C. The temperature of the liquid when heat transference is complete is 36° C. Calculate the specific heat of the liquid and point out what assumption has been made in the calculation. [Specific heat of copper = 0·1.] [L.U.

16. Describe how you would compare approximately the rates at which a given gas ring and an electric hot plate can each supply heat to a vessel placed on it.

Assuming that there are no heat losses, what volume of coal gas, having a calorific value of 480 B.Th.U. per cub. ft., will be needed to heat 30 gall. of water, contained in a copper boiler weighing 60 lb., from 52° F. to boiling point? Calculate the cost of heating this water by gas costing 10d. per therm, if there is a 50 per cent. loss in the heating apparatus. (1 gall. of water weighs 10 lb.; specific heat of copper = 0·1.) [J.M.B.

17. Distinguish between the *thermal capacity* (or heat capacity) of a body and the *specific heat* of a substance.

A cube of iron weighing 63 grm. and a cube of glass of the same size, weighing 22 grm., are both heated to 150° C. They are then transferred to exactly similar calorimeters each containing 35 c.cm. of water at 6° C. The final temperatures are 27° C. for the calorimeter containing iron and 24° C. for that containing glass. If the specific heat of iron is 0·11, calculate (*a*) the water equivalent of each of the calorimeters, (*b*) the specific heat of glass.

It was observed that the calorimeter containing glass took a few minutes to reach its final temperature, whereas that into which the iron was transferred reached 27° C. very quickly. Explain this observation. [J.M.B.

18. Explain the meaning of (*a*) *a degree Fahrenheit*, (*b*) *a therm.*

Given a piece of iron of mass 100 grm. and specific heat 0·11, how would you find the approximate temperature of an oven?

Name *one* advantage and *one* disadvantage associated with the use of a copper head on a soldering iron. [J.M.B.

19. Explain the terms *thermal capacity* and *water equivalent*, and describe an experiment by which you would determine the value of one of these quantities for a calorimeter. Show clearly how the result of the experiment would be calculated.

20. Describe carefully how you would determine the specific heat of a piece of metal. Explain fully the precautions you would take to avoid errors and the method by which you would calculate the result.

How could you modify the method so as to find the specific heat of a solid such as salt which dissolves in water?

21. Given a piece of metal of known specific heat, how could you use it (*a*) to find the specific heat of a liquid and (*b*) to determine the temperature inside an oven? In each case make clear the steps in calculating the result.

22. What is meant by the *calorific value* of a solid fuel or foodstuff? Draw and describe a form of apparatus for determining such a calorific value and give a description of the way in which the determination is made.

What is the importance of knowing the calorific values of foodstuffs and in what units are they usually stated?

23. 50 grm. of water at 80° C. are poured into a vessel containing 40 grm. of water at 12° C. The temperature of the mixture is 46° C. Calculate the water equivalent of the vessel.

24. Using gas of thermal value 500 B.Th.U. per cu. ft., what volume of gas will be consumed in bringing half a gallon of water at 60° F. to boiling point? Assume that one-third of the heat generated by burning the gas escapes into the air and neglect heat gained by the vessel.

If the price of the gas is 20 pence a therm, what is the cost of heating the water? (One gallon of water = 10 lb.)

25. Three equal calorimeters each contain equal masses of water at 15° C. 100 grm. of copper at 100° C. dropped into the first raises the temperature to 18° C.; 60 grm. of aluminium at 100° C. dropped into the second raises the temperature to 18·6° C., and 200 grm. of lead at 100° C. dropped into the third raises the temperature to 17° C. Show that the specific heats of copper, aluminium, and lead are approximately in the ratio $1 : 2 : \frac{1}{3}$.

26. An electric heater immersed in 200 grm. of water contained in a calorimeter of 20 grm. water equivalent raises the temperature from 15° C. to 20° C. in 2 min. When 200 grm. of water and 250 grm. of iron were placed in the calorimeter the heater raised the temperature from 15° C. to 19·4° C. in 2 min. Calculate the specific heat of the iron.

27. A piece of metal heated to 100° C. is dropped into 200 grm. of water at 14° C. in a copper calorimeter weighing 80 grm. and raises the temperature to 17·5° C. What is the thermal capacity of the piece of metal?

If the experiment is repeated with 200 grm. of paraffin at 14° C. in place of the water the temperature rises to 20·25° C. What is the specific heat of the paraffin?

CHAPTER XX

CHANGE OF STATE. LATENT HEAT

The same substance may exist as a solid, a liquid, or a gas. Thus, water is familiar to us as solid ice, liquid water, or gaseous steam, and can be changed from one state to another, in that order, by raising its temperature. In the case of pure substances these changes of state take place at definite temperatures. This was assumed in the case of water in connexion with the " fixed points " of a thermometer.

Melting Point

The temperature at which a substance turns from solid to liquid or liquid to solid is known as its **melting point** or **freezing point** according to the direction of the change being considered.

Support a thermometer with its bulb in the middle of a beaker and fill up the beaker with crushed ice. The thermometer will register 0° C. Warm the beaker with a very small Bunsen flame and note the temperature from time to time. As soon as sufficient water is formed keep the mixture of ice and water well stirred. It will be found that the thermometer registers 0° C. as long as any ice remains, but begins to rise as soon as all the ice has melted.

Support a thermometer dipping into water in a wide test-tube. Stand the test-tube in a beaker containing a freezing mixture. Keep the water well stirred and watch the thermometer. It will be found to fall rapidly until it registers 0° C. when ice will begin to form. It will then remain steady until all the water has changed to ice. Then it will slowly fall below 0° C. to the temperature of the freezing mixture.

FIG. 183.

Similar experiments may readily be carried out with naphthalene ("moth balls"). Melt naphthalene in a wide test-tube until a depth of 2 in. has been obtained Allow it to solidify with a thermometer bulb embedded in the middle of it. Now support the tube so that it dips in water in a beaker which is kept at a temperature a few degrees above 80° C. (Fig. 183). Read the thermometer every half-minute. It will be found to rise until the naphthalene begins to melt. Then it will remain at one temperature until all has melted after which it will rise again to the temperature of the water in the beaker.

Remove the tube with the melted naphthalene from the water and allow it to cool in the air, reading the thermometer every half minute again. Observations similar to those on the freezing of water will be made. Note that the stationary temperature during solidification is the same as that during melting. The two sets of readings may be shown on a graph as illustrated in Fig. 184.

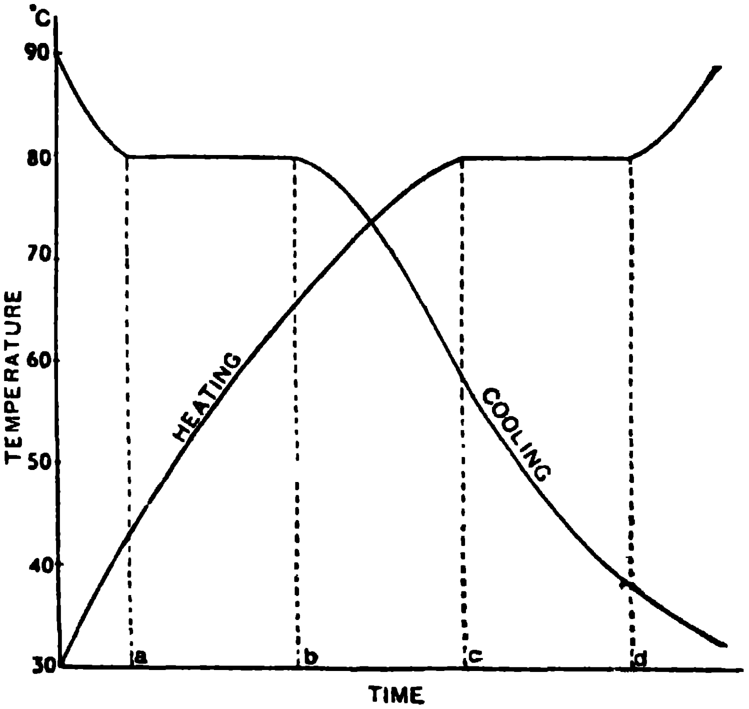


FIG. 184.

These experiments show (1) that the temperature at which a substance freezes is the same as that at which it melts, and (2) once a substance is at its melting point, its temperature does not change while freezing or melting is proceeding.

It is evident that the temperature corresponding to the horizontal parts of the two curves in Fig. 184 is the melting point of naphthalene. The construction of such heating or cooling curves is a good way of finding melting points of substances.

Latent Heat

During the period represented by *cd* in Fig. 184, the naphthalene would be at a lower temperature than the water in the beaker, and so would be receiving heat from it although its temperature was not rising. Similarly, during the period *ab*, it would be giving out heat to the air although its temperature was not falling. It is evident that a similar absorbtion and liberation of heat without temperature change took place during the melting of ice and the freezing of water. This phenomenon always occurs during a change of state, and heat which is given out or absorbed by a substance without change of temperature is called **latent heat**; that associated with melting or freezing is called **latent heat of fusion**. Quantitatively, **the latent heat of fusion of a substance** is defined as **the quantity of heat which is absorbed by unit mass of the substance at its melting point while it changes from solid to liquid without change of temperature.**

Latent Heat of Fusion of Ice

Colin Wooden 1958

A rough measurement of this may be made by a method similar to that by which Joseph Black (1728-1799) first investigated it.

Put a quantity of ice into a beaker, drying each piece with filter-paper as it is put in. Stand a thermometer in the beaker and place a Bunsen burner with a small steady flame below it. Note the time taken for all the ice to melt. Continue the heating for as long after the ice has melted as it took in melting, and note the temperature to which the water formed is raised. It will probably be found to be something between 70° C. and 80° C. That means that, simply to melt ice without raising the temperature above 0° C., takes as much heat as will raise the temperature of the resulting water from 0° C. to 70° C. or 80° C.

Applying this to a single gramme of the ice, to raise the resulting water from 0° C. to 70° C. would require 70 calories. Hence, a gramme of ice already at 0° C. has to be supplied with 70 to 80 calories just to melt it, the resulting water still being at 0° C. The correct value is about 80 calories.

This explains why a pond freezes so slowly unless the air temperature falls a long way below freezing point, for even when the water is down to its freezing point, 80 calories of heat must escape from it to the air for every gramme that freezes. Similarly, when it melts, every gramme that melts must take 80 calories from the air to change it to water still at 0° C.

The value of the latent heat of fusion of ice can be found more accurately by a calorimeter experiment. Weigh the calorimeter. Put some water in it and weigh it again. Warm the water to about 5 degrees above air temperature and then shield it in the usual way. Take the temperature of the water. Add small dry chips of ice one at a time, stirring and letting each chip melt completely before adding the next. Continue until the temperature is about 5 degrees below air temperature. Note the temperature to which the water has been cooled and weigh the calorimeter and its contents again to find the mass of ice added.

RESULTS

Mass of calorimeter (copper)	= 50·25 grm.
∴ Water equivalent of calorimeter ...	= 5·025 grm.
Mass of calorimeter and water	= 182·76 grm.
∴ Mass of water	= 132·51 grm.
Air temperature	= 16° C.
Temperature of water	= 21° C.
Temperature after addition of ice	= 10° C.
Final mass of calorimeter and contents ...	= 199·57 grm.
∴ Mass of ice added	= 16·81 grm.

The original water and calorimeter cooled from 21° C. to 10° C., and so lost $11 \times (132{\cdot}51 + 5{\cdot}025)$ calories $= 11 \times 137{\cdot}535 = 1512{\cdot}885$ calories. The ice melted and then the water formed from it rose from 0° C. to 10° C. If the latent heat of fusion of ice is L calories per grm.:—

$$\text{Heat gained by ice in melting} = 16{\cdot}81\,L \text{ calories.}$$

$$\text{Heat gained by water formed from it} = 16{\cdot}81 \times 10 \text{ calories;}$$

$$\therefore \text{Total heat gained} = 16{\cdot}81\,L + 168{\cdot}1 \text{ calories.}$$

$$\text{Heat gained} = \text{Heat lost;}$$

$$\therefore 16{\cdot}81\,L + 168{\cdot}1 = 1512{\cdot}885 \textit{ or } 16{\cdot}81\,L = 1344{\cdot}785;$$

$$\therefore L = 80{\cdot}0, \textit{ i.e.} \text{ Latent heat of fusion of ice} = 80 \text{ calories per grm.}$$

The reason why the calorimeter is first warmed above air temperature and finally cooled below it is that the melting of the ice takes some considerable time. When the calorimeter is above air temperature it will lose heat to the air, and while it is below air temperature it will gain heat from the air. An attempt is made to balance this loss and gain so that error due to it will be eliminated from the results.

EXAMPLES.—(1) *Taking the latent heat of fusion of ice as* 80 *calories per grm., what mass of ice at its melting point could be just melted by* 500 *grm. of boiling water?*

500 grm. of water falling from 100° C. to 0° C. lose 500 × 100 calories.

80 calories will just melt 1 grm. of ice;

$$\therefore \text{ Mass of ice melted} = \frac{500 \times 100}{80} = 625 \text{ grm.}$$

(2) *If* 10 *grm. of ice at its melting point are added to* 100 *grm. of water at* 40° *C., what will be the resulting temperature of the water?*

Let the resulting temperature be t° C.

Heat lost by warm water = 100 (40 − t) calories.

Heat gained by ice during melting = 10 × 80 cals.

Water formed from ice then rises from 0° C. to t° C., gaining 10 × t cals.

Heat gained = Heat lost.

$$(10 \times 80) + (10 \times t) = 100\,(40 - t);$$

$$\therefore\ 800 + 10t = 4000 - 100t; \quad \therefore\ 110t = 3200; \quad \therefore\ t = 29\tfrac{1}{11}.$$

Resulting temperature is $29\frac{1}{11}$° C.

(3) *A piece of iron weighing* 250 *grm. is heated to* 100° *C. and then dropped into a hole in a block of ice. If* 34·5 *grm. of ice is melted, what is the specific heat of the iron?*

Heat gained by ice in melting = 34·5 × 80 calories.

Let s be the specific heat of iron.

Then Heat lost by iron in falling to 0° C. = 250 × s × 100 calories.

Heat gained = Heat lost;

$$\therefore\ 250 \times s \times 100 = 34{\cdot}5 \times 80;$$

$$\therefore\ s = \frac{34{\cdot}5 \times 80}{250 \times 100} = \frac{2760}{25000} = {\cdot}110;$$

$\therefore$ Specific heat of iron = ·110.

Boiling Points

Fit a wide-mouthed flask as shown in Fig. 185. Partly fill with water, and heat, with constant stirring, from time to time reading the thermometer which dips into the water. The temperature will be found to rise until the water begins to boil, and then will remain constant so long as the boiling continues. Raise the thermometer so that it is in the steam instead of dipping into the water The steam will be found to have the same temperature as the boiling water.

This apparatus may be used for finding the **boiling point** of a liquid. For more accurate work a *hypsometer* (page 168) may be used. Owing to the formation of convection currents it is difficult to ensure that all parts of a heated liquid are at the same temperature, and unless stirring is very thorough, the temperature shown by the thermometer may vary with different positions of the bulb. For that reason it is better to have the thermometer surrounded by the vapour and clear of the liquid when taking a boiling point. Other precautions should be taken as when marking the upper fixed point of a thermometer (page 168).

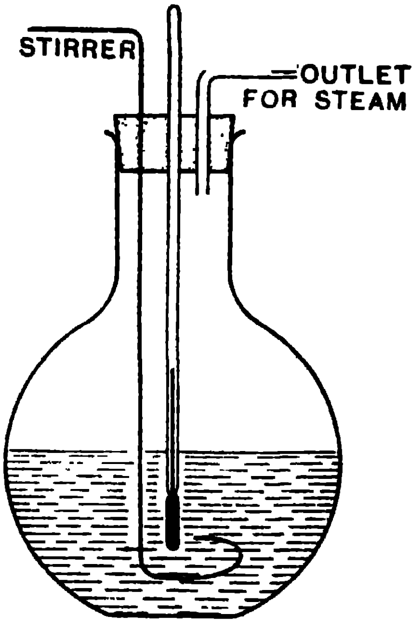


FIG. 185.

Latent Heat of Vaporisation

Since a liquid has a constant temperature while boiling, although heat is still being supplied to it, a substance has a latent heat of vaporisation as well as a latent heat of fusion. The latent heat of vaporisation of a substance is the quantity of heat [illegible] to change unit mass of it at its boiling point from liquid to vapour [illegible] ithout change of temperature.

In the case of water a rough estimate of this may be made by a method similar to the original method of Black. Place some water in a beaker and note its temperature. Put a small steady flame under it and note the time taken to bring it to the boil. Continue heating until it has all boiled away, and note the time this takes after boiling commenced. Suppose the original temperature was 16° C., the time taken to bring the water to the boil was 2½ min., and a further 16¼ min. were required to boil the water completely away. Then, to turn the water to steam after bringing it to its boiling point takes $\frac{16{\cdot}25}{2{\cdot}5}$ times as much heat as to raise it from 16° C. to 100° C.

Apply this to one gramme of the water. The heat required to raise its temperature from 16° C. to 100° C. is 84 calories. Therefore, the heat required to turn 1 grm. of water at its boiling point to steam is—

$$\frac{84 \times 16{\cdot}25}{2{\cdot}5} = 546 \text{ calories} = \text{Latent heat of vaporisation.}$$

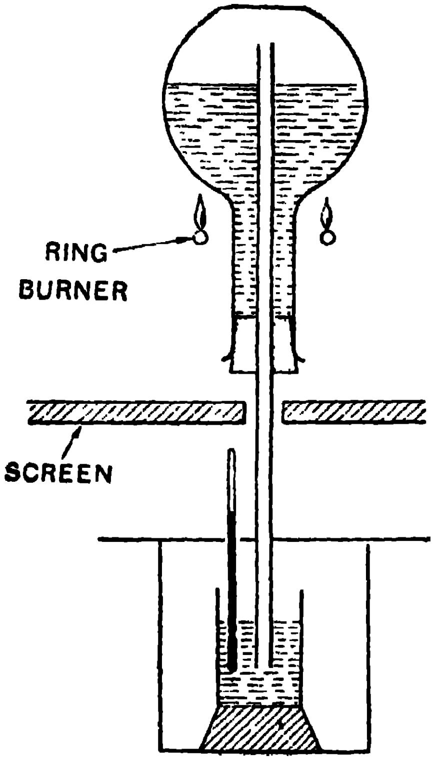


FIG. 186.

A more accurate calorimetric method is as follows. Steam is generated in an inverted flask, fitted with a long straight tube and heated by a ring gas burner. The tube passes through a hole in an asbestos or wooden screen (Fig. 186).

A calorimeter is weighed, partly filled with water and weighed again. It is then shielded in the usual way. The temperature of the water is taken. When steam issues freely from the tube, the calorimeter is placed so that the tube dips into it. Steam condenses in the cold water and raises its temperature. When the temperature has been raised 20 or 30 degrees the calorimeter is removed, allowed to cool, and weighed again to find the mass of steam condensed.

RESULTS

Mass of calorimeter (copper) ... ... = 88·2 grm.
∴ Water equivalent of calorimeter ... = 8·82 grm.
Mass of calorimeter + water ... ... = 188·2 grm.
∴ Mass of water ... ... = 100·0 grm.
Temperature of water ... ... = 20° C.
Temperature to which raised ... ... = 47° C.
Final mass of calorimeter and contents = 193·0 grm.
∴ Mass of steam condensed ... ... = 4·8 grm.
Heat gained by water and calorimeter (100 + 8·82) × 27 calories

If L is the latent heat of vaporisation of water, heat lost by steam condensing = 4·8 L calories.

The resulting water cooled from 100° C. to 47° C. and the loss of heat during this cooling = 4·8 × 53 calories.

$$\text{Heat gained} = \text{Heat lost};$$

$$\therefore\ 108{\cdot}82 \times 27 = 4{\cdot}8\,L + 4{\cdot}8 \times 53;$$

$$\therefore\ 2938 = 4{\cdot}8\,L + 254{\cdot}4;$$

$$\therefore\ 4{\cdot}8\,L = 2683{\cdot}6; \qquad L = \frac{2683{\cdot}6}{4{\cdot}8} = 559.$$

The latent heat of vaporisation of water is 559 calories per grm. An accurate value is 537 calories per grm.

The following points concerning the experiment should be noted:—

(1) The passage of the steam down the tube through the boiling water tends to prevent it from condensing before it reaches the calorimeter.

(2) The screen below the burners prevents direct radiation of heat from the burners and flask to the calorimeter.

(3) Some heat will escape from the calorimeter while it is being heated by the steam, causing a small error in the results.

EXAMPLE.—50 *grm. of dry ice are contained in a vessel. Steam is passed in until the resulting water is at* 30° *C. Calculate the mass of steam condensed, neglecting the heat absorbed by the vessel. Take the latent heat of vaporisation of water as* 540 *calories per grm. and latent heat of fusion of ice as* 80 *calories per grm.*

To melt the ice would require 50×80 calories.

To raise the resulting water to 30° C. would require 50×30 calories;

$\therefore$ Total heat required $= 50\,(80 + 30) = 5500$ calories.

Let the mass of steam condensed be x grm.

In condensing it would lose $540x$ calories.

Resulting water cooling to 30° C. would lose $70x$ calories;

$\therefore$ Total heat lost $= 610x$ calories;

$$\therefore\ 610x = 5500; \quad \therefore\ x = \tfrac{5500}{610} = 9 \text{ (approx.)};$$

$\therefore$ Mass of steam condensed $= 9$ grm.

Applications of Latent Heat of Vaporisation

It will be noted that the latent heat of vaporisation of water involves a large quantity of heat. A gramme of steam at 100° C. in condensing will lose without change of temperature nearly $5\frac{1}{2}$ times as much heat as a gramme of water loses in cooling from 100° C. to 0° C. This explains why steam is so effective as a heating agent, and why scalding by steam is likely to be much more severe than scalding by boiling water.

It may often be noticed that the air temperature rises considerably during a heavy fall of snow. The water vapour in the air is, of course, below its boiling point, but in changing to snow, it will give up both its latent heat of vaporisation and its latent heat of fusion, so that every gramme of vapour converted to snow liberates over 600 calories of heat into the air.

All liquids in evaporating take up a considerable quantity of latent heat. This may be taken partly from the surroundings and partly from the remaining liquid, so that the temperature of a liquid usually falls if some of it evaporates without heat being supplied to it.

This may be illustrated with ether which evaporates very readily. Put a little ether in a thin beaker and stand the beaker in a drop of water on a piece of wood. Place a thermometer in the ether and blow a stream of air through it. The ether will evaporate quickly and its temperature will be found to fall rapidly below 0° C., so that in a minute or two the beaker is frozen to the piece of wood.

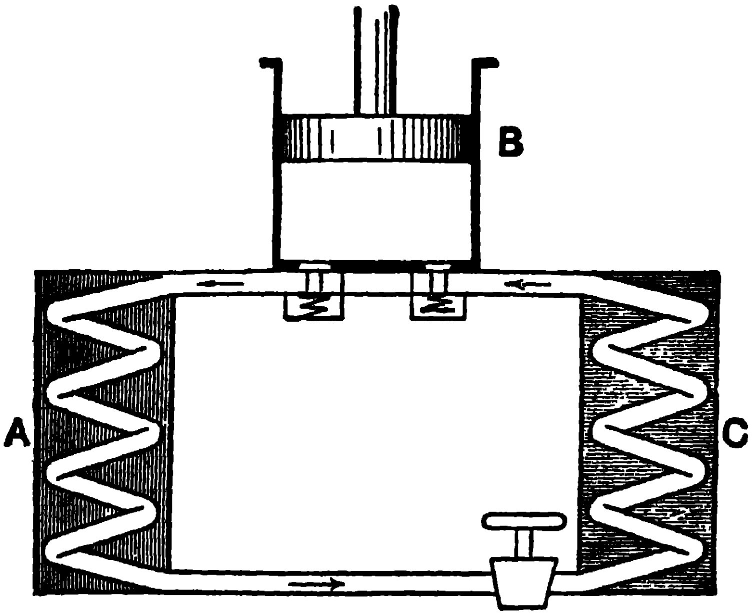


FIG. 187. FROM EWING'S "MECHANICAL PRODUCTION OF COLD."

The action of butter coolers is based on this effect. The butter is placed in a glass vessel inside a porous earthenware vessel which is partly filled with water. Water slowly oozes through the pores of the outer vessel and evaporates into the air, taking its latent heat of vaporisation from the remaining water. Thus the temperature of the latter is reduced and the butter kept cool.

The same principle is used in refrigeration. Ammonia gas is pumped by B into the coil in A under sufficient pressure to liquefy it (Fig. 187). Its latent heat of vaporisation is taken up by cold water which is kept running through A. The liquefied ammonia is then driven through a valve into the coil in C where the pressure is lower. Under the reduced pressure the ammonia evaporates in this coil taking its latent heat from the brine with which C is filled. The gas is drawn off from the second coil by B and circulated again. Every time it passes through C the temperature of the brine will be reduced, and since the freezing point of brine is considerably lower than that of water, it may be brought to a temperature considerably below 0° C. The cold brine is then pumped through pipes to places where freezing is required. In ice rinks, for instance, the brine tubes are embedded in the floor below the ice which is thus kept frozen.

Change of Volume with Change of State

Most substances contract when changing from liquid to solid. This may be seen if melted paraffin-wax is allowed to solidify in an evaporating basin. When it is solid, the surface will be found to have become concave owing to the contraction of the wax beneath.

Water is exceptional in expanding when it freezes. That it does expand is shown by the fact that ice floats on water at 0° C., and so must be less dense than it. The fact is also convincingly shown if a bottle with a screw stopper is completely filled with water and left out-of-doors in frosty weather with the stopper screwed in. When the water freezes the bottle will be broken by the expansion of its contents. The bursting of water pipes during severe frosts is due to this expansion as the water in the pipe changes to ice.

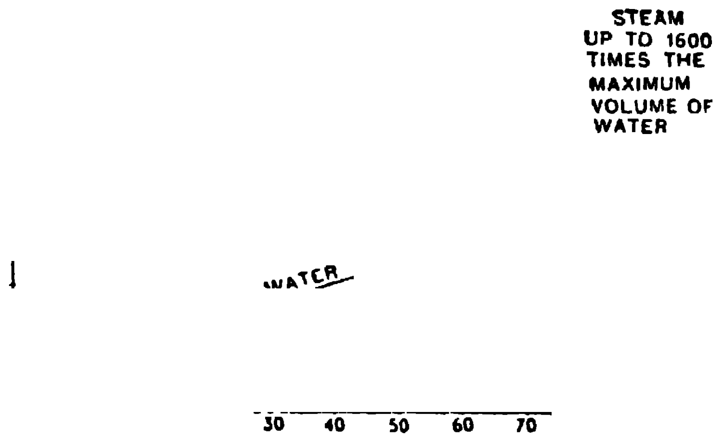


FIG. 188.

It can be shown that, approximately, 10 volumes of water at 0° C. become 11 volumes of ice at the same temperature. It follows that ice at its melting point has a density $\frac{10}{11}$ times that of ice-cold water. Hence, when ice floats the volume of water displaced will be $\frac{10}{11}$ of the volume of the ice; that is, only $\frac{1}{11}$ of the volume of the ice is above the surface of the water. If ice is cooled below 0° C. it contracts like other solids.

There is always a very large increase in volume when a liquid changes to vapour. In the case of water, the volume of steam at

100° C. is about 1600 times the volume of the water to which it would condense.

Fig. 188 represents graphically, but not to scale, the changes of volume which would occur if a mass of ice at − 10° C. were heated until it became steam.

Effect of Pressure on Freezing Point

If two pieces of ice are pressed together and the pressure then released, they will be frozen together, even if the experiment is carried out in a moderately warm room. The reason for this is that pressure lowers the freezing point of water so that, when the blocks are pressed together, a little of the ice melts and forms a layer of water between the blocks. This water will be at 0° C., so that when the pressure is released it freezes again.

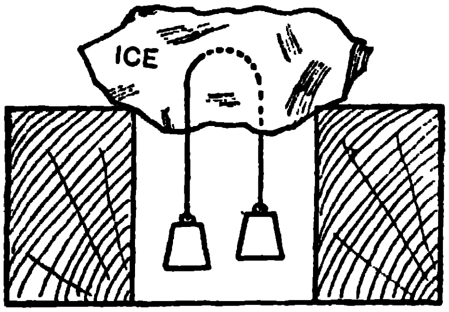


FIG. 189.

This lowering of the freezing point by pressure is connected with the expansion of water on freezing. Increase of pressure will make this expansion more difficult, and so keep the water liquid until its temperature is below 0° C. The effect is very small, a pressure of more than 100 atmospheres being required to reduce the freezing point by 1° C.

Substances which contract on solidifying will have their freezing points raised by pressure which will assist the contraction and so help the liquid to solidify at a temperature higher than its normal freezing point.

The effect in the case of ice is well shown by an experiment due to Tyndall, which is illustrated in Fig. 189. A lump of ice is supported on two blocks, and heavy weights are attached to a wire passing over it. The wire will gradually pass through the block, but the block will still be one solid piece when the wire has gone right through it.

The explanation is that, owing to the small area on which the weights are supported, there is a very great pressure just below the wire. This lowers the freezing point just below the wire, and a little ice melts. The wire sinks through the resulting water which is thus released from the pressure and freezes again above the wire. This process continues until the wire has gone right through the block.

The wire should be of good conducting material, preferably copper, so that the latent heat liberated by the water as it freezes again may be conducted away as quickly as possible and heat may be conducted to the ice below the wire to melt it.

The making of a snowball depends on this property of ice. When the snow is pressed in the hands, its freezing point is lowered, a little melts, and when the pressure is released, bringing the freezing point up to 0° C. once more, the water which was formed freezes again and binds the particles of snow together. If snow is at a temperature much below 0° C. satisfactory snowballs cannot be made from it, for the pressure exerted by the hands is not sufficient to bring its freezing point below its actual temperature and so cause some to melt.

In skating, the whole weight of the body is supported on the edge of the skate, and the resulting pressure causes a little ice to melt below the skate edge, so enabling it to grip. Skating is difficult on very cold ice, as the pressure is not then sufficient to cause melting.

The movement of glaciers is partly due to the effect of pressure on melting point. The bottom layer of ice tends to melt owing to the pressure of that above it. The resulting water flows out from under the glacier, and when released from the pressure, freezes again on the front of the glacier.

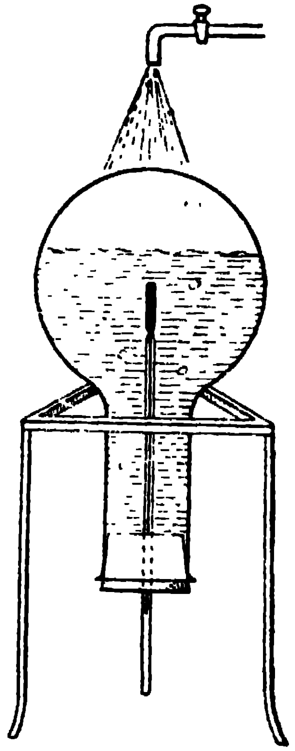


FIG. 190.

Effect of Pressure on Boiling Point

Increased pressure raises the boiling point of water. Here again it opposes the expansion involved in the process and so keeps the water liquid at temperatures above 100° C. Conversely reduced pressure lowers the boiling point.

This is well shown by the experiment illustrated in Fig. 190. Water is boiled in a strong round-bottomed flask. When steam has been issuing freely for a minute or two the flame is removed and the flask tightly corked, the cork carrying a thermometer. The flask is then supported in the position shown. When boiling stops, a little cold water is poured over the flask, and the boiling recommences. This may be repeated a number of times, and the water can be set boiling again when the thermometer reading is several degrees below 100° C.

The explanation of this is that the cold water condenses some of the vapour in the upper part of the flask and so reduces the pressure on the warm water which thus has its boiling point lowered.

The effect on boiling point is much greater than that on freezing point, a difference of 1° C. being caused by a change of about 3 cm. in pressure. One result of it is that water boils at a considerably lower temperature at high altitudes than at sea level owing to the reduction of atmospheric pressure with height. The change of boiling point of water may therefore be used in determining altitudes.

In some industrial processes, for example the vulcanising of rubber, materials have to be raised to temperatures, above 100° C., at which they would burn in air. This can be accomplished by heating them in water enclosed in a very strong boiler to which a strong airtight lid can be screwed down. The steam generated increases the pressure in the boiler and so raises the boiling point of the water to the required temperature.

In locomotive boilers, too, steam is generated under high pressures, and so becomes much hotter than it would be if generated at atmospheric pressure.

Effect of Dissolved Substances on Freezing and Boiling Points

The presence of a dissolved substance lowers the freezing point and raises the boiling point of a liquid. The effect on freezing point may be observed by placing a wide test tube containing dilute salt solution in a freezing mixture. Support a thermometer with its bulb in the brine which should be kept well stirred. It will be found that freezing does not commence until the thermometer registers a temperature below 0° C. Also, as the freezing progresses the temperature continues to fall. The reason for this is that pure ice separates from the solution so that the remaining liquid becomes more concentrated and this causes an increased lowering of the freezing point. Accurate experiments show that the lowering of the freezing point is proportional to the concentration of the solution.

If a concentrated solution of salt is used it will be found that salt and not ice first separates. A solution containing about 25 per cent. of salt solidifies as though it were a single substance at — 21° C., which is the lowest temperature at which any solution of salt will remain liquid.

This lowering of the freezing point is the basis of the use of salt for thawing ice or snow on pavements or the use of mixtures of ice and salt as freezing mixtures. Some of the salt dissolves in the film of water on the particles of ice. This forms a solution with a lower freezing point than that of the ice, some of which then melts, withdrawing its latent heat of fusion from the solution which falls in temperature. By using a mixture with about 25 per cent. of salt the temperature can be lowered to — 21° C. A similar result is obtained if calcium chloride is used instead of salt and a mixture of four parts of calcium chloride to three parts of ice will give a temperature of — 55° C.

The effect on boiling point may be shown by boiling salt solution in the apparatus illustrated in Fig. 185. The bulb of the thermometer must be kept in the solution as the steam will be water vapour at the boiling point of water. It will be found that the temperature of the solution rises above 100° C. before it boils and that, owing to the increased concentration as water evaporates from the solution, the temperature continues to rise as the solution boils. This will continue until the remaining solution is saturated. Salt will then be deposited and the saturated solution having a constant concentration will boil at a constant temperature. The rise in boiling point like the fall in freezing point is proportional to the concentration of the solution.

QUESTIONS ON CHAPTER XX

1. State what is meant by (*a*) the melting point, and (*b*) the boiling point of a substance.

Describe briefly how you would determine the melting point of paraffin-wax and the boiling point of alcohol.

2. Mercury freezes at — 39° C. and boils at 357° C.

State how the temperature would change as time went on if mercury at — 42° were heated with a small steady flame until it had been boiling for a few minutes.

Sketch a graph to illustrate your answer and state what deductions might be made from the graph.

3. Calculate the quantities of heat required in the following cases. Take latent heat of fusion of ice as 80 calories per grm., the specific heat of ice as 0·5, and the latent heat of vaporisation of water as 540 calories per grm.

(i) To convert 10 grm. of ice at 0° C. to water at 50° C.

(ii) To convert 15 grm. of ice at 0° C. to steam at 100° C.

(iii) To convert 9 grm. of ice at — 10° C. to water at 90° C.

4. Calculate the resulting temperatures in the following cases. Take values of constants as in Question 3.

(i) 10 grm. of ice at 0° C. added to 100 grm. of water at 40° C.

(ii) 20 grm. of ice at − 5° C. added to 150 grm. of water at 50° C.

(iii) 5 grm. of steam at 100° C. passed into 150 grm. of water at 15° C.

(iv) 5 grm. of steam at 100° C. passed into a vessel containing 20 grm. of ice at 0° C. (Neglect heat gained by vessel.)

5. The latent heat of fusion of ice is 80 calories per grm. The latent heat of vaporisation of water is 540 calories per grm. Explain the meanings of these statements.

Express the two quantities named in B.Th.U. per lb. Give reasons for your answers.

6. Explain the following:—

(i) Why an iceberg floats with about $\frac{10}{11}$ of its mass below water level.

(ii) Why the air is usually very cold for miles around an iceberg.

(iii) Why steam pipes warm a building more efficiently than hot-water pipes.

(iv) Why potatoes cannot be cooked in an open saucepan at the top of a high mountain.

7. Explain the principle made use of in refrigeration, and describe a simple form of refrigerating machine. [L.U.

8. 40 grm. of ice (at its melting point) is heated till it is all converted to steam at normal atmospheric pressure.

State concisely what changes take place (*a*) in temperature, (*b*) in volume, (*c*) in state.

Give any information you can concerning the quantity of heat required for the complete process. [L.U.

9. Explain the meaning of *latent heat.*

A closed lead pipe is full of water at 10° C. Describe and explain the changes which take place when the pipe is placed in a freezing mixture. [L.U.

10. A thermometer is placed in a copper vessel which is half filled with small pieces of dry ice. State and explain what you would observe if a constant heat source such as a small Bunsen flame were placed underneath the vessel.

If you were provided with a suitable clock, indicate briefly what other information could be obtained from the experiment. [L.U.

11. Define a *British Thermal Unit.*

A copper vessel of mass 1·2 lb. contains 4·5 lb. of water at a temperature of 77° F.; and when 0·78 lb. of dry ice at the melting point are put in, the temperature after the ice has all melted is 50° F. Calculate from the data the latent heat of fusion of ice in British Thermal Units per pound. [Sp. Ht. of copper = 0·1.] [L.U.

12. Define the latent *heat of vaporisation of water.*

Describe how you would determine this quantity experimentally. State the sources of error that are likely to arise in your method, and the precautions you would take to eliminate them. [L.U.

13. Distinguish between *specific heat* and *latent heat.*

A calorimeter of water equivalent 5·4 grm. contains 45 grm. of water at a temperature of 13° C. In this 3 grm. of dry steam at 100° C. are condensed. Calculate the resulting temperature of the water. [Latent heat of steam = 540 cal. per grm.] [L.U.

14. Define *Latent heat of fusion* and *Latent heat of vaporisation.*

Give the essential details of an experiment you have carried out to determine *one* of these quantities for ice or water.

Pieces of dry ice are added to a litre of water at 40° C.; the lowest temperature reached is 20° C. What weight of ice has been melted and what is the total volume of water at the end of the experiment? [L.U.

15. Describe *one* experiment to show that when ice melts there is a change of volume, and *one* to show that there is absorbtion of heat.

Explain the following observations: (*a*) exposed water pipes may be burst during a severe frost, though the burst is not usually discovered till after the thaw has started; (*b*) snow on the shady side of the garden may take a long time to disappear after a thaw has started. [J.M.B.

CHAPTER XXI

PROPERTIES OF VAPOURS

Evaporation

Although a liquid has a definite boiling point at which it rapidly turns to vapour, it may **evaporate** *at any temperature.* Water puddles on the roads dry up even on cold days, and liquids left in open vessels gradually disappear.

A number of factors affect the rate at which evaporation takes place. Among the most important of these are:

(1) Temperature.—A liquid will disappear from an open vessel much more rapidly if it is kept warm than if it is cold, even though it is not raised to its boiling point. Naturally, when the liquid has its latent heat of vaporisation supplied to it by being warmed, it can evaporate more quickly than when it has to withdraw that heat gradually from its surroundings.

(2) Dryness of the Air.—This is dealt with more fully later, but it may be noted that, if two dishes of water are placed on the bench and one is covered with a bell-jar, evaporation takes place more rapidly from the uncovered one. In that case the vapour formed can rapidly escape from the vicinity of the dish; but in the other case it is retained by the bell-jar, so that the air around the dish rapidly becomes very damp.

(3) Motion of the Air.—Washing hung out on the line dries more quickly on a windy day than on a still day. In the experiment on page 240, air was blown through the ether to make it evaporate rapidly. This is connected with factor 2. The motion of the air carries the vapour away from the liquid and brings fresh dry air into contact with it.

(4) Pressure.—The effect of pressure on the boiling point of a liquid has already been considered. If a strong flask containing water is connected to an air pump and the air pumped out of it, the water will be seen to evaporate very quickly under the reduced pressure.

(5) Nature of the Liquid.—Under similar conditions a liquid with a low boiling point evaporates more rapidly than one with a high

boiling point. Mercury (B.P. 357° C.) hardly evaporates at all under ordinary atmospheric conditions, while alcohol (B.P. 78° C.) and ether (B.P. 35° C.) evaporate much more rapidly than water.

Evaporation and Boiling

It is instructive to heat some water slowly in a beaker and watch carefully what happens. As the water becomes warm, small bubbles will form on the bottom and sides of the beaker and escape from the water. These can be shown to consist mainly of air which has been dissolved by the water and is driven out by the heating.

Long before the water boils, wisps of steam will be seen to rise *from the surface* and escape into the air, and these little clouds become more dense as the temperature rises.

Finally, large bubbles are seen to form *inside the liquid*, to rise to the surface and burst, liberating large quantities of steam, and to throw the whole of the liquid into violent agitation. It is only during the last stage that the water is said to boil.

It has previously been noted that the temperature will rise continuously until the boiling commences, but will then remain stationary. On the other hand, if evaporation takes place without heat being supplied to the liquid, the temperature falls (page 240).

From these observations the following comparison between boiling and evaporation can be drawn up.

BOILING V. IMP.	EVAPORATION
1. Takes place only at a fixed temperature for a given liquid at a given pressure.	1. May take place at any temperature.
2. Temperature remains constant so long as liquid is boiling.	2. Temperature may change during evaporation.
3. During boiling vapour escapes from the interior of the liquid.	3. Vapour escapes only from the surface of the liquid during evaporation.

Vapour Pressure

Set up two simple barometers side by side (Fig. 191). If they have been carefully prepared the mercury will stand at the same level in both.

By means of a small curved pipette, insert a drop of water into the mouth of one tube (*b*). It will float up through the mercury into the vacuum at the top of the tube. There it will immediately evaporate. The mercury will be depressed a little, showing that the vapour formed is exerting a pressure in the space above it.

Introduce successive drops of water into the tube and the above observations will be repeated, showing that as more water vapour is formed in the space, it exerts a greater pressure.

Finally a stage will be reached when further drops introduced do not evaporate but form a water layer floating on the mercury. The introduction of more water at this stage does not cause further depression of the mercury, showing that no further increase in the pressure of the water vapour takes place. Thus water will evaporate into a space until the vapour formed by it exerts a certain pressure, and then evaporation ceases. The space is then said to be **saturated with vapour**, and the pressure exerted by the vapour is said to be the **saturation or maximum vapour pressure of the liquid** *under the given conditions*. This saturation pressure is evidently measured by the difference between the heights of the mercury columns in the two tubes.

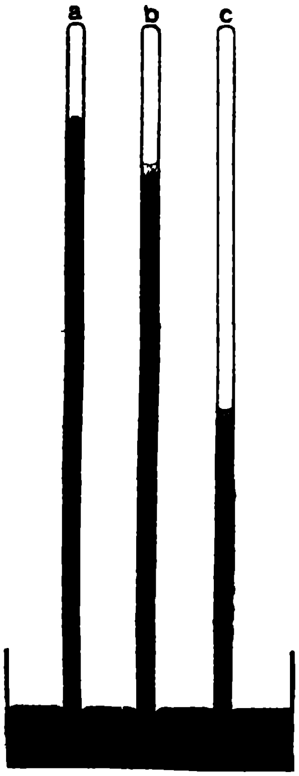


Fig. 191.

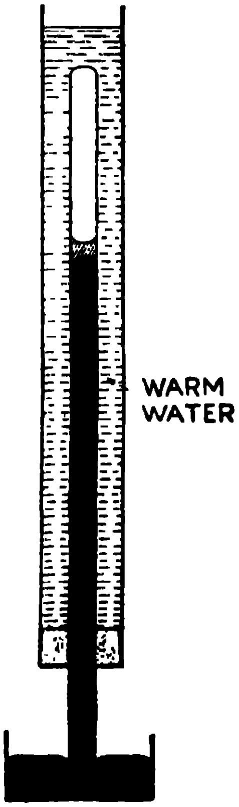


Fig. 192.

If a third barometer (*c*) is set up alongside the other two and ether introduced into it instead of water, a much greater depression of the mercury column will be produced, showing that different liquids have different maximum vapour pressures under the same conditions.

Vapour Pressure and Temperature

Fit a wide tube around the barometer tube into which water has been introduced, and fill it with warm water (Fig. 192). The water which had remained liquid in the barometer tube will evaporate and depress the mercury still further. Add more drops. Several of them will evaporate and cause further depressions of the mercury, but eventually a state of saturation will be reached once more when water will remain liquid in the barometer space. and the pressure of the vapour will remain constant. If still hotter water is placed in the jacket, more water will be required to saturate the space and a higher saturation pressure will be obtained. From this it is seen that *the saturation vapour pressure of a liquid increases with rise of temperature*, and it is clear that, in stating a saturation pressure, the temperature at which it was measured should be given.

Vapour Pressure and Boiling Point

Mercury is introduced into a U-tube with one short closed arm until the closed arm and the bend are completely filled (Fig. 193). The pressure of the atmosphere down the open limb will be sufficient to support a column of mercury filling the closed limb. A little water is now run in and made to float to the top of the closed arm by tilting the tube. The tube is then supported in a beaker of water which is heated. As the temperature rises some of the enclosed water will evaporate and drive the mercury round the bend. If, at any stage, all the enclosed water evaporates, more must be introduced. When the water in the beaker is boiling it will be found that the mercury stands at the same level in both arms of the U-tube. This shows that the vapour in the closed arm is exerting a pressure equal to that of the atmosphere. Thus it is found that *the saturation vapour pressure of a liquid at its boiling point is equal to the external pressure on it.* This enables us to give an exact definition of the boiling point of a liquid. The boiling point of a liquid is the

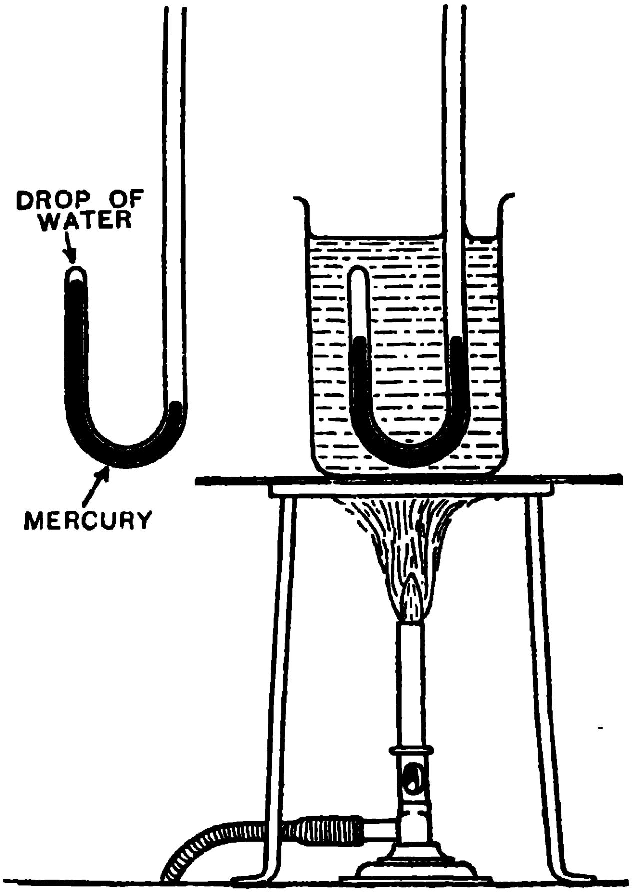


FIG. 193.

temperature at which its saturation vapour pressure is equal to the pressure of the atmosphere.

This is explained by the observation on page 240 that during boiling bubbles of vapour escape from the interior of the liquid. This cannot happen if the pressure of the vapour in the bubbles is less than that of the atmosphere, as in that case the bubble would collapse under the external pressure. But the pressure inside the bubble cannot exceed the saturation vapour pressure of the surrounding liquid. Hence, until that liquid has reached a temperature at which its saturation vapour pressure equals the pressure of the atmosphere, boiling cannot take place.

The connexion between these observations and the change of boiling point with pressure should be obvious. The U-tube experiment just described may be adapted for finding the boiling point of a liquid of which very little can be obtained.

The presence of dissolved gas in a liquid assists the process of boiling. The tiny bubbles of gas provide spaces into which the liquid can vaporise and so promotes the formation of the vapour bubbles. In the absence of dissolved gas a liquid often " boils with bumping," that is, it rises considerably above its true boiling point and then a large volume of vapour is suddenly formed and causes a violent disturbance in the liquid. This is repeated time after time. It can be prevented, and steady boiling can be secured, by dropping a little porous material, such as pipeclay, into the liquid, as air is carried into the liquid in the pores of the material.

Moisture in the Atmosphere

The atmosphere always contains a certain amount of water vapour which varies from day to day. Usually evaporation into the air is taking place from exposed water surfaces. If, however, the amount of moisture in the air becomes sufficient to exert the maximum vapour pressure of water at the temperature of the air, this evaporation will stop. If the temperature of the air then falls, the vapour in it will be exerting more than its saturation pressure for the new temperature, and some of it will condense.

This can usually be seen if a tumbler containing very cold water is taken into a warm room. The outside of the tumbler is quickly clouded by the condensation of water on it, since it cools the air immediately surrounding it to a temperature below that at which the vapour present produces saturation.

This process is largely responsible for the feeling of discomfort experienced in a crowded ill-ventilated room. Normally evaporation of moisture from the body is taking place, and this moisture takes its latent heat of vaporisation from the body, producing a cooling effect. If the air surrounding the body becomes saturated with moisture this evaporation stops and the temperature of the body rises.

It will also be seen that the " dryness of the air " mentioned on page 248 depends on its temperature as well as on the amount of water vapour in it. Hot air is much less moist than cold air containing the same proportion of water vapour, since it requires much more vapour to saturate it.

Dew Point

If air is cooled it will always be possible to reach a temperature at which the moisture present just saturates it. This temperature is called the **dew point** for the particular time at which it was determined. *Note that dew point is not a fixed point, but that it describes the condition of the atmosphere at a particular time.*

To determine the dew point of the air of the laboratory, partly fill a bright tin can, a cocoa tin will do, with cold water. Insert a thermometer. Add small chips of ice, stir well, and let each ice chip melt before the next is added. Note the reading of the thermometer as soon as the outside of the tin is misted by condensed moisture. Continue the stirring without adding more ice, so that the water gradually warms up to air temperature again, and note the thermometer reading as the mistiness just disappears. The average of the two readings is taken as the actual dew point.

Formation of Dew, Mist, Clouds, etc.

In the section on radiation it was pointed out that the surface of the earth is warmed by radiation from the sun which does not heat the air through which it passes. When the surface of the earth is hot, the layer of air in contact with it will be warmed by conduction from it, and then convection currents will spread this heat through the lower layers of the atmosphere. Conversely, during the night, the surface of the earth tends to lose heat by radiation into space. This radiation does not heat the air as it passes through it, but the air in contact with the earth's surface will lose heat by conduction to the earth and so be cooled.

(1) DEW AND MIST.—If the air in contact with the ground is cooled by the above process until it is below its dew point, moisture will be condensed and deposited on the cold surfaces. Thus *dew* is formed. If the cooling is very rapid and extends for a considerable distance from the surface of the ground, tiny drops of moisture will be condensed throughout the mass of cooled air and a *mist* results. Dew is usually more copious on grass than on surrounding surfaces because the grass itself tends to give off water vapour, and if the air above it is already saturated this vapour is condensed as it enters the air. The conditions which favour the formation of dew and mist are:—

(*a*) The air should contain a large amount of moisture so that little cooling will bring it to its dew point.

(*b*) The sky should be cloudless. This favours radiation from the earth. Clouds have a blanketing effect and tend to reflect radiation back to the earth.

(*c*) The air should be still. If a wind is blowing the air in contact with the earth's surface is constantly being changed and no portion of it remains in contact with the earth long enough to be cooled to its dew point.

(2) HOAR FROST.—In very cold weather *hoar frost* may be formed on the ground instead of dew. This may happen in two ways. Firstly, dew may be formed and then frozen owing to a further fall of temperature. If, however, the air is very dry, it may not have a dew point above freezing point, and so is still unsaturated when cooled to the latter temperature. Cooling below freezing point will produce saturation at some point, and if the cooling proceeds beyond that point, some of the moisture must condense. Since it is already below its freezing point it condenses as particles of ice instead of as drops of water.

(3) CLOUDS.—The formation of a *cloud*, like that of mist, is due to a large mass of air being cooled below its dew point, but the cooling is produced in a different way.

If you open the valve of an inflated cycle tyre and hold your finger in the escaping air you will find that it is very cold. The expansion of a gas without heat being supplied to it always has a cooling effect.

When damp air rises, either as a convection current or through a wind being deflected upwards by a hillside, it expands because the pressure on it is falling. This expansion produces cooling, and if the air is cooled below its dew point, condensation will take place. The condensed particles may be very small so that they tend to fall very slowly, and the movements of the air may keep them suspended for a long time.

(4) RAIN AND HAIL.—When the tiny droplets in a cloud run together to form large drops which are too heavy to be supported by the movements of the air *rain* occurs. *Hail* is formed by rain drops passing through very cold layers of air on their way to the ground and becoming frozen.

(5) SNOW.—*Snow* is formed under conditions similar to those for the formation of hoar frost. That is, vapour is condensed directly to ice crystals in a layer of air where the dew point is lower than freezing point. As these crystals fall through other layers of moisture laden air, further crystals condense on them forming a mass of crystals with a large amount of air entrapped between them.

(6) FOG.—*Fog* is formed when a large mass of air containing dust is cooled below its dew point. The dust particles form surfaces on which the condensed moisture collects and continue to float in the air.

Hygrometry

The measurement of the dampness of the air is termed **hygrometry**, and it is an important factor in weather forecasting. From the foregoing it should be clear that the nearness or otherwise of a state of saturation in the air is of more importance than the actual amount of moisture present. Hence the fraction

$$\frac{\text{Mass of water vapour present in a given volume of air}}{\text{Mass required to saturate that volume at air temperature}}$$

is taken as measuring the **hygrometric state** or **relative humidity** of the air. That fraction multiplied by 100 is termed the **percentage relative humidity.** The mass of vapour present in a given volume will be proportional to its pressure, so relative humidity (R.H.) may also be expressed as:—

$$\text{R.H.} = \frac{\text{Pressure of vapour present in the air}}{\text{Saturation pressure at air temperature}}.$$

Further, the vapour actually present produces saturation at dew point, so the above can be expressed as:—

$$\text{R.H.} = \frac{\text{Saturation pressure at dew point}}{\text{Saturation pressure at air temperature}}.$$

The latter expression is generally used in determinations of relative humidity. Tables have been constructed giving the saturation pressures of water vapour at various temperatures. Hence the relative humidity of the air may be found by noting the temperature of the air, determining the dew point by the method given on page 253 and looking up the corresponding saturation pressures from the tables. For example, on a certain day the temperature of the air was 16° C. and the dew point was 6° C. Tables give saturation pressures of aqueous vapour as 13·64 mm. at 16° C. and 7·01 mm. at 6° C.;

$$\therefore \text{ Percentage relative humidity} = \frac{7{\cdot}01}{13{\cdot}64} \times 100\% = 51{\cdot}4\%.$$

This means that the air was a little more than half saturated.

Instruments designed for the accurate determination of dew point are known as hygrometers. One of the simplest is that due to Regnault illustrated in Fig. 193 (*a*). It consists of two wide tubes, each having a polished silver thimble fitted on its lower end. The one on the left contains a quantity of ether into which the bulb of a thermometer dips, and is fitted with tubes as shown so that a current of air can be drawn through the ether causing evaporation and consequent cooling. This cools the air surrounding the tube and when the layer of air in contact with it is cooled to dew point moisture will be deposited on the silver thimble and dim its surface. The temperature of the ether is then read. The stream of air is then stopped and the ether gradually warms up to air temperature.

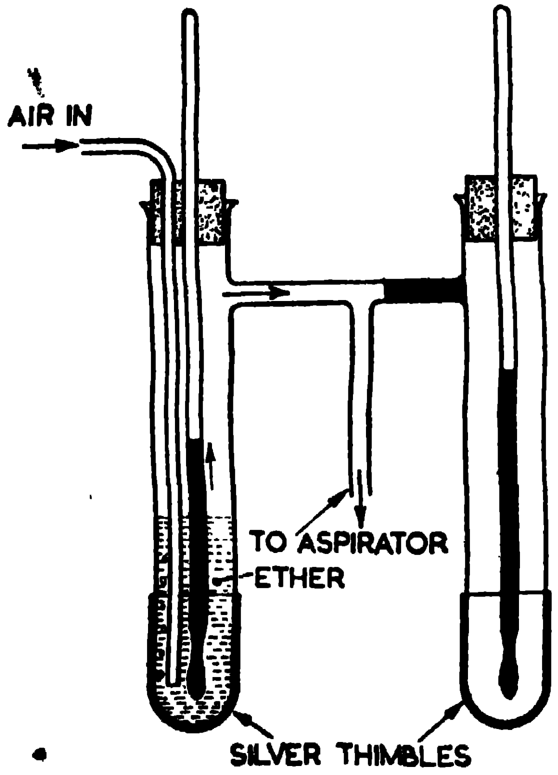


FIG. 193 (*a*).

The thermometer is read again when the dew disappears from the silver surface and the average of the two temperatures recorded is taken as the dew point. The tube on the right contains no ether so the surface of its silver thimble remains bright throughout the experiment and by comparison makes it easier to see the clouding of the other surface immediately it takes place. The air temperature at the time of the experiment can be read from the thermometer in the right-hand tube.

Wet and Dry Bulb Thermometers

The hygrometric state can be determined by using a pair of thermometers, one of which has its bulb surrounded by cotton fabric which dips into water (Fig. 194). Water evaporates from the fabric around the bulb, withdrawing latent heat of vaporisation from the thermometer and so causing it to give a lower reading than the dry bulb instrument. The drier the air, the more rapidly does this evaporation take place and the more the wet bulb reading is depressed. Thus a big difference between the readings of the two thermometers indicates dry air, while a small difference indicates that the air is nearly saturated. Special tables are obtainable which enable the pressure of vapour present in the air to be determined from the air temperature and the difference between the wet and dry bulb readings.

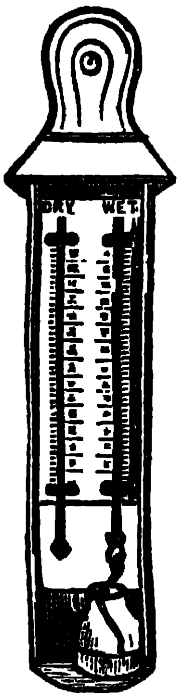


FIG. 194.

It is not necessary to go into details of these special tables in this book: if you are interested, look up *Glaishel's Factors*, *Smithsonian Table* or *Apjohn's Formula* in some advanced book on Heat.

QUESTIONS ON CHAPTER XXI

1. Explain the process of evaporation and distinguish between evaporation and boiling.

State factors which affect (*a*) the rate of evaporation of a liquid, (*b*) the temperature at which it boils. [L.U.

2. Explain each of the following:—

(i) Wet clothes usually dry more quickly on a warm day than on a cold day.

(ii) They dry more quickly on a windy day than on a still day.

(iii) They dry very slowly on a rainy day even if kept indoors.

3. Explain (*a*) why warm moist air causes more discomfort than hotter dry air, (*b*) why petrol spilled on the hand feels colder than water at the same temperature, (*c*) why it often rains on the windward side of a range of hills when it is fine on the other side.

4. State and explain what would be the effect on the readings of a barometer if (*a*) a few bubbles of air, (*b*) a little water were left in the tube. How could you find out whether it was air or water in the space at the top of the tube of a barometer showing these effects ?

5. What is meant by *saturation vapour pressure ?*

Describe *one* experiment to verify the statement that the saturation vapour pressure of water at the temperature at which water boils is equal to the atmospheric pressure.

Explain clearly the difference between evaporation and boiling. [J.M.B.

6. Describe the various observations which may be made if water in a beaker containing a thermometer is slowly heated until it boils. Hence show the differences between evaporation and boiling.

Some alcohol (B.P. 78° C.) was heated in a flask and the temperature rose to 81° C. There was then a large burst of vapour from the alcohol and the temperature fell to 78° C. Explain these observations and state how the alcohol might have been made to boil steadily at 78° C.

7. Water is introduced, drop by drop, into the space above the mercury in a barometer tube until a liquid film forms on the surface of the mercury. State and explain carefully what happens during this procedure.

Describe further what will occur if the barometer tube is surrounded by steam, there being always sufficient water in the barometer tube to maintain a liquid film on the top of the mercury column. [L.U.

8. Describe an experiment to show that the temperature at which water boils depends on the pressure on the water.

Describe how you would determine the normal boiling point of a liquid of which only a small quantity was available. How would you modify your observations when the atmospheric pressure during the experiment was 74 cm. of mercury ? [J.M.B.

9. Why is the temperature recorded by a thermometer usually lowered when a piece of damp muslin is wrapped round the bulb (called a wet-bulb) ?

Describe an experiment in support of your explanation.

On what does the extent by which the temperature is lowered depend ?

Is it possible for such a " wet bulb " not to be lowered in temperature ? Give reasons for your answer. [J.M.B.

10. A woollen fabric when immersed in a highly volatile liquid, such as petrol, for cleaning purposes and afterwards hung in the air becomes covered with hoar frost. Explain fully, with reasons, why this occurs. [L.U.

11. What do you understand by the percentage or relative humidity of the atmosphere ? Describe how the dew point is determined and how from a knowledge of this temperature the percentage or relative humidity may be obtained. [L.U.

12. Explain the formation of dew, pointing out the conditions most favourable for its deposition.

A deposition of moisture is found sometimes on the inside and sometimes on the outside of a window pane. Explain the conditions under which these deposits are likely to occur. [L.U.

CHAPTER XXII

MECHANICAL EQUIVALENT OF HEAT

The performance of work against opposing friction is always associated with the production of heat. Machine bearings, if not well lubricated, become very hot; water is run on to a grindstone to prevent excessive heating of the metal being ground; and primitive man usually obtained fire by utilising the heat produced when pieces of wood are rubbed together.

Heat is also produced when work is done in compressing a gas, as may be noticed from the rise in temperature of a bicycle pump when a tyre is being blown up. Conversely, when a gas expands, and thereby does work against the pressure on it, as was noticed in the last chapter, heat disappears and the gas is cooled.

An electric lamp or electric fire illustrates the fact that electrical energy may be used to produce heat, while the thermopile (page 204) shows that heat may produce electrical energy.

Heat is also frequently produced when kinetic energy disappears. If a nail on an iron anvil is continuously hammered with a heavy hammer it becomes very hot.

These illustrations support the view that **heat is a form of energy** since, apparently, various forms of energy may be transformed into heat, and heat may be transformed into other forms of energy.

Caloric Theory

Early investigators of heat considered that it was a material fluid, called caloric, which flowed from one body to another. The flow of heat into a body did not increase its weight, so caloric had to be considered to be weightless. Much of what has been dealt with in previous chapters is most readily described by thinking of heat flowing from one body to another, and the caloric theory was useful in developing ideas of definite quantities of heat, specific heats, etc.

Count Rumford

Count Rumford was an American engineer who, about 1798, was appointed by the Elector of Bavaria to superintend the boring of cannon. He was impressed with the great amount of heat which was liberated during the boring, and felt that explanations based on the caloric theory were unsatisfactory.

One of these explanations was that during the boring heat flowed into the metal from its surroundings. Rumford arranged for a piece of metal to be bored while it was immersed in a tank of water. The water was quickly brought to boiling point, indicating that the metal gave heat out to its surroundings instead of receiving heat from them.

Another explanation was that the boring altered the thermal capacity of the metal. If the thermal capacity were reduced, the heat already present would raise the metal to a higher temperature. Rumford disproved this by showing that the specific heat of the chips bored out was the same as that of a piece of the metal before boring.

Further, by using a blunt borer, Rumford showed that a great quantity of heat could be produced when only a little metal was removed by it, and that, apparently, this production of great quantities of heat could be continued so long as the boring continued. On the caloric theory a point should have been reached when the metal had been " emptied " of heat and no more could be got from it.

As a result of these experiments Rumford came to the conclusion that the caloric theory was not true, and that heat was probably a form of energy.

Sir Humphry Davy

A year or so later Sir Humphry Davy came to similar conclusions after experiments in which blocks of ice were rubbed together. He found that the blocks melted when rubbed together. This happened even when they were rubbed by mechanical means in vacuum, so that heat could not flow in from the surroundings. Moreover the thermal capacity of a mass of water was known to be greater than that of an equal mass of ice, so as there was no fall of temperature, the water produced must have contained more heat than the original ice.

James Prescott Joule

If heat is a form of energy, transformations between heat and other forms of energy must be subject to the law of conservation, that is, the quantity of heat produced and the quantity of mechanical energy, electrical energy, or other form of energy producing it must be equivalent. It follows from this that there must be a fixed quantity of mechanical energy which will always produce 1 unit of heat if transformation is complete. This was verified by James Prescott Joule who announced his results in 1843.

Joule's apparatus is illustrated in Fig. 195. A calorimeter containing water was fitted with paddles on a spindle which could be turned by two falling weights attached to strings wrapped round a drum connected with the spindle. The calorimeter, as indicated in the lower sectional drawing was divided into compartments by a number of baffle plates through which the paddles would just pass. Thus the water could not be set into a swirling motion and so gain kinetic energy when the paddles rotated and work would be done by the paddles against the resistance of the water. The drum could be disconnected from the spindle while the weights were being wound up, so that work was done on the water only when the weights were falling.

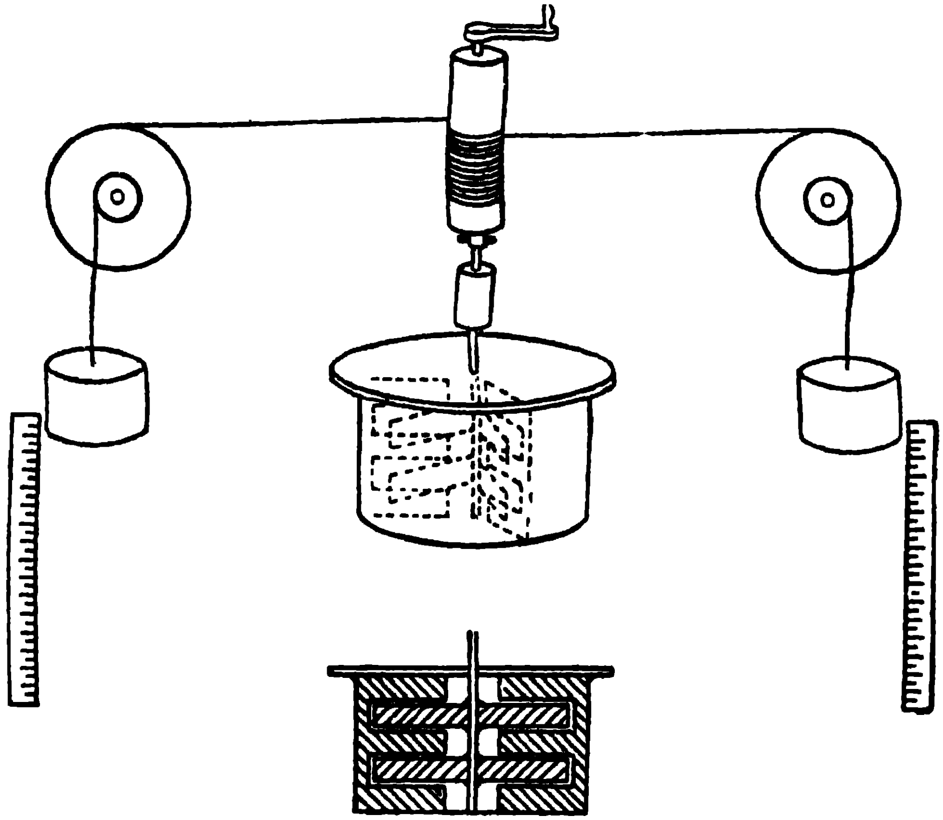


FIG. 195.

A known weight of water was placed in the calorimeter, the water equivalent of which was known. The temperature was taken by means of a thermometer passing through the cover of the calorimeter. The weights were wound up, the spindle connected, and the weights allowed to fall driving the paddles through the water. This was repeated a number of times and the temperature of the calorimeter rose. The distance through which the weights fell each time was measured.

Suppose each weight had a mass of M lb., and that they fell d feet each time and were allowed to fall n times. Then the loss of potential energy each time they fell was $2Md$ foot-pounds, and the total loss of potential energy was $2Mdn$ foot-pounds. If the mass of water plus the water equivalent of the calorimeter was W lb., and the rise of temperature $t°$ F., the quantity of heat generated was Wt British thermal units. Hence it could be said that the disappearance of $2Mdn$ foot-pounds of energy resulted in the production of Wt British thermal units. Therefore 1 British thermal unit would be produced by $\frac{2Mdn}{Wt}$ foot-pounds of energy. In a large number of experiments Joule found a practically constant value for this expression, and so established the equivalence of heat and mechanical energy.

The number of mechanical units of energy that can be converted into 1 unit of heat is called the mechanical equivalent of heat or Joule's Equivalent. The value Joule found for it was 772·7 ft.-lb. per British thermal unit. Later and more accurate work gives the value 778 ft.-lb. per British thermal unit. Expressed in metric units the mechanical equivalent of heat is $4{\cdot}18 \times 10^7$ ergs per calorie. In formulae this constant is generally represented by J, so that provided corresponding units are used for H, E, and J, we may write $H = \frac{E}{J}$ or $E = JH$, where H and E are equivalent quantities of heat and mechanical energy.

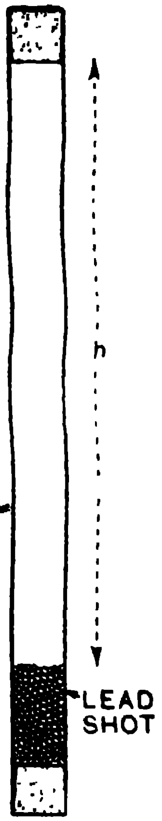


Fig. 196.

Laboratory Method of Finding J

Fit a long, wide tube with a cork at each end (Fig. 196). Weigh out about 200 grm. of lead shot and pour it around the bulb of a thermometer in a beaker to take its temperature. Then place it in the tube. Measure the distance marked h in Fig. 196. Note that this is the average distance the shot will fall if the tube is inverted. Now invert quickly 50 times taking care that the tube is brought upright and the shot fall the whole length of the tube at each inversion. Quickly pour the shot round the thermometer bulb again and take its temperature which will be found to have risen. A typical result is given below—

Wt. of shot = 200 grm. 1st temperature = 16° C.

Distance fallen each time = 98 cm. Number of times fallen = 50.

2nd temperature = 20° C.

Since the specific heat of lead = ·03,

$$\text{Heat gained by the lead} = 200 \times 4 \times \cdot 03 \text{ calories.}$$

$$\text{Loss of potential energy each time the lead fell} = 200 \times 98 \times 980 \text{ ergs;}$$

$$\therefore \text{ Total loss of potential energy} = 50 \times 200 \times 98 \times 980 \text{ ergs.}$$

$$\therefore \text{ 1 calorie is equivalent to } \frac{50 \times 200 \times 98 \times 980}{200 \times 4 \times \cdot 03} \text{ ergs}$$

$$= \frac{4802000}{\cdot 12} = \frac{4{\cdot}802}{1{\cdot}2} \times 10^7 \text{ ergs.}$$

i.e. 1 calorie is equivalent to = 4·002 × 10^7 ergs.

It will be noted that the lead really need not be weighed since its mass occurs in both numerator and denominator of the above fraction.

Steam Engines

Joule's equivalent is of fundamental importance to engineers since most engines are contrivances for converting heat into mechanical energy for doing work. Thus in a steam engine the combustion of the fuel in the furnace converts a quantity of chemical energy into heat and results in the production of gaseous products of combustion at a very high temperature. These hot gases flowing through the boiler tubes transfer much of their heat to the water in the boiler and so generate steam at high pressure, thus converting much of the heat into potential energy. By means of valve mechanism a quantity of this high pressure steam is admitted into the cylinder and the supply is then cut off. The steam expands and falls in pressure, driving the piston along the cylinder. Thus its potential energy is transformed into kinetic energy in the moving parts of the engine.

Most modern steam engines are "double acting," that is the steam is admitted alternately on the two sides of the piston. Thus the expanding steam on the one side drives out the steam on the other side through an exhaust valve which is connected to a condenser. The condensing of the steam on the exhaust side reduces the pressure on that side below that of the atmosphere and so allows a bigger expansion of the working steam than if its pressure were only allowed to fall to that of the atmosphere.

EXAMPLE.—*What minimum weight of coal of calorific value* 11,000 *B.Th.U. per pound must be burned per hour in the furnace of a locomotive working at* 100 *h.p.? Assume a* 50 *per cent. mechanical efficiency and neglect heat losses.*

100 h.p. is a rate of working of 100 × 550 ft.-lb. per sec.;

$$\therefore \text{ Energy output of engine per hr.} = 100 \times 550 \times 60 \times 60 \text{ ft.-lb.}$$

As mechanical efficiency is 50 per cent.,

$$\text{Energy supplied per hr.} = 100 \times 550 \times 60 \times 60 \times 2 \text{ ft.-lb.}$$

$$\text{This is equivalent to } \frac{100 \times 550 \times 60 \times 60 \times 2}{778} \text{ B.Th.U.}$$

$$\therefore \text{ Weight of coal burned per hr.} = \frac{100 \times 550 \times 60 \times 60 \times 2}{778 \times 11000}$$

$$= 46 \text{ lb. (approx.).}$$

Actually considerably more than this would be required, as there are very great heat losses. The gases from the furnace are still hot when they have passed through the boiler and escape up the chimney. There is a certain amount of radiation from both furnace and boiler, and the steam is not reduced to air temperature when it has finished expanding in the cylinders. The output of a steam engine is only the equivalent of 10 to 12 per cent. of the heat supplied to it.

Internal Combustion Engines

These have a greater thermal efficiency than steam engines since the fuel is actually burned in the cylinders, and losses in transferring the heat from the point where it is produced to the point where the work is to be done are avoided.

In a *four-stroke petrol engine*, the first down stroke of the piston draws a mixture of air and petrol into the cylinder. The next upstroke compresses this mixture which is then exploded by an electric spark. The great heat developed raises the gaseous products of the explosion to a very high pressure which drives the piston down. As the piston comes up again it sweeps the used gases out of the cylinder. These four operations are then repeated.

In the Diesel engine air only is drawn in by the first stroke. This is compressed by the next stroke, towards the end of which oil is forced in under pressure. The heat generated by the compression of the air ignites the oil and no magneto or coil is needed. The next two strokes are similar to those of a petrol engine.

Even in internal combustion engines there are considerable heat losses as the exhaust gases are still at a high temperature when they leave the cylinders and much heat is conducted and radiated from the engine. The output of a petrol engine is equivalent to about 25 per cent. of the heat supplied, and that of a Diesel engine about 35 per cent.

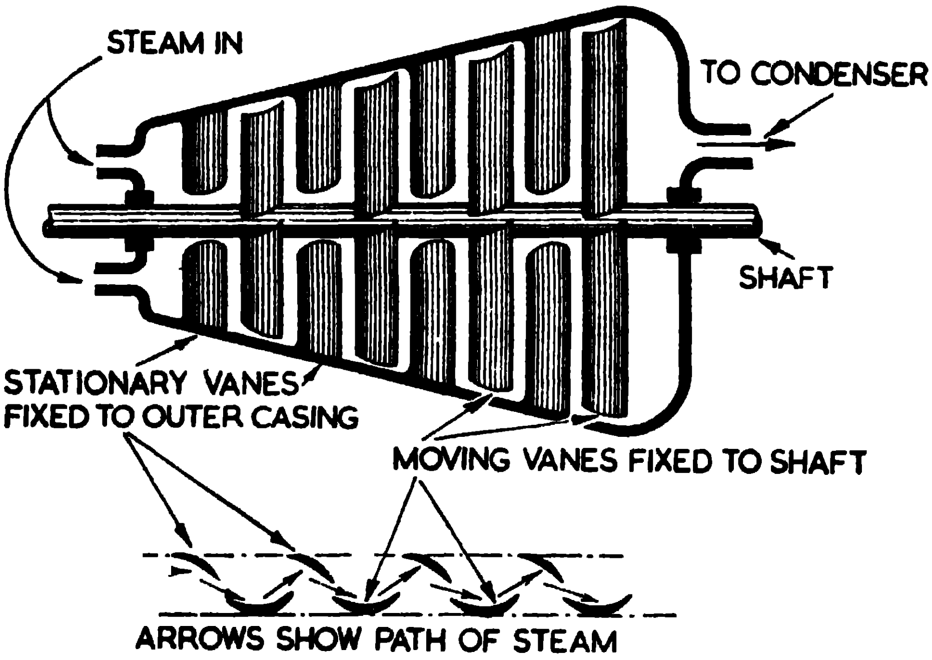

STEAM IN
TO CONDENSER
SHAFT
STATIONARY VANES FIXED TO OUTER CASING
MOVING VANES FIXED TO SHAFT
ARROWS SHOW PATH OF STEAM

Fig. 196 (*a*).

Steam Turbines

In many steam engines nowadays the turbine arrangement is used instead of the cylinder and piston. Here again the heat produced by burning the fuel generates high pressure steam. This is admitted into one end of the stator, a fixed casing [Fig. 196 (*a*)]. Its expansion as it enters gives it a considerable velocity so that some of its potential energy is transformed into kinetic energy. A ring of fixed vanes inside the stator deflects the steam so that it strikes a ring of other vanes on the rotor, the rotating part, at a suitable angle and some of its kinetic energy is transferred to the rotor making it rotate. The rotor vanes deflect the steam to another set of stator vanes which in turn deflect it on to further rotor vanes, and these processes are repeated a number of times. The stator casing widens out from the end where the steam enters, allowing the vanes to be made larger as the distance from the steam inlet increases. Thus as the steam passes from between one set of vanes to the spaces between the next set there is further expansion and more of its potential energy is transformed into kinetic energy. By the time the steam reaches the far end of the stator a large proportion of its energy has been so transformed and transferred to the rotor. Such turbine engines have a considerably higher efficiency than the cylinder and piston type.

Kinetic Theory

Many of the matters dealt with in preceding chapters are explained by a theory of matter known as the Kinetic Theory. This theory accepts the idea of the atomic and molecular structure of matter but makes the following important additional assumptions:—

(1) The particles constituting any portion of matter are not in contact but are separated by distances larger than the actual diameters of the particles.

(2) These particles mutually attract one another with forces which become smaller as the distances between the particles increase.

(3) The particles are in continuous movement and thus possess kinetic energy.

(4) Heating a body usually increases the velocity of motion of its particles and hence their kinetic energy, *i.e.* the heat energy supplied is converted into kinetic energy of motion of the particles and the temperature of the body corresponds to a definite average kinetic energy of those particles.

Solids, Liquids, and Gases

It has already been noted that these are three states of matter and that a substance may be changed from one state to another by heating or cooling and that absorption or liberation of latent heat takes place during the process.

In the solid state a body has a definite volume and is almost incompressible, *i.e.* extremely large pressures are needed to reduce its volume appreciably. It also has a definite shape independent of the space in which it is placed and usually considerable force is needed to separate portions of it. Because of the last property it is said to possess considerable *cohesion.* In the liquid state also matter possesses a definite volume and has little compressibility, but it has no definite shape and automatically takes the shape of any vessel into which it is placed. Also a liquid shows much less cohesion than a solid. In the gaseous state matter has neither fixed volume nor fixed shape. A quantity of gas introduced into any closed space immediately spreads out to fill the whole of that space. Also a gas shows considerable compressibility, obeying Boyle's Law, and has practically no cohesion.

The differences in compressibility indicate that the particles are much more closely packed in solids and liquids than in gases, and more closely in solids than in liquids. In solids the distances between particles are so small that the forces of mutual attraction are very great, making it difficult to move one particle bodily away from the others and so giving great cohesion. Also, particles do not easily move from place to place within a solid and the motion of each is a vibration around a mean position. In a liquid the greater distance apart of the particles reduces their mutual attraction and particles are free to move from place to place and are constantly shooting in all directions within the liquid. This freedom of movement accounts for the liquid immediately taking the shape of its containing vessel. But particles cannot easily escape from the liquid. Particles on the free surface are attracted by the particles below them and there is no corresponding upward force so they tend to be pulled inwards. It is this inward pull on surface particles which gives rise to surface tension. In gases the particles are so far away from each other that their mutual attractions are negligible and each particle is perfectly free to move so that they will spread into any free space offered to them. These free movements of particles in liquids and gases explain the phenomena of diffusion described in Chapter XIII.

Latent Heat

In a change of state from solid to liquid or liquid to gas the particles of the substance are driven further apart and hence work is done against the attractive forces between them. Part of the heat energy supplied is used in doing this work instead of increasing the kinetic energy of the particles and hence that quantity of heat does not result in a rise of temperature of the substance.

Evaporation

It was mentioned in an earlier paragraph that particles cannot easily escape from the free surface of a liquid, but some do escape and so cause evaporation. In a liquid, and in a gas, though the particles will have a constant average kinetic energy at a constant temperature, all do not have the same kinetic energy. Owing to constant collisions with one another the velocity of some is increased and that of others decreased. Some particles shooting towards the surface will break through it, but immediately they do so the attraction of the remaining liquid will tend to pull them back and most will return to the liquid. The distance through which this retaining attraction is appreciable is quite small and if an escaping particle has sufficient velocity to carry it beyond this range of attraction it will become free of the effect of the bulk of the liquid and become a gas particle free to move in space. Since rise in temperature increases the average velocity of the particles, it increases the number with the necessary velocity for escape and so increases the rate of evaporation. Those particles with the most kinetic energy will be the ones most likely to escape and their loss results in a lower average kinetic energy of the particles in the bulk of the liquid. This explains the cooling due to evaporation when heat is not supplied from outside the liquid.

The kinetic theory also explains the phenomenon of vapour saturation. If a liquid is evaporating into a closed space the evaporated particles are constantly having their directions of motion changed by collisions with one another and with the walls of the containing vessel. In time this will direct some of the particles towards the liquid surface and when they enter the range of attraction they will be pulled back into the liquid. At first there are very few particles in the space above the liquid and so their rate of return is lower than that of escape, and the concentration of particles in the gas space increases and raises the pressure in that space. This increases the rate of return and when a certain concentration has been reached in the gas space the number of particles returning to the liquid in a given time will be equal to the number escaping during that time. Hence the concentration and pressure of the vapour particles becomes constant. Raising the temperature of the liquid increases the escape rate and so a higher concentration and pressure of vapour must be reached to restore equilibrium. In this connexion it should be noted that the pressure exerted by a gas arises from the bombardment of the retaining surfaces by the gas particles.

Cooling of Gas by Expansion

Although the particles of a gas are so far apart that their mutual attractions have little effect on their movements, such mutual attractions are not altogether absent and when a gas expands work has to be done against these forces in increasing the distance apart of the particles. If heat is not supplied from outside, the energy for doing this work is supplied from the kinetic energy of the particles themselves. Thus their average kinetic energy is reduced and there is a fall in temperature.

QUESTIONS ON CHAPTER XXII

Take the mechanical equivalent of heat as 778 ft.-lb. per B.Th.U. or $4{\cdot}2 \times 10^7$ ergs per calorie, and g as 980 cm. (per sec.)^2.

1. What was the caloric theory and what is the theory which has displaced it?

Briefly describe the part played by Rumford and Davy in disproving the caloric theory.

2. Describe Joule's experiment on the conversion of mechanical energy to heat. What was the special importance of his experiments?

3. What is meant by the mechanical equivalent of heat?

If a piece of lead (specific heat 0·03) falls a distance of 80 ft., how much will its temperature rise on striking the ground? Assume that 50 per cent. of the heat generated remains in the lead.

4. A waterfall is 250 m. high. Assuming that the water retains 75 per cent. of the heat generated at the end of its fall, find how much the temperature of the water at the foot of the fall is above that of the water at the top of the fall.

5. Explain what is meant by the statement that "heat is a form of energy." Describe two experiments to illustrate the truth of this statement, one of the experiments being of such a nature that a quantitative relation between heat and energy can be obtained. [L.U.

6. Define a *unit quantity of heat* and a *unit of work*.

Describe an experiment with which you are acquainted to show that when heat is produced by the expenditure of work there is a definite numerical relation between the amount of work performed and the quantity of heat produced. Show how this relation is deduced from the observations made. [L.U.

7. Describe two experiments which illustrate that heat is a form of energy. Explain the meaning of the statement that the "Mechanical equivalent of heat is 4·18 joules per calorie." [L.U.

SECTION III—LIGHT

CHAPTER XXIII

PROPAGATION OF LIGHT

We know that for objects to be visible the presence of light is necessary. We are becoming familiar with photo-electric arrangements by which light sets various mechanisms in operation. This suggests that light is a form of energy. There is additional evidence for this in the fact that light can promote chemical actions such as the changes in the coating of a photographic film when it is exposed.

We may therefore describe light as *that form of energy which affects the eye in such a way as to produce the sensation of sight.* In Chapter XXXI. it is shown that this energy is transmitted by radiation.

Luminosity

Some bodies, such as the sun, a candle flame, or the glowing filament of an electric lamp, emit light which they, themselves produce. Such bodies are said to be **luminous.** Others, such as a brick or a flower, do not produce light, and are *non-luminous*. Luminous bodies are seen when light which they give out enters the eye. Non-luminous bodies become visible only when light from luminous bodies falls on them. Thus a brick in a closely shuttered room is invisible, but becomes visible when an electric lamp is switched on in the room. In the latter case, light from the lamp falls on the brick and some is thrown off again from its surface. If some of this light enters the eye the brick is seen. It should be noted that every case of seeing involves the entry into the eye of light coming from the object seen.

Transmission of Light

Some substances, such as glass and clear water, transmit most of the light falling on them in such a way that objects may be seen quite clearly through them. They are said to be **transparent.** Others, such as wood and metals, allow no light to pass through them and are said to be **opaque.** A third class allows the passage of light but in such a way that the objects from which the light comes cannot be seen through them. Thus a room with frosted glass windows receives light from outside but objects outside cannot be seen through the windows. Substances of this class are said to be translucent. Examples of translucent substances are ground glass, tissue paper, waxed paper, "cloudy" liquids.

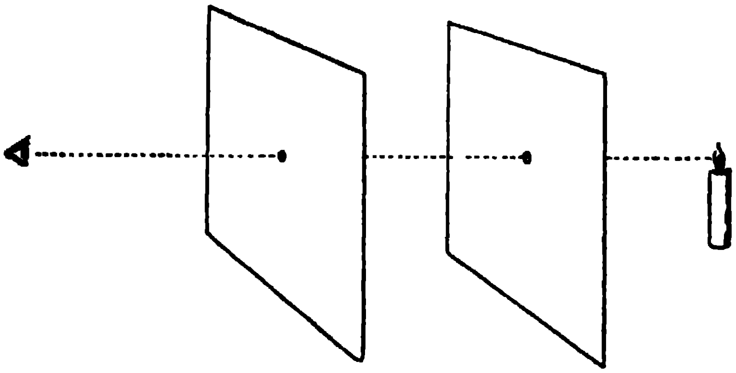


Fig. 197.

Rectilinear Propagation

One of the most obvious properties of light is that beams of light travel in straight lines. Beams of sunlight passing through gaps in clouds, and beams from searchlights, pocket torches, or car headlamps may all be observed to have straight boundaries. Also, without some arrangement for redirecting the light from them, we cannot see objects round corners which would be possible if light could follow curved paths.

Fig. 197 illustrates a definite experiment to demonstrate that light travels in straight lines. Each of two sheets of cardboard is pierced by a small hole and placed near a candle flame. It will be found that the flame is visible through both screens only when it is on the straight line which passes through the two holes.

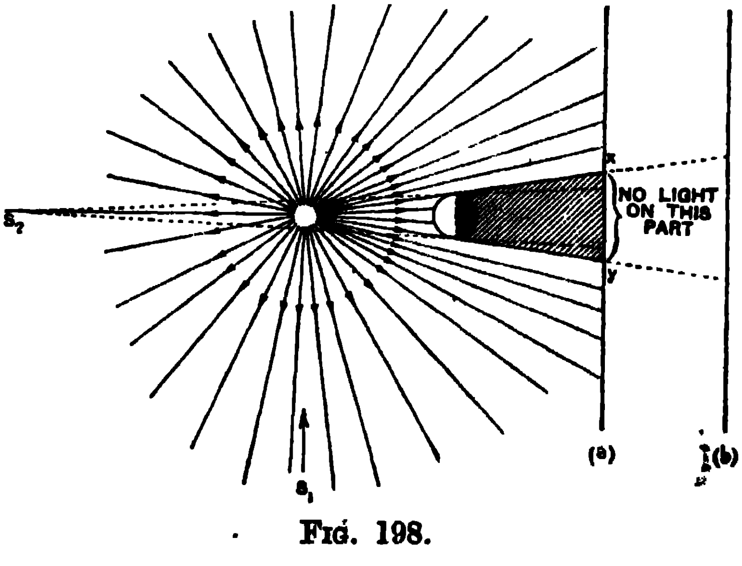


Fig. 198.

Shadows

The formation of sharp shadows of opaque objects is further evidence that light travels in straight lines. If, in a darkened room, such an object is placed between a white screen and a very small source of light, such as a pocket torch lamp without either lens or reflector, a clear shadow, of the same shape as the object but larger than it, will be seen on the screen. The identity of shape and the sharpness of the edge of the shadow indicate that the beams of light which pass the edge of the object do not bend into the space behind it.

Further, if the distance between the lamp and the object is increased, the shadow becomes smaller, while, if the distance between the object and the screen is increased, the shadow becomes larger. All these observations may readily be explained from Fig. 198 in which it is assumed that light spreads in straight lines in all directions from the small source of light.

Fit an electric lamp inside a box. Cut a hole in the front of the box and cover it with tissue paper. When the lamp is switched on light will spread out from each point on the paper over the hole which may thus be regarded as a source of light larger than a point. If an opaque object is placed between this source and a screen, a shadow with a perfectly dark centre surrounded by a less dark region will be formed. The totally dark part is called the **umbra** of the shadow, and the tly dark part is its **penumbra**.

the object is larger than the source of light, the umbra will be than the object and will increase in size as the screen is moved urther from the object. Fig. 199 illustrates the formation of such a shadow. No light spreading from point *a* can reach the screen between the points *x* and *y*. Similarly no light from *b* can reach the part of the screen between *p* and *q*. Hence the part of the screen between *x* and *q* receives no light from any part of *ab* and is totally dark. Light from

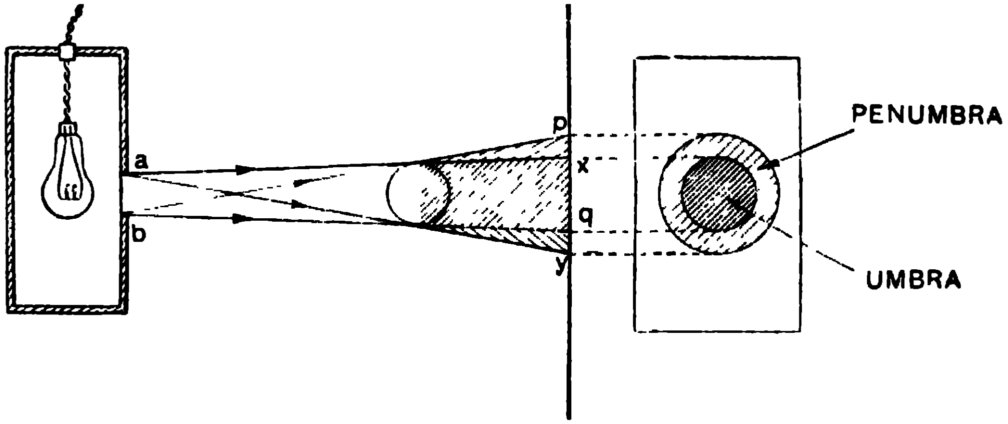


FIG. 199.

the upper part of *ab* but not from its lower part can reach *px* which is thus less brightly illuminated than the part of the screen above *p* but is not in total darkness. Similar remarks apply to *qy*. The part of the figure on the right shows the appearance of the shadow, taking into account the solidity of the arrangement which is only shown in section in the main diagram.

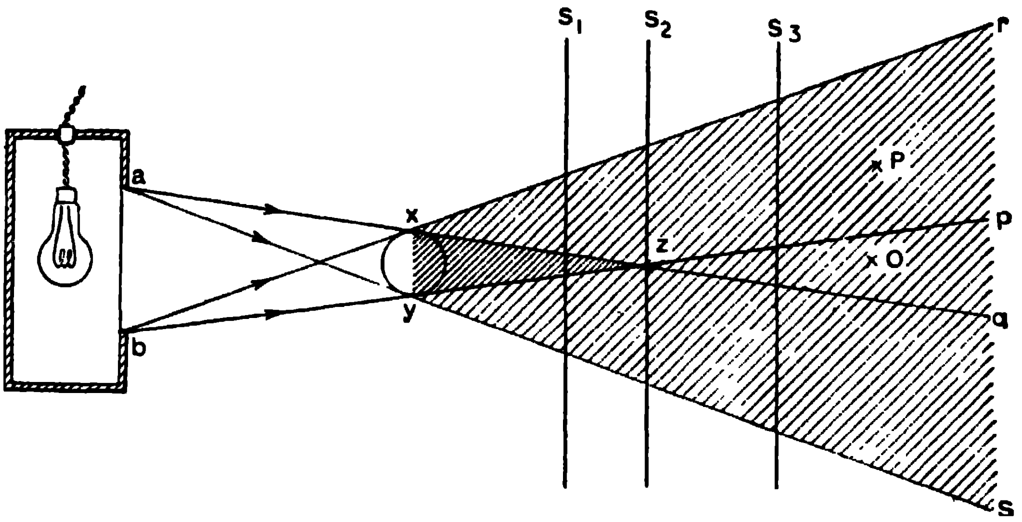
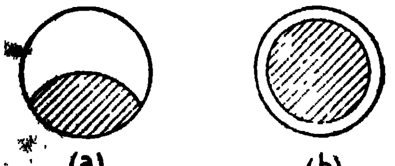


Fig. 200.

If the source of light is larger than the object, the umbra will be smaller than the object, and when the screen is moved sufficiently far back, it will disappear altogether, the whole shadow becoming penumbra. This is illustrated in Fig. 200. No light from any part of the source could penetrate into the cone *zxy*. Any point in the lightly shaded portion of the diagram would receive light from some part of the source but not from all of it. Thus, on a screen placed at S_1 there would be seen a small umbra surrounded by a wide penumbra. If the screen were moved back to S_2, the umbra would be reduced to a point, while at S_3 there would be no umbra at all.



(a) (b)

Fig. 201.

It is useful to consider what would be visible from various parts of Fig. 200. An eye within the cone *zxy* could not receive light from any part of *ab*, the whole of which would, therefore, be obscured by *xy*. At P no light from the lower part of the source could be received, but some of the upper part would be visible. Fig. 201 (*a*) illustrates the appearance of *ab* from

P, taking into account the solidity of the arrangement. At O no light could be received from the central portion of *ab*, but both upper and lower portions would be visible. Taking into account solidity, this means that the central portion of *ab* would be obscured, but that a bright ring would be seen surrounding this dark portion, as shown in Fig. 201 (*b*).

Eclipses

The sun is a luminous body, and the earth and the moon are non-luminous bodies. Moonlight is light from the sun which has been reflected by the surface of the moon, so that only those parts of the moon on which the sun's light is falling are visible. This explains the phases of the moon. At new moon the moon is between the sun and the earth, so that its illuminated side is turned away from the earth. At full moon the moon is on the opposite side of the earth to the sun, so that the whole of its illuminated side is visible from the earth. The sun is larger than either the earth or the moon, so both the latter bodies cast shadows into space of the type shown in Fig. 200.

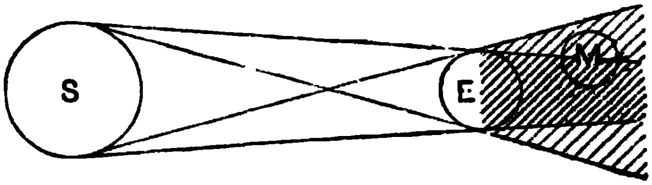


FIG. 202.
ECLIPSE OF THE MOON.

(1) ECLIPSES OF THE MOON.—At full moon the moon may enter the earth's shadow (Fig. 202), and any part of it which passes through the umbra of that shadow will be obscured as the light from the sun will be cut off from it. If the three bodies are directly in line when the moon passes through the shadow, the whole of it will enter the umbra so that the moon will be completely obscured, giving a *total eclipse*. If they are slightly out of line, only part of the moon will enter the umbra and be obscured, and a *partial eclipse* will be observed. Usually, at full moon they are so much out of line that the moon does not touch the earth's shadow, and no eclipse takes place.

(2) ECLIPSES OF THE SUN.—At new moon the earth may pass through the moon's shadow so that parts of the sun will be obscured to observers on the earth. Since the earth is larger than the moon the whole of it cannot pass through the umbra. At points of the earth which do come within the umbra (B, Fig. 203), the whole of the sun will be obscured, and a *total eclipse* will be observed. At places which

only pass through the penumbra part of the sun will always be visible, a *partial eclipse* will occur (A and C, Fig. 203).

The distance between the earth and the moon varies, so that sometimes during eclipses parts of the earth pass through the part of

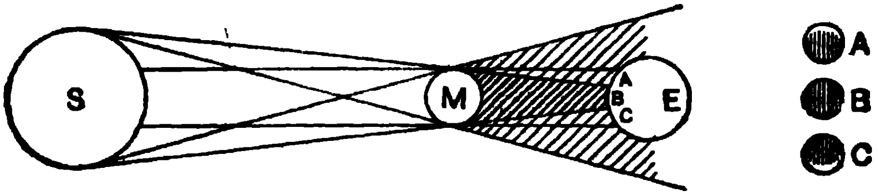


Fig. 203. Eclipse of the Sun.

moon's shadow corresponding to the region *zpq* in Fig. 200. As explained on page 275, at such places the centre of the sun will be obscured but a bright ring will be seen surrounding the central dark portion. Such an eclipse is said to be *annular* (Latin *annulus*, a ring). Owing to the fact that the three bodies are seldom in line, it is only occasionally that the sun is eclipsed at new moon.

Pinhole Images

Remove the lid of a cardboard box and paste tissue or grease-proof paper in place of it. Prick a small hole in the centre of the opposite side of the box. Point the hole at a lighted lamp in a dark room. An upside down (inverted) image of the lamp will be seen on the paper.

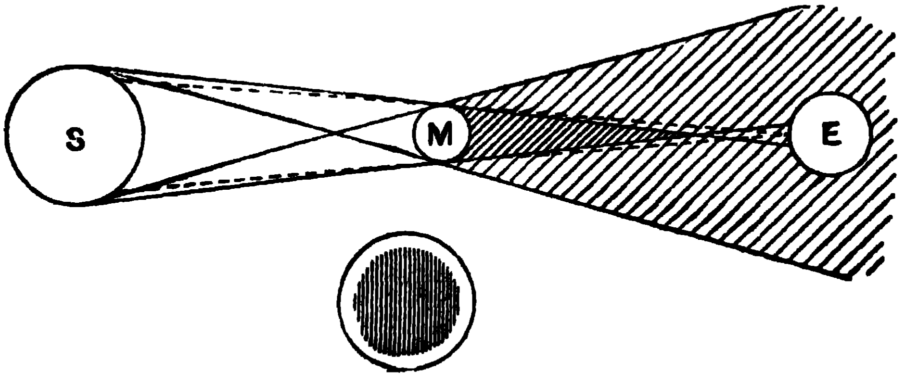


Fig. 204. The dotted lines show two rays from the sun proceeding to an observer on the earth.

The formation of this image is illustrated in Fig. 205. Beams of light spread out in all directions from each point on the object. One narrow beam from the top of the object will pass through the hole at A and light up a spot near the bottom of the paper, B. Similarly, one beam from the bottom of the object will light up a spot near the top of B. In between spots of light corresponding to other points on the object will be formed. These spots, blending into one another, light up a part of the paper with the same shape as the object, but owing to the crossing of the beams at A, this image is upside down.

Further study of these images may be made by making the pinhole camera in two separate parts, one of which will slide into the other so that the distance between A and B may be varied. It will be found that, as this distance increases, the image becomes larger and less bright. The spreading of the beams from A clearly accounts for the increase in size. The diminution of brightness is due to the fact that the same quantity of light as before is now spread over a greater area of the paper.

If the hole is enlarged, the image will become brighter but its edges will be less clearly defined. The larger hole will let in more light which accounts for the increased brightness. The separate spots of light of which the image is formed will, however, be larger, and this will produce visible irregularities in the outline.

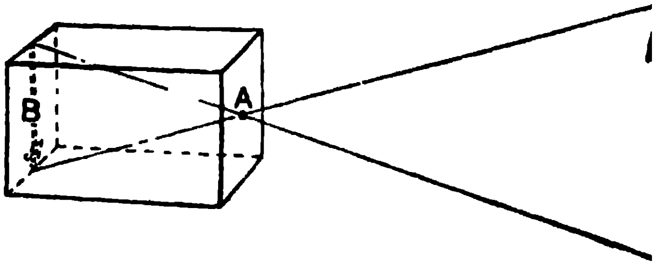


Fig. 205. The Pinhole Camera.

Experiments may also be made with holes of different shapes. This will make no difference to the image so long as the hole is small, as while the separate spots of light will have the same shape as the hole, there will still be a spot for each point on the object, and so long as these are small enough to blend properly, the image will have the same shape as the object. In this connexion note the circular patches of light which may often be observed in the shadow cast by a tree. Small openings between the leaves act as "pinholes" and form images of the sun which are not affected by the shapes of the holes.

Photographs may be taken with a camera which has a pinhole in place of the usual lens. Long exposures are necessary since very little light is admitted to the plate or film. Such cameras, however, are useful for architectural photographs. A lens, unless it has been specially corrected, will distort vertical lines. There is no such distortion in a pinhole image.

QUESTIONS ON CHAPTER XXIII

1. Explain the terms *luminous*, *non-luminous*, *transparent*, *opaque*, *translucent* mentioning in each case a body to which the term would apply.

2. Give three common observations illustrating that light travels in straight lines, and describe a definite experiment to show the truth of that statement.

3. Explain with the aid of diagrams the different types of shadows which may be formed. In your explanation make clear the meaning of the terms *umbra* and *penumbra*.

4. Explain the formation of pinhole images. Discuss the effect on them of (*a*) increasing the distance between pinhole and screen, (*b*) enlarging the hole, (*c*) altering the shape of the hole, (*d*) making two holes about $\frac{1}{4}$ in. apart.

5. Answer the following by means of scale drawings:—

(*a*) What would be the lengths of the diameters of the umbra and penumbra of the shadow of a metal ball 8 in. diameter placed 2 ft. from a source of light which is 6 in. in diameter, the screen being 1 ft. from the ball?

(*b*) A small hole is made in the window shutter of a room 10 ft. wide, and an image of a tree outside the room is cast on the opposite wall. If the image is 4 ft. high and the tree is 30 ft. from the window, what is the height of the tree?

6. Explain the formation of shadows, pointing out the differences in the nature of the shadow formed as the source of light increases in size.

Show from your diagrams how it is possible to obtain three different types of eclipse. [L.U.

7. Explain the occurrence of an eclipse of the moon. When is this (*a*) total; (*b*) partial?

Suggest why the moon can often be seen throughout totality as a dull, copper-coloured disc. [L.U.

CHAPTER XXIV

REFLECTION. PLANE MIRRORS

While we cannot normally see round corners, mirrors are often placed at road junctions and at corners in corridors so that approaching traffic round the corner can be observed. Light falling on the surface of the mirror is thrown off in a new direction making this possible. This throwing off of light falling on them by surfaces is called **reflection.** Non-luminous bodies only become visible when they throw off light in this way.

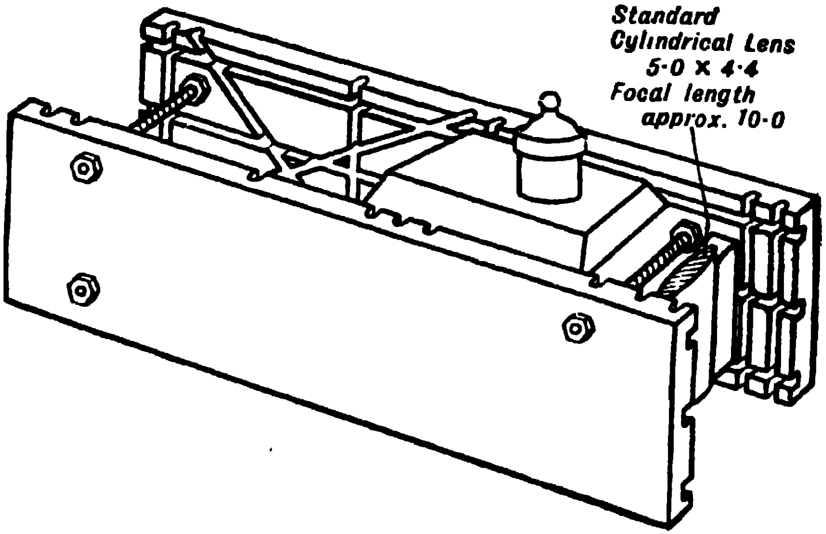


FIG. 206. RAY BOX.

Note that on a diagram we show the path of light by a straight line. This is referred to as a ray of light and the name is often used in certain cases where we have used the name *beam*: a beam is a collection of adjacent rays from a source.

Ray Boxes

Fig. 206 illustrates an arrangement which enables a study of the paths of beams of light to be made. It consists of a box with a sliding top to which a lamp is fixed. A lens is placed in one end of the box and screens with openings in them may be placed in front of the lens to let out narrow beams of light. Each beam will light up a narrow strip of the bench on which the box is placed, and so its path may be traced. If a screen with a number of parallel vertical slits is used, a number of beams will be obtained, and by altering the position of the lamp these may be made to follow parallel, converging, or diverging paths as desired. Various forms of apparatus based on this principle are obtainable.

Regular and Diffuse Reflection

Set a ray box to give parallel beams and then substitute a screen with a single slit for the one with a number of slits. Stand a slip of mirror across the beam and note that it is reflected as a quite definite beam [Fig. 207 (*a*)]. Substitute a roughish piece of white cardboard. A definite beam will not be reflected, but a patch in front of the card will be lighted up by parts of the beam being thrown off in various directions [Fig. 207 (*b*)]. Reflection of the first type is said to be regular; that of the second type is called diffuse reflection. A surface which causes good regular reflection is called a mirror.

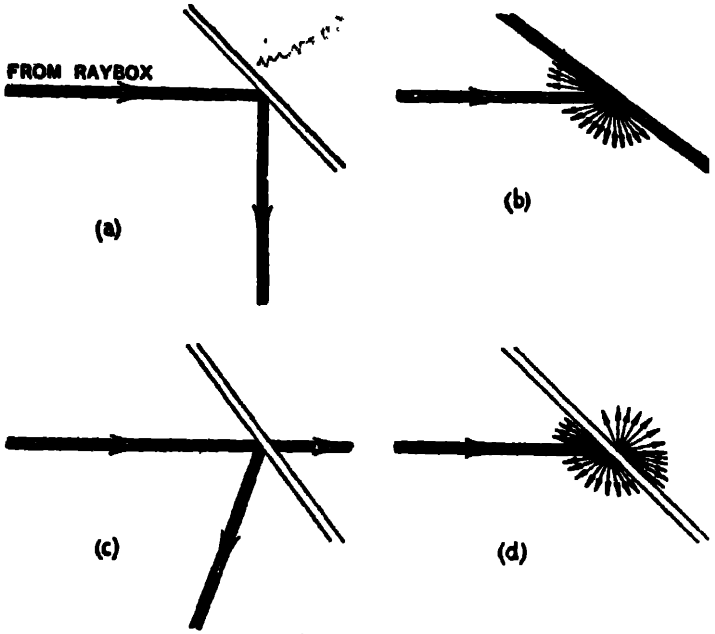


Fig. 207.

Try the effect of other surfaces, such as a fairly smooth tile, a polished sheet of metal, polished and rough pieces of wood. It will be found that smooth polished surfaces give good regular reflection, while rough surfaces tend to cause diffuse reflection.

If a clear sheet of glass is used a considerable portion of the light passes through the sheet and continues as a clear beam on the other side, but part is reflected [Fig. 207 (*c*)]. For this reason mirror glass is given an opaque backing—usually a thin coating of metal—which will reflect nearly all the light falling on it. It should be noted that with such mirrors, reflection will take place mainly from the back surface, though some light will be reflected from the front surface.

If a piece of ground glass is placed across the beam some of the light will be found to be diffusely reflected. Part will be transmitted, but it, too, will be diffused [Fig. 207 (*d*)].

When a dead black surface is used light will be found to be neither reflected nor transmitted to any visible extent. Such surfaces absorb light almost completely just as they absorb heat radiation. The interiors of cameras, telescopes, and other optical instruments are painted dead-black so that unwanted reflections from them will not take place.

The remainder of this chapter will be mostly concerned with regular reflection, but the fact should not be overlooked that diffuse reflection plays a very important part in life. It is light diffusely reflected from their surfaces which makes non-luminous bodies visible. Also light enters a room which is not facing the sun owing to its being diffusely reflected from objects outside. Further, diffuse reflection from walls and ceiling tend to produce a uniform distribution of the light in the room.

Laws of Reflection

The beam of light which travels towards a mirror is called the **incident beam**; that which travels away from it is the **reflected beam**. The point where the beam meets the mirror is its *point of incidence*. Any line perpendicular to the face of a mirror is said to be a *normal* to it. If the normal at the point of incidence is drawn, the angle between it and the incident beam is called the **angle of incidence.** The angle between the reflected beam and the normal at the point of incidence is the **angle of reflection.**

Mount a mirror slip along the diameter joining the 90° graduations of a circular card graduated in degrees. Place it so that the beam from the ray box meets the mirror just at the centre of the circle. Then, as shown in Fig. 208, the angle of incidence and the angle of reflection may be read from the circular scale. In this way find the values of the angle of incidence and angle of reflection in a number of cases, and verify the law that **the angle of reflection is equal to the angle of incidence** in every case. Verify also as particular cases that when the incident beam travels along a normal to the mirror, the reflected beam goes back along the same path, and that when the angle of incidence is 45°, the reflected beam is perpendicular to the incident beam.

A second law states that **the incident beam, the reflected beam, and the normal at the point of incidence are all in the same plane.** This is verified by noting that if the mirror is placed perpendicularly to the bench and the incident beam travels along the surface of the bench, the reflected beam also travels along the bench surface and is not directed either upwards into the air nor downwards into the bench. The two laws together definitely fix the direction along which any given incident beam will be reflected.

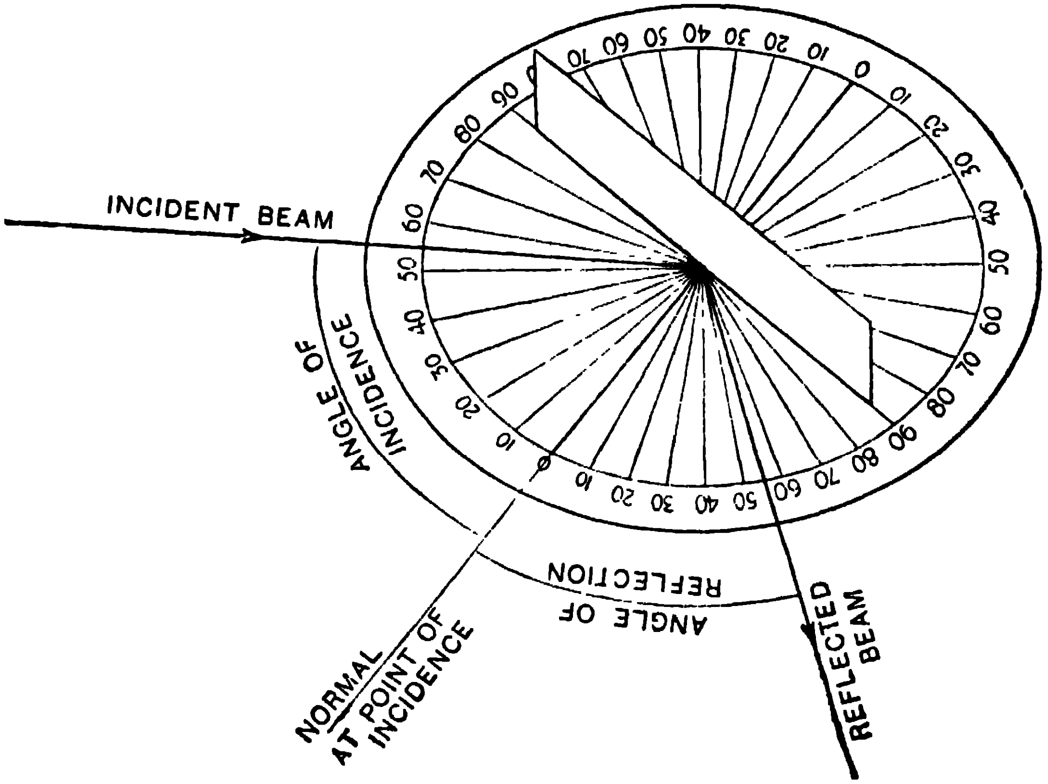


Fig. 208.

The laws of reflection also apply in diffuse reflection, but in that case, owing to the roughness of the reflecting surface, parts of the incident beam which are parallel to one another meet various parts of the surface at different angles of incidence, and so, as illustrated in Fig. 209, are reflected in various directions so that the beam is scattered instead of being reflected as a single beam.

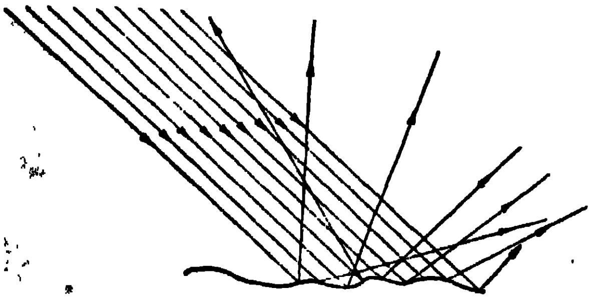


Fig. 209.

Images formed by Plane Mirrors

IMAGES.—When we look into a plane mirror we seem to see objects which are really in front of the mirror in positions behind the mirror. When we look through a telescope distant objects seem to be in positions nearer to us than they really are, and when we use a magnifying glass the object we are examining seems to be further away than its true position. **Whenever an object seems to be in some position differing from its true position it is said that an image of it exists at its apparent position.** The formation of an image is always due to the fact that, on their journey from the object to the eye, the beams of light are changed in direction so that they appear to come from points other than those from which they started.

IMAGE OF A POINT FORMED BY A PLANE MIRROR.—In Fig. 210 O represents a point on an object from which light is spreading and which stands in front of a mirror MN. One beam of light from O meets the mirror at P and is reflected along PQ making ∠ XPQ = ∠ XPO. Similarly another beam from O meets the mirror at R and is reflected along RS, so that ∠ YRS = ∠ YRO. If the two beams PQ and RS enter an eye placed as shown in the diagram they will seem to be coming from I. Hence O will appear to be at I, that is there is an image of O at I.

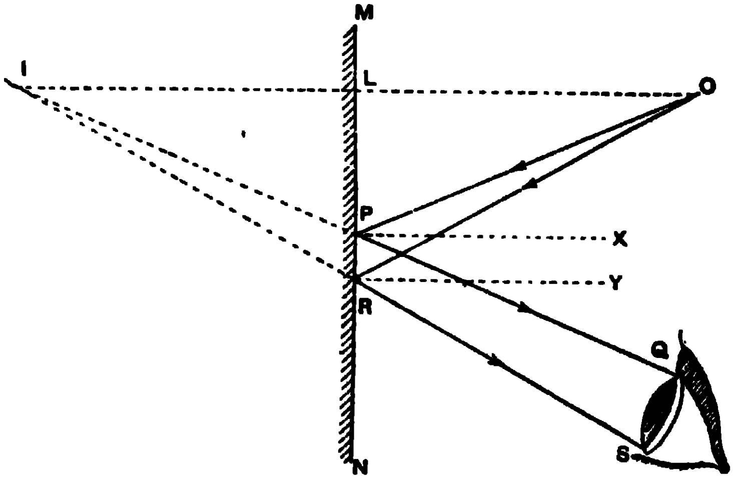


FIG. 210.

If an accurate figure is drawn it will be found that IO is at right angles to MN and that IL = LO. Hence the figure indicates that **the image of a point formed by a plane mirror is as far behind the mirror as the point is in front of it, and the line joining the point to its image is at right angles to the mirror.** This may be verified by the following experiment. Draw a line MN across a sheet of paper. Place a plane mirror upright along this line. At O, two or three inches from the mirror, place a pin upright. Stick in two other pins at P and Q, placing them so that when you sight along them they appear to be in line with

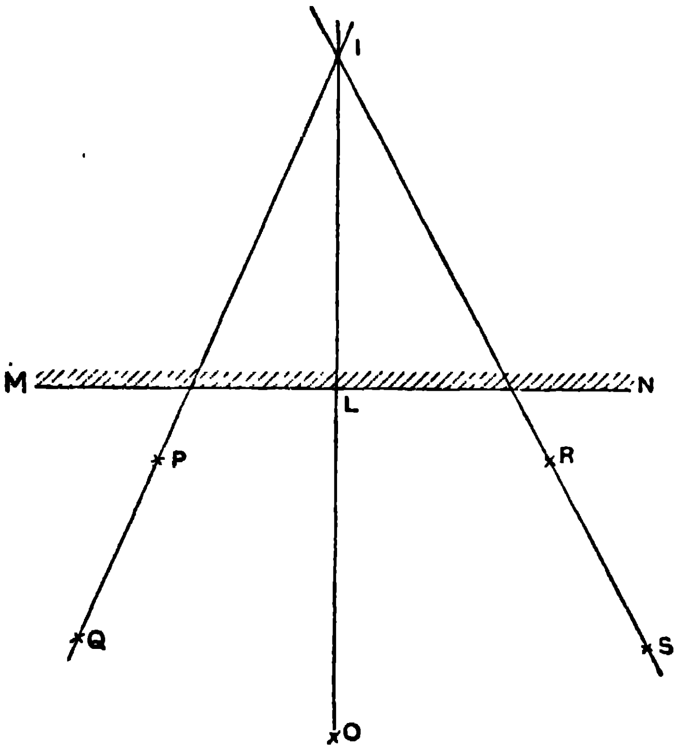


FIG. 211.

the image of O. Similarly place two other pins at R and S in line with the image of O. Remove pins and mirror. Join up pin pricks at Q and P and at R and S. Produce lines QP and RS to meet at I. Since the image of O was along both these lines it must be at I. Measurement will show that IL = LO and IO is perpendicular to MN.

IMAGE OF A WHOLE OBJECT FORMED BY A PLANE MIRROR.—Fig. 210 was concerned only with the image of a point. By considering an object as made up of a number of points, the image of a whole object may be constructed. Fig. 212 shows an object AB placed in front of a mirror. If AX is drawn perpendicular to MN and produced until XA′ = XA, A′ will be the position of the image of A. Similarly B′, the position of the image of B, may be found. Intermediate points on AB would have images between A′ and B′, so A′B′ is the image of AB.

The figure also shows how the extreme points of the image are seen by an eye placed at E. A′ is joined to the limits of the eye opening by lines cutting the mirror at points P and Q which are then joined to A. Thus a beam of light APQ is reflected from the mirror into the eye and seems to come from A′. Simi-[illegible] a beam by which B is to be seen at B′ may be constructed.

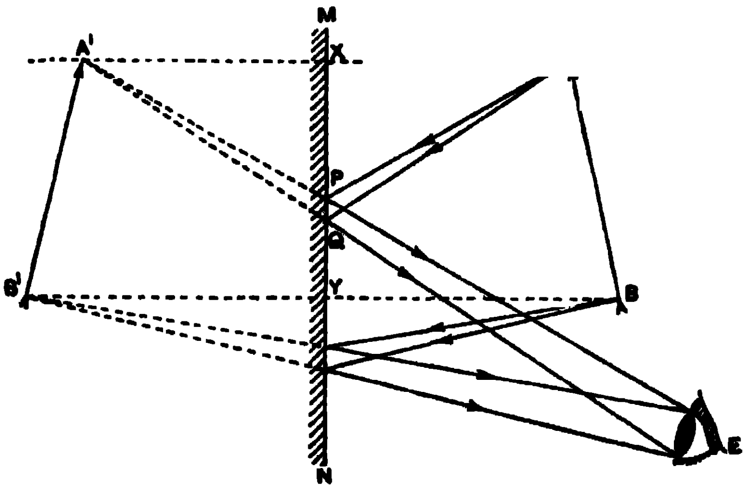


FIG. 212.

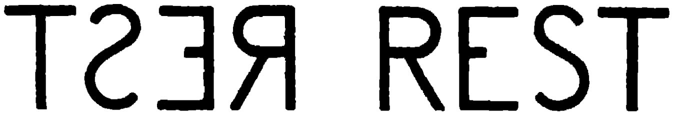


Fig. 213. Lateral Inversion.

The diagram shows that the image is the same size as the object and is right way up. This is readily verified by looking at images of objects in plane mirrors. A plane mirror image is, however, laterally inverted. This means that the left-hand side of the image is the image of the right-hand side of the object. This will be obvious to you if you stand in front of a mirror and raise your right hand to your right ear. It is also seen if you hold some print or writing in front of a mirror. The image shows the letters inverted as in Fig. 213.

Real and Virtual Images.—Place a convex lens at a distance of a yard or two from a lighted lamp, and look at the latter through the lens from a distance of several yards on the other side of it. A small inverted image of the lamp will be seen between the lens and the eye (Fig. 214). If a cardboard screen is moved outwards from the lens on the side opposite to the lamp, a position will be found where the image is clearly marked out by light falling on the screen. Thus the light which forms this particular image really passes through the points on the image. When this occurs the image is said to be real. If Fig. 212 is considered it is clear that the light does not really pass through the points forming the image in a plane mirror but only *appears* to come from those points. Images of this type are said to be virtual. It is clearly useless to try to place a screen to receive a virtual image.

Rotation of a Mirror

Draw two intersecting lines, AB and CD, on a sheet of paper, and draw a third line, OP, from their point of intersection as shown in Fig. 215. Stand a mirror along AB and fix two pins at R and Q on OP. Place two others at S and T so that they are in line with the

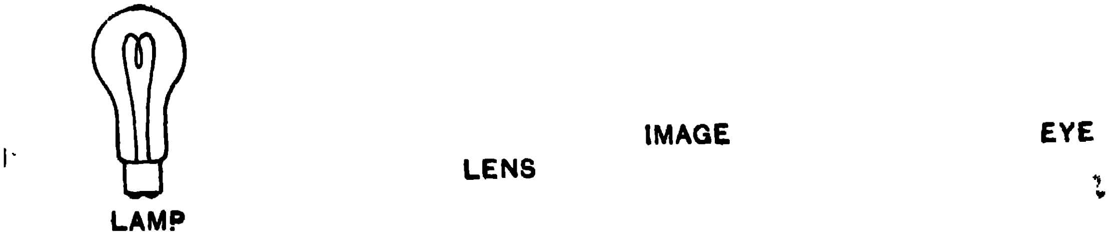


Fig. 214.

images of R and Q, then a beam incident along OQ is reflected along ST. Turn the mirror until it lies along CD. The pins at S and T must now be moved to U and V to be in line with the images of Q and R so that the reflected beam now travels along UV. Measure the angles APC and VPT. Your results should verify the statement that, *if the incident beam has a constant direction and the mirror is rotated, the reflected beam will rotate through twice the angle through which the mirror turns.*

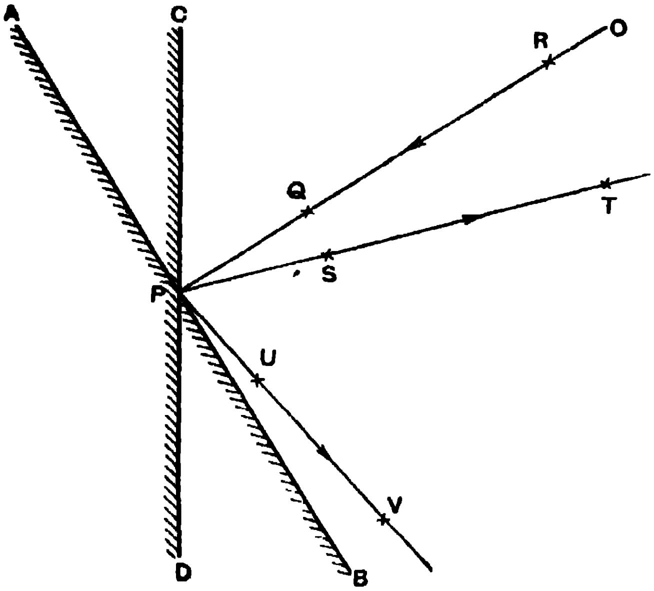


Fig. 215.

Inclined Mirrors

If two mirrors are placed facing one another along lines meeting at an acute angle and a small object is placed between them, a number of images of the object will be seen. If the angle between the mirrors is gradually widened it will be found that the number of images depends on the angle, the number diminishing as the angle becomes greater. The following special cases may readily be verified:—

Angle Between Mirrors	No. of Image Positions
30°	11
45°	7
60°	5
90°	3
120°	2

This is in accordance with the rule that if the number of degrees, x, in the angle is an exact factor of 360, the number of image positions is $\frac{360}{x} - 1$.

If narrow mirror slips are used and a tall pin is used as the object, the actual positions of the images may be found. Move another pin, which can be seen above the mirrors, behind them until it seems to be in the same place as one of the images. To find if this pin and the image are really in the same place move the head slightly from side to side. If the pin and image are nearly, but not quite together, one will seem to move with respect to the other when the head is moved. This apparent movement of one body with respect to another is called parallax. When there is no parallax between the pin and the image they really coincide.

If all the image positions are determined in this way and a circle with its centre at the point of intersection of the mirrors is drawn passing through the object, it will be found that all the images lie on that circle.

The formation of these multiple images can be explained from Fig. 216, which represents the case of two mirrors at 60°. By reflection at M_1 an image of O is formed at I_1 in such a position that OI_1 is perpendicular to M_1 and $OX = I_1X$. Thus I_1 will be on the circle already mentioned, and arc PI_1 = arc PO. Now I_1 is in front of M_2, so an image of I_1 is formed at I_2, arc QI_2 being equal to arc QI_1. Similarly I_2 is in front of M_1, and an image of it is formed at I_3. I_3 is behind both mirrors, so no image of it can be formed.

Another series of images i_1, i_2, i_3, is formed, i_1 being a direct image of O in M_2. Accurate construction shows that i_3 and I_3 coincide, so that there are only 5 image positions. If the angle between the mirrors is not an exact factor of 360° images will be formed at two separate positions in the space behind the mirrors.

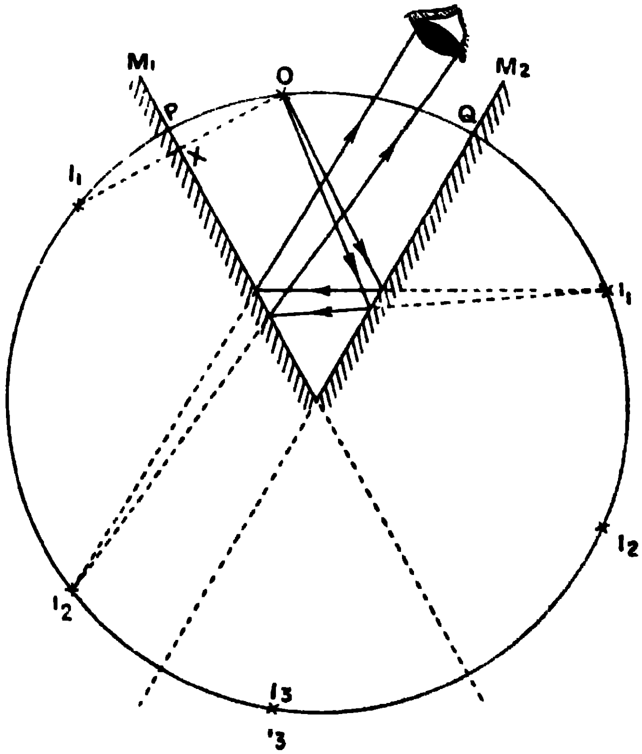


FIG. 216.

Fig. 216 also shows how to draw the path of beams by which an eye at E sees i_2, the light from O being first reflected from M_2 and then from M_1. In drawing such beams first join the image being seen to the eye and then work back to the object.

Parallel Mirrors

You have probably observed a shop window in which mirrors have been placed at each side so that they face one another and are parallel. Repeated images of the contents of the window give the impression of a window extending to a very great distance each way. By setting up a pair of parallel mirror slips with a pin between them and determining the positions of a number of images as on page 287, it will be found that all the images are on a straight line which passes through the object and is normal to both mirrors. Fig. 217 illustrates the formation of these images, I_2 being an image of I_1, and so on. It will be noted that no image can be behind both mirrors, and therefore, theoretically, there is no end to either of the series. Actually, a little light is diffused and absorbed at each reflection, so that the later images in the series become very faint and disappear.

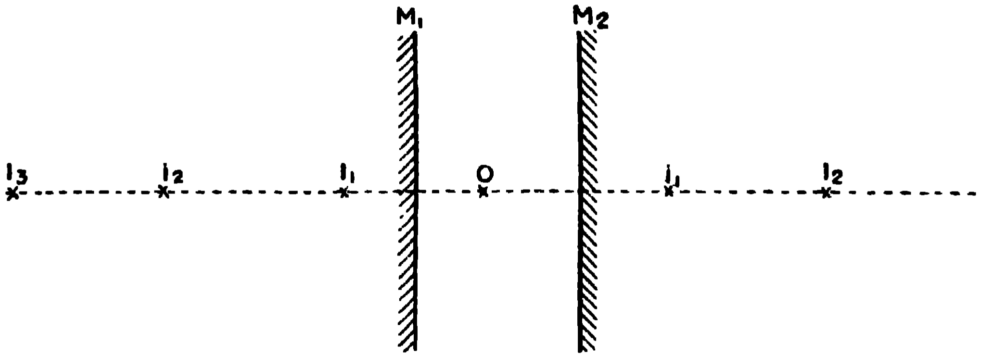


FIG. 217.

Uses of Plane Mirrors

The use of plane mirrors for seeing round corners at cross roads and in corridors has already been mentioned. Since a beam is turned through a right angle when incident on a mirror at 45°, it is best to set the mirror at an angle of 45° to the road or corridor.

The same principle is applied in the *periscope*. This contains two mirrors facing one another and each fixed at 45° to the framework. A beam of light from an object at A (Fig. 218) entering the periscope horizontally will meet the mirror M_1 at an angle of 45°, and so will be turned through a right angle and pass vertically down the tube. Thus it also meets M_2 at 45° and is again turned through a right angle and reflected horizontally to the eye at E. Thus A may be seen from E in spite of the obstacle X but will appear to be at A^1.

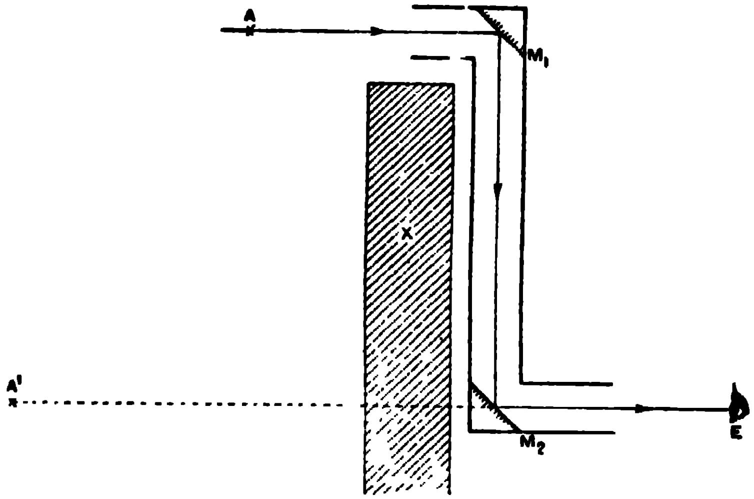


FIG. 218. PERISCOPE.

Fig. 219 shows the same principle used to give the illusion of seeing through a brick.

The fact that the image is as far behind the mirror as the object is in front of it is sometimes utilised in eye testing. The test card should be read from a distance of 24 ft. If the testing room is not as long as that the one being tested may be placed 12 ft. from a mirror on the wall and the card placed above his head. The image of the card is then 24 ft. from him.

The rotation principle is applied in the sextant, which is used by navigators to find the altitude of the sun, that is, the angle between a horizontal line and one joining the observer to the sun. The construction of a sextant is shown in Fig. 220. AB is a fixed mirror with the upper half silvered and the lower half clear. DE is another mirror facing AB but mounted on a pivoted arm, the end of which carries a vernier travelling over a circular scale of degrees. AB is observed through a small telescope fixed to the framework. The two mirrors are parallel when the rotating arm gives a zero reading on the scale. With this setting, the instrument is held in such a position that the horizon is seen both by direct vision through the lower part of AB and by reflection

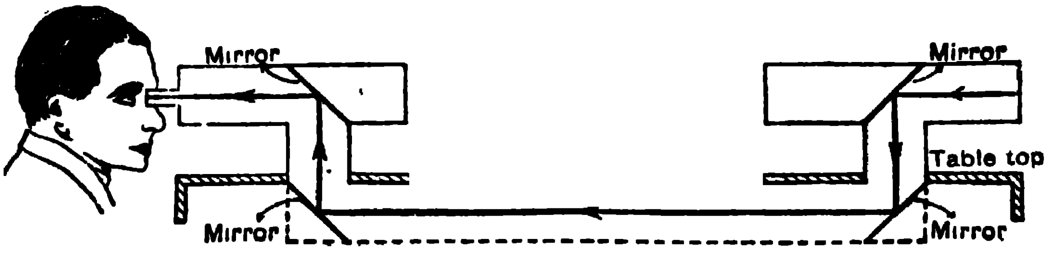
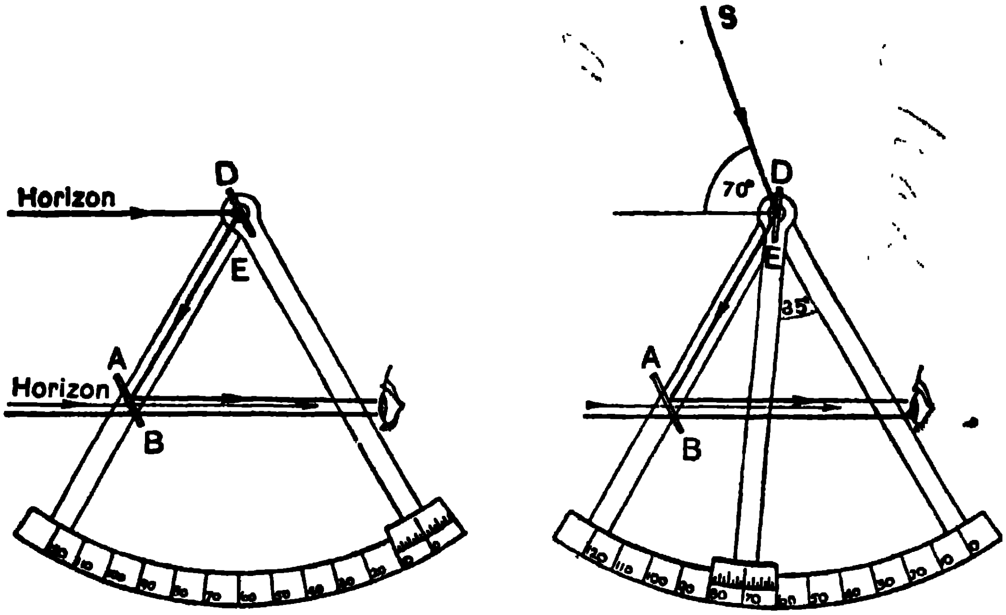


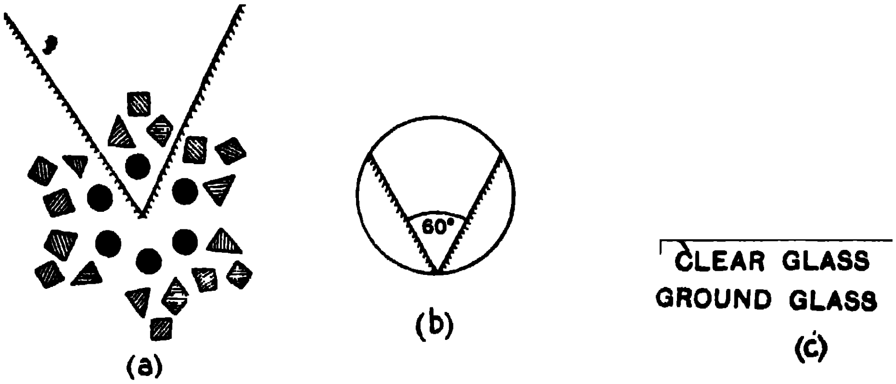
FIG. 219. "SEEING" THROUGH A BRICK.

from the two mirrors as shown in Fig. 220 (*a*). The arm is then rotated until an image of the sun is seen by reflection while the horizon is still seen by direct vision [Fig. 220 (*b*)]. The angle between the horizontal and DS is then twice the angle through which the arm has been rotated. So that direct readings may be made, the scale is graduated to give readings which are double the angle through which the arm has rotated.



(*a*) FIG. 220. SEXTANT. (*b*)

Inclined mirrors are employed in the kaleidoscope which though frequently sold as a toy can be of use to designers. To follow the principle of the instrument, stand two mirrors inclined to one another at 60° and place a few fragments of coloured substances between them. Note the symmetrical pattern that is formed by the repeated images [Fig. 221 (*a*)].



(a) (b) (c)

FIG. 221. KALEIDOSCOPE.

In the instrument as generally used, two mirrors at an angle of 60° are enclosed in a tube as shown in section in Fig. 221 (*b*). At one end of the tube is an opening to which the eye is applied. At the other end is mounted a ring which can be turned carrying a disc of ground glass on the outside and a disc of clear glass on the inside [Fig. 221 (*c*)]. Between these two discs are a number of pieces of coloured glass. On looking into the tube, the symmetrical pattern formed by the images of the pieces of glass is seen. When the ring carrying the discs is turned the pieces of glass fall into fresh positions and give fresh patterns.

QUESTIONS ON CHAPTER XXIV

1. Draw a diagram to illustrate the terms *incident beam, reflected beam, angle of incidence,* and *angle of reflection.*

State the relation between the angle of incidence and the angle of reflection.

2. State how you could show by experiment that smooth surfaces reflect light regularly but rough ones cause diffuse reflection.

Draw diagrams to show why this is the case.

3. Why are mirrors backed with opaque material? When you look obliquely into a mirror made of thick plate glass, two images of an object may be seen, there being a faint image in front of main bright image. Explain this.

4. State the laws of reflection and describe how you would verify them by experiment.

5. Explain what is meant by an image and give illustrations to show the difference between real and virtual images.

6. State the position of an image of an object formed by a plane mirror.

Draw a diagram, based on the laws of reflection, which will show that the image has the position you have stated.

How would you verify your statements by experiment?

7. Explain briefly the construction and action of (*a*) a periscope, and (*b*) a kaleidoscope.

8. How would you place mirrors (*a*) to enable you to see the back of your head, (*b*) to enable you to see along the whole length of a corridor which has a corner making an angle of 120°, (*c*) to throw as

much daylight as possible into a room which has a tall building opposite and near to its window?

Illustrate your answers by diagrams.

9. Describe an experiment you have performed or witnessed in illustration of the laws of *reflection of light.*

What facts do you know concerning the image of an object which is produced by a plane mirror? [L.U.

10. Describe and explain the action of a sextant. [L.U.

11. Two mirrors are arranged at 90° to each other with their reflecting surfaces facing inwards. Draw the path of a ray which suffers reflection at both surfaces, and find the relation between the directions of incidence and emergence of the ray.

If an object (*e.g.* an arrow) is viewed after two reflections of this kind what is the nature of the image seen and how does it appear with respect to the object?

Draw a diagram to illustrate your answer, and trace the path of a cone of rays from one point on the object to an eye placed between the mirrors. [L.U.

12. Explain, illustrating your answer with a diagram, how a narrow room can be made to appear very spacious by means of mirrors placed on opposite walls.

CHAPTER XXV

CURVED MIRRORS

You have probably noticed mirrors with curved surfaces being used as shaving mirrors, driving mirrors, and reflectors in searchlights and headlamps. The properties of such mirrors are dealt with in this chapter.

Spherical Mirrors

The mirrors which will be considered are those which form parts of the surfaces of spheres, and so are called spherical mirrors. They are of two types. Those in which the reflecting surfaces bend inwards are called **concave mirrors**, and those in which they bend outwards are **convex mirrors**. The point which is the centre of the sphere of whose surface the mirror forms a part is called its **centre of curvature.** The mid-point of the actual mirror surface is called its **pole.** The straight line passing through the centre of curvature and the pole is the **principal axis** of the mirror. A section made by a plane passing through the centre of curvature and the pole is a **principal section.** If the mirror is only a small part of the surface of the sphere it is said to be of **small aperture.** The aperture is often said to be measured by the distance between two opposite points on the edge of the mirror, but whether the mirror is of large or small aperture is really shown by the ratio of this distance to the diameter of the sphere.

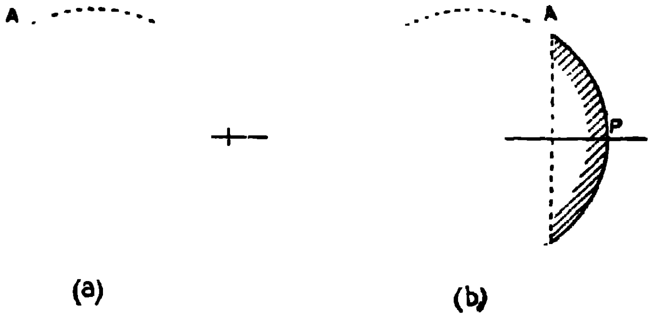


FIG. 222.

(*a*) CONCAVE, (*b*) CONVEX, C = CENTRE OF CURVATURE.

Most diagrams of spherical mirrors show principal sections, but since all principal sections of a given mirror are alike, results obtained for one will apply to all.

Cylindrical mirror strips may be used in ray box experiments to illustrate the properties of spherical mirrors. A narrow section perpendicular to the axis of such a mirror is similar to a principal section of a spherical mirror.

Law of Reflection at a Spherical Surface

Stand a concave mirror slip on a sheet of paper and mark along its reflecting surface. Find by geometrical construction the centre, C, of the arc so drawn.

From some point, A (Fig. 223), on the arc draw a line passing through C. Replace the mirror slip and arrange the ray box to give a single narrow beam meeting the mirror at A [Fig. 223 (*a*)]. If XA is its path and AY the path of the reflected beam, it will be found that $\angle$ YAC = $\angle$ XAC. Test this for several different directions of the incident beam, always making it meet the mirror at A.

Similar results may be obtained with a convex mirror [Fig. 223 (*b*)].

Thus, if the line joining the centre of curvature to a point on the surface is regarded as the normal at that point, the law that the angle of reflection equals the angle of incidence applies to spherical as well as to plane mirrors. These results give a method, which should be clear from Fig. 223, by which the path of a reflected beam can be constructed.

[*Note.*—The radius through A is the normal to the surface at that point since it is perpendicular to the tangent plane at that point, and a tangent plane has the same direction as the curved surface at its point of contact.]

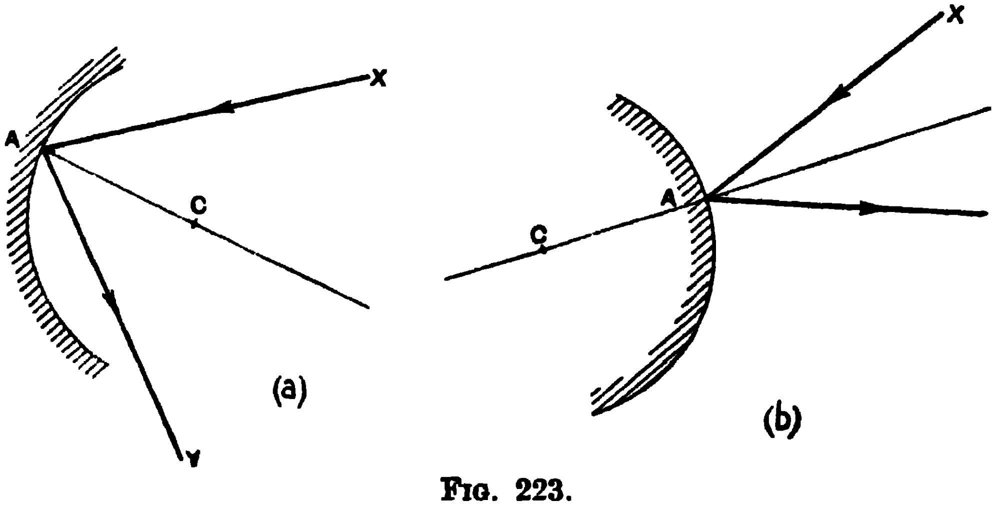


FIG. 223.

Action on Beams Parallel to the Principal Axis

Mark the position of the mirror and its centre of curvature as above. Also mark the position of the pole and draw in the principal axis. Arrange the ray box to give a number of parallel beams and direct

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them on the mirror in a direction parallel to the principal axis (Fig. 224).

In the case of the concave mirror those beams near the principal axis will be found to be reflected through one point, F, on it. Those falling on the outer part of the mirror will be reflected to cross the axis between F and the pole. If the position of F is marked it will be found to be half-way between the pole and the centre of curvature.

In the case of the convex mirror the beams will diverge after reflection. Mark the paths of the reflected beams and produce them backwards. The central ones will be found to diverge from a point half-way between the pole and the centre of curvature.

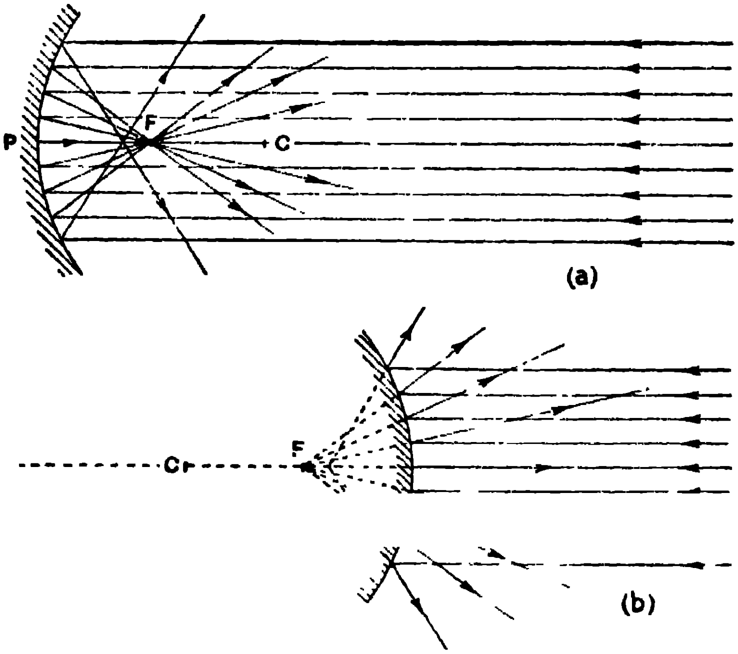


FIG. 224.

From this we can say that **beams parallel and near to the principal axis are reflected through (concave mirror) or so as to appear to come from (convex mirror) one point on the principal axis.** The point so defined is the **principal focus** of the mirror and its distance from the pole is the **focal length.** The focal length of a mirror is half its radius of curvature.

Geometrical Proof that F is Half-Way between P and C

Let PQ (Fig. 225) be a beam incident at Q on a concave mirror with centre C and pole A. Also let $PQ \parallel CA$. (Note that $\parallel$ means "parallel to.")

Join CQ. Construct $\angle CQF = \angle CQP$. Then QF is the path of the reflected beam. Let it cut AC at F.

$$\angle PQC = \angle CQF \text{ (const.) and } \angle PQC = \angle QCF \; (PQ \parallel CF);$$

$$\therefore \angle CQF = \angle QCF; \quad \therefore QF = FC.$$

If Q is near A, QF approx. = AF; $\therefore$ AF approx. = FC;
i.e. F is the mid-point of AC.

A similar proof can be given for convex mirrors.

Spherical Aberration and Caustic Curves

Because beams parallel but not near to the principal axis are not reflected through or from the principal focus, many results which are true for mirrors of small aperture are not altogether true for those of large aperture. This is said to be due to spherical aberration.

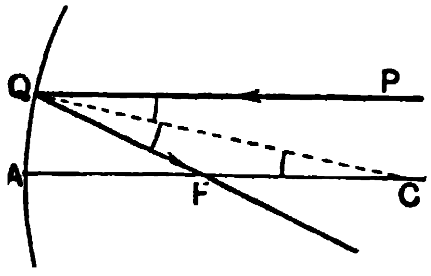


FIG. 225.

In Fig. 224 (*a*), all the reflected beams cross the axis between P and F. If, therefore, a broad beam of parallel light falls on a concave mirror, an area enclosed by a double curve, as shown in Fig. 226, is very brightly illuminated. This bright area may often be observed on the surface of a cup of tea, owing to reflection from the inner surface of the cup. The double curve bounding it is called a caustic curve.

Further Ray Box Experiments

Arrange the ray box to produce a set of beams crossing one another at the point marked as the centre of curvature in the previous experiments. Replace the mirror. With both types of mirror the reflected beams will retrace the paths of the incident beams.

Now arrange so that the beams cross at the point formerly marked as the principal focus. With each kind of mirror the reflected beams will be parallel to the principal axis (Fig. 224 (*a*) and (*b*) with arrows reversed). This result illustrates the important principle that the path of any beam of light is reversible.

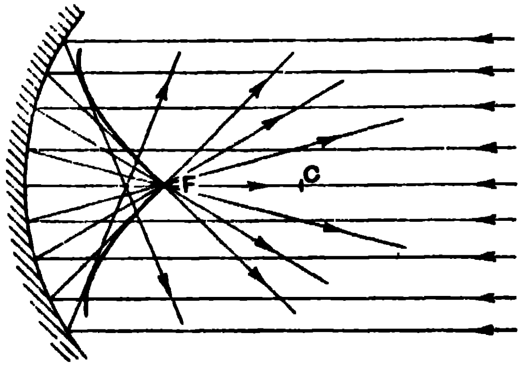


FIG. 226. CAUSTIC CURVE.

It also explains why concave mirrors are used as reflectors in headlamps, etc. If the reflecting mirror is of small aperture and the lamp is placed at its principal focus, the light reflected from it will form a parallel beam and loss of intensity due to spreading of the light will be avoided. Actually a parabolic mirror, that is one whose principal section is a parabola, is generally used since all beams which come from the principal focus are reflected parallel to the axis by such a mirror (Fig. 227).

Image Formation by Spherical Mirrors

Stand large concave and convex mirrors vertically and gradually approach them, noting the changes which take place in the image of your face as you approach. With the concave mirror at a considerable distance a small (diminished) upside down (inverted) image in front of the mirror will be seen. As you move towards the mirror the image moves forward from the mirror and becomes larger until it is almost as large as your face. Then, for some distance, no image will be seen, after which an enlarged (magnified) right way up (erect) image appears behind the mirror.

With the convex mirror the image is always erect, diminished, and behind the mirror.

(1) REAL IMAGES FORMED BY CONCAVE MIRRORS.—Some of the cases mentioned in the last paragraph may be further investigated by using a brightly illuminated object in a darkened room. A suitable object is formed by replacing the lens of a ray box by a sheet of ground glass and placing in front of it a piece of cardboard from which a triangular opening has been cut. Fine wire gauze fixed over this opening assists in finding positions where images are sharply focused.

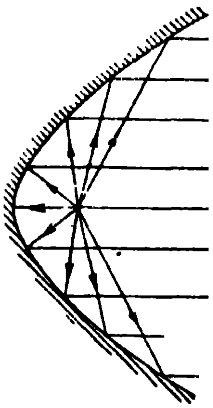


FIG. 227. PARABOLIC MIRROR.

Stand the mirror facing the object at a distance of four or five feet and turn it a little to one side so that the light it reflects is thrown on to a vertical screen (Fig. 228). Move the screen backwards and forwards until it is in a position where a clear image of the object is seen on it. Note that it is diminished and inverted and at a distance from the mirror less than that of the object. Clearly this is a real image (see page 285).

Move the object nearer to the mirror in steps of two or three inches at a time. For each object position find the corresponding image position. The image moves in the opposite direction to the object, increasing in size but remaining less than the object until a position is reached where image and object are equidistant from the mirror and equal in size. After this the image distance becomes greater than the object distance. The image remains real and inverted but is now magnified. At last the image is thrown to a very great distance. Then it reappears as an upright magnified image behind the mirror. This image is obviously virtual. These observations should be compared with those made in bringing the face up to the mirror.

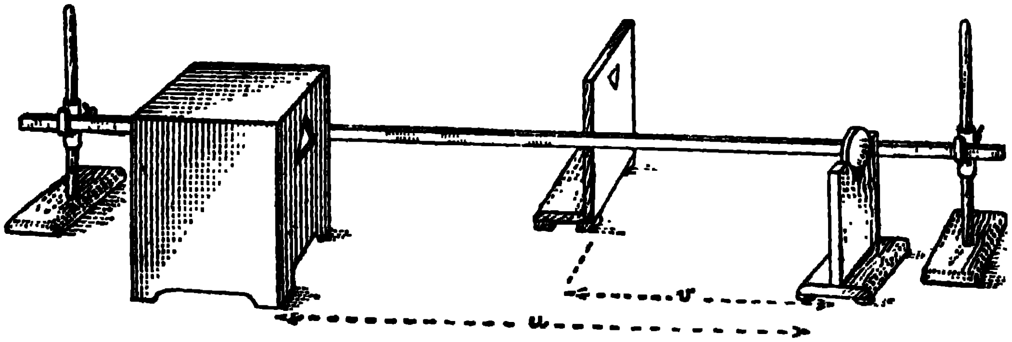


FIG. 228.

For a number of positions measurements of corresponding object and image distances from the mirror and heights of object and image should be made and used to verify the relations

$$\text{(i)}\quad \frac{1}{\text{Image distance}} + \frac{1}{\text{Object distance}} \text{ is constant.}$$

$$\text{(ii)}\quad \frac{\text{Height of image}}{\text{Height of object}} = \frac{\text{Image distance}}{\text{Object distance}}.$$

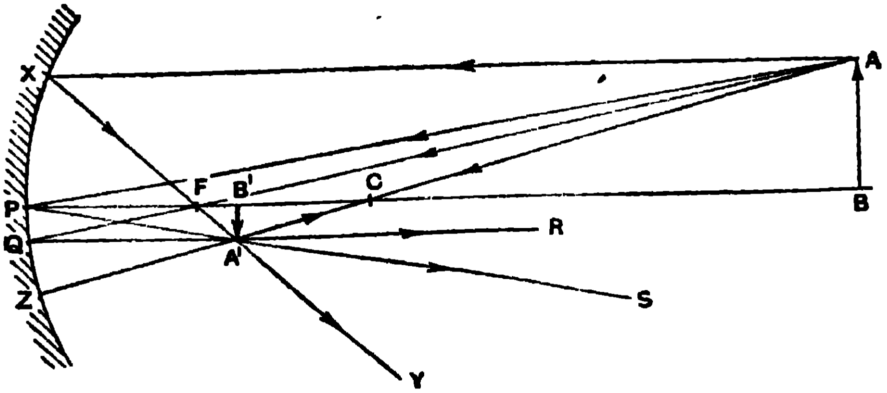


FIG. 229.

Construction of Images

For constructional purposes the following summary of the results of the experiments on pages 293-6 should be noted:—

(1) Beams parallel and near to the principal axis are reflected through the principal focus by a concave mirror and so as to appear to come from the principal focus by a convex mirror.

(2) Beams which pass through the principal focus of a concave mirror or which are directed towards the principal focus of a convex mirror are reflected parallel to the principal axis.

(3) Beams which pass through the centre of curvature of a concave mirror or which are directed towards the centre of curvature of a convex mirror are reflected back along their original paths.

Let AB be an object standing on the principal axis of a concave mirror with centre of curvature C and pole P. Mark the principal focus, F, half-way between P and C (Fig. 229).

From A draw AX, parallel to PC, meeting the mirror at X. A beam travelling along AX would be reflected through F along XY.

Draw AC produced to meet the mirror at Z. A beam travelling along AZ would be reflected back along ZA.

Draw AF produced to meet the mirror at Q. A beam travelling along AQ would be reflected parallel to PC, along QR. Join AP. Draw PS making $\angle$ SPC = $\angle$ APC. A beam travelling along AP would be reflected along PS.

All the beams diverging from A, after reflection, pass through A^1. If they enter an eye after this crossing they seem to be diverging from A^1 and therefore the image of A is at A^1. Similarly images of points between A and B would be formed between A^1 and B^1, so that A^1B^1 is the image of AB.

Note that only two of the beam paths need be traced to fix the position of A^1. In making any construction choose the two which are most convenient. If the beams diverging from a point on the object do not cross in front of the mirror after reflection, but cross behind the mirror when produced backwards, the point evidently has a virtual image at the point of intersection.

By means of such constructions verify the following cases and compare them with your experimental results.

CONCAVE MIRRORS

(1) Object at distance greater than radius. Image between F and C. Real, inverted, and diminished.

(2) Object at C. Image at C. Real, inverted, and same size as object.

(3) Object between C and F. Image beyond C. Real, inverted, and magnified.

(4) Object at F. Image at infinity. (Beams diverging from a point on the object are parallel after reflection, and so do not meet either way at any measurable distance.)

(5) Object between F and P. Image behind mirror. Virtual, erect, and magnified.

The mirror is used in position 5 as a shaving mirror, so that an upright magnified image of the face can be seen.

CONVEX MIRROR

All positions of object. Image behind mirror. Virtual, erect, diminished.

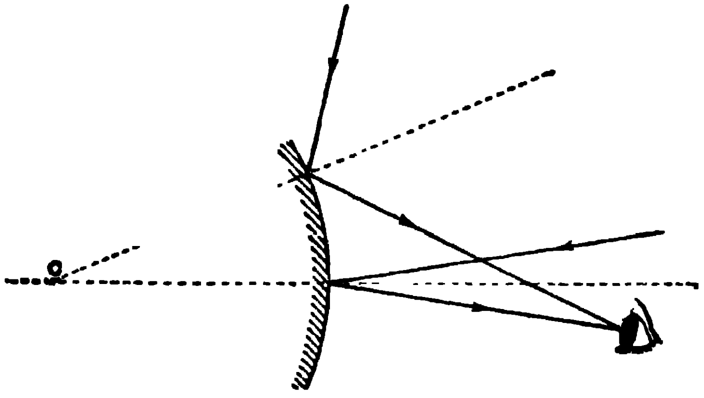


FIG. 230.

Since convex mirrors form diminished images, images of a very wide field of vision may be seen in a small mirror. This is the reason for their use as driving mirrors. Fig. 230 shows how such a mirror can reflect beams from widely separated objects to the same eye.

CONSTRUCTION OF BEAMS BY WHICH IMAGES ARE SEEN.—Fig. 231 (*a*) and (*b*) illustrate constructions for image positions as explained above, and also for the paths of beams by which images can be seen from particular positions. The latter construction is made in the way already explained for plane mirrors (page 284).

Mirror Calculations

Many mirror problems can be solved by means of accurately drawn scale diagrams, but in some cases it is convenient to calculate positions of images, etc.

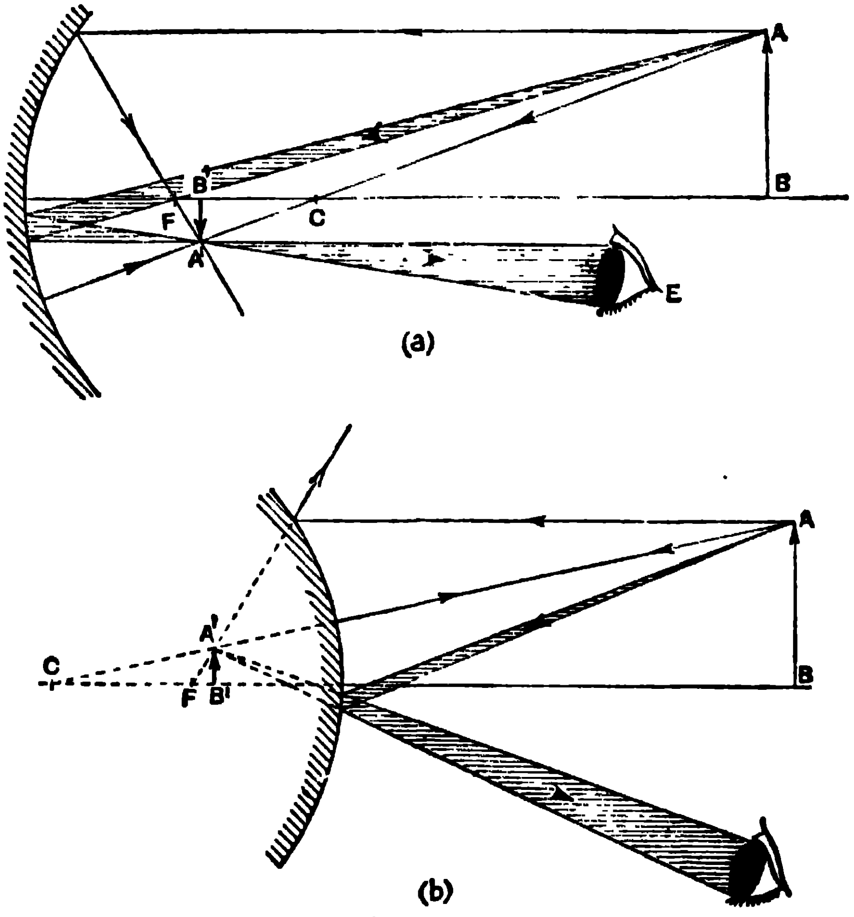
The results obtained by experiment on page 298 may be written—

$$\frac{1}{v}+\frac{1}{u}=\frac{1}{f}$$

$$\frac{\text{Height of image}}{\text{Height of object}} = \frac{v}{u},$$

where v = distance of image from mirror, u = distance of object from mirror, and f is a constant. If the focal length of the mirror is determined by method 1 on page 303, it will be found to be equal to the value of f in the above equation. A geometrical proof of this is given at the end of the chapter.

The equations can also be shown to apply to cases where virtual images are formed and to convex mirrors if the following convention of signs is followed:—



(a)

(b)

Fig. 231.

All distances are measured from the mirror. Real objects, images, and foci have positive distances. Virtual images and foci have negative distances.

It follows from the above that focal lengths of concave mirrors are positive and those of convex mirrors negative. The following worked examples should make the use of these equations clear.

Examples.—(1) *An object 2 in. high is placed at a distance of 3 ft. from a concave mirror of radius 12 in. Calculate the position, size, and nature of the image.*

Both u and f will be positive and $f = 6$ since focal length is half radius.

$$\frac{1}{v} + \frac{1}{u} = \frac{1}{f}; \quad \therefore \frac{1}{v} + \frac{1}{36} = \frac{1}{6}; \quad \therefore \frac{1}{v} = \frac{1}{6} - \frac{1}{36} = \frac{5}{36};$$

$$\therefore v = \tfrac{36}{5} = 7{\cdot}2.$$

$$\frac{\text{Height of image}}{2 \text{ in.}} = \frac{7{\cdot}2}{36}; \quad \therefore \text{Height of image} = 2 \times \frac{7{\cdot}2}{36} = {\cdot}4 \text{ in.}$$

Since v is positive, the image is *real* and is therefore *inverted* and 7·2 in. in *front* of the mirror. It is ·4 in. high.

(2) *An image is formed 5 cm. behind a convex mirror of 10 cm. focal length. Calculate the position of the object.*

As the image is behind the mirror it is virtual so that v is negative. Also f is negative for a convex mirror.

$$\frac{1}{v} + \frac{1}{u} = \frac{1}{f}; \quad \therefore \frac{1}{-5} + \frac{1}{u} = \frac{1}{-10};$$

$$\frac{1}{u} = -\frac{1}{10} \quad \frac{1}{5} = +\frac{1}{10}; \quad \therefore u = 10.$$

The object is 10 cm. in *front* of the mirror.

(3) *A mirror yields an image 8 cm. behind it of an object 40 cm. in front of it. Find the type of mirror and calculate its focal length.*

Here u is positive and v is negative.

$$\frac{1}{v} + \frac{1}{u} = \frac{1}{f}; \quad \therefore \frac{1}{-8} + \frac{1}{40} = \frac{1}{f}; \quad \therefore -\frac{4}{40} = \frac{1}{f};$$

$$\therefore f = -\frac{40}{4} = -10.$$

The focal length of the mirror is — 10 cm., and since it is negative the mirror is convex.

Measuring Mirror Constants

If either radius or focal length is found the other is known since radius is twice focal length.

(A) Concave Mirrors

(1) Beams from a point on a very distant object falling on a concave mirror will be nearly parallel, and so will be converged to a point very near the principal focus. Hence, if the mirror is stood facing the sun a screen is moved until the smallest possible image of the sun is formed on it, the distance from the pole of the mirror to the image is the focal length of the mirror.

(2) When the object is at the centre of curvature the image coincides with it (page 299). Using the illuminated object, adjust its distance from the mirror until a clear image is formed on the surface containing the object. The distance between the pole and the object is then the radius of curvature.

(3) Proceeding as on page 298, measure a series of corresponding object and image distances and in each case calculate f from $\frac{1}{v} + \frac{1}{u} = \frac{1}{f}$.

$\left(\text{Note that } \frac{1}{f} = \frac{u+v}{uv}, \text{ so } f = \frac{uv}{u+v}\right)$

Average the values of f found.

(B) Convex Mirrors

A convex mirror never gives a real image of a real object, so the above methods are not applicable to it.

The positions of virtual images formed by convex mirrors can be determined as follows.

In front of a convex mirror, AX, stand a narrow slip of plane mirror, BY, and a pin, PC. An image of the upper part of PC will be seen in AX and one of its lower part in BY. Adjust the distance between the mirrors until the two parts of the image coincide. (Use the parallax test.) Then both mirrors are giving an image of the pin at the same point D. Measure AC and BC.

FIG. 232.

$$\text{Now } u = \text{AC and } v = \text{AD}.$$

From the properties of plane mirrors, $\text{BD} = \text{BC}$; $\therefore\ \text{DC} = 2\text{BC}$.

$$v = \text{AD} = \text{DC} - \text{AC} = 2\text{BC} - \text{AC}.$$

Hence v can be found from the measurements made and f can be calculated from $\frac{1}{v} + \frac{1}{u} = \frac{1}{f}$, remembering that a negative sign should be used in substituting for v.

Geometrical Proof of Mirror Formulae

In Fig. 233 A′B′ is the image of AB.

$$\triangle\text{'s ABC and A'B'C are similar;} \quad \therefore\ \frac{\text{BC}}{\text{B'C}} = \frac{\text{AB}}{\text{A'B'}}$$

$$\text{Also } \triangle\text{'s XYF and A'B'F are similar;} \quad \therefore\ \frac{\text{YF}}{\text{FB'}} = \frac{\text{XY}}{\text{A'B'}} = \frac{\text{AB}}{\text{A'B'}},\ (\text{XY} = \text{AB});$$

$$\therefore\ \frac{\text{YF}}{\text{FB'}} = \frac{\text{BC}}{\text{B'C}}.$$

But YF is approx. = PF if X is near P;

$$\therefore\ \frac{\text{BC}}{\text{B'C}} = \frac{\text{PF}}{\text{FB'}} \quad \cdots\cdots\cdots\cdots\cdots\cdots\text{(i)}$$

Now let $\text{PC} = r$, $\text{PF} = f$, $\text{PB} = u$, and $\text{PB'} = v$. Then $r = 2f$, $\text{BC} = u - r = u - 2f$, $\text{B'C} = r - v = 2f - v$, and $\text{FB'} = v - f$. Substituting in (i).

$$\frac{u - 2f}{2f - v} = \frac{f}{v - f};$$

$$\therefore\ 2f^2 - fv = uv - uf - 2fv + 2f^2;$$

$$\therefore\ uf + fv = uv.$$

$$\text{Dividing throughout by } ufv,\ \frac{1}{v} + \frac{1}{u} = \frac{1}{f}.$$

Also △'s A′PB′ and APB are similar;

$$\therefore\ \frac{\text{A'B'}}{\text{AB}} = \frac{\text{PB'}}{\text{PB}} = \frac{v}{u},\ i.e.\ \frac{\text{Ht. of image}}{\text{Ht. of object}} = \frac{v}{u}.$$

A similar proof can be given for a convex mirror if proper signs are given to each measurement.

QUESTIONS ON CHAPTER XXV

1. Draw diagrams to illustrate the terms *centre of curvature*, *pole*, *principal axis*, and *aperture* in the case of both concave and convex spherical mirrors.

2. State the laws of reflection and describe experiments to show that these laws apply to reflection by spherical surfaces.

A point of light is on the principal axis of a concave mirror at a distance of 6 in. from the pole. The radius of curvature of the mirror is 1 in. Construct the paths of two beams diverging from the point of light and being reflected by the mirror, and hence find where its image is formed.

3. Describe experiments to show the effect of concave and convex mirrors on beams of light parallel to their principal axes.

Use the results of your experiments to define what is meant by the principal focus of aspherical mirror.

4. Explain what is meant by *real* and *virtual* images. How would you demonstrate the formation of both kinds of image by a concave mirror?

Describe how you could use a concave mirror to throw a magnified image of a small transparent picture on the wall of a room.

Construct a diagram to illustrate your answer.

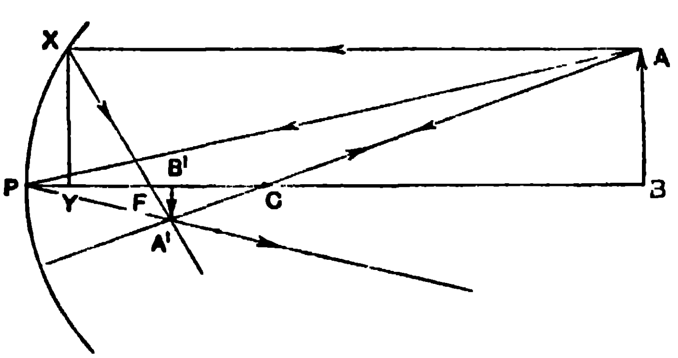


FIG. 233. SEE GEOMETRICAL PROOF OPPOSITE.

5. Describe how you would verify by experiment the relations $\frac{1}{v}+\frac{1}{u}=\frac{1}{f}$ and $\frac{\text{Ht. of image}}{\text{Ht. of object}}=\frac{v}{u}$ in the case of a concave mirror.

6. Find by calculation and by drawing the position, size, and nature of the image in each of the following cases:—

Concave Mirrors

(*a*) Object distance 10 cm., radius of curvature 6 cm., height of object 2 cm.

(*b*) Object distance 6 cm., focal length 4 cm., height of object 1 cm.

(*c*) Object distance 1 in., radius of curvature 3 in., height of object $\frac{1}{2}$ in.

Convex Mirror

(*d*) Object distance 6 in., focal length 1 in., height of object 3 in.

(*e*) Object distance 12 cm., radius of curvature 6 cm., height of object 1·5 cm.

7. Describe and explain two practical uses of concave mirrors and one of convex mirrors. In each case construct a diagram to illustrate your answer.

8. Distinguish between *real* and *virtual* images.

When an object is placed 20 cm. from a concave mirror a real image magnified three times is obtained. What is the radius of curvature of the mirror? Find also where the object must be placed to give a virtual image of the same magnification. Illustrate your answers by diagrams drawn to scale. [L.U.

9. Explain, with the aid of diagrams, how upright images of an object may be formed by plane, concave, and convex reflecting surfaces. Point out how these images may be distinguished from one another. [L.U.

10. Compare the images formed when an object 1 in. high is placed 6 in. in front of (*a*) a plane mirror, (*b*) a concave mirror of radius of curvature 8 in. Illustrate your answer with diagrams drawn to scale and explain the construction of the diagrams. [L.U.

11. A pin 4 cm. in height is fixed vertically on the principal axis of a concave spherical mirror whose radius of curvature is 36 cm.

Draw a careful diagram to scale, ¼ full size, showing how the image of the pin is produced by the mirror. The distance of the pin from the mirror is 27 cm.

Explain how you make use of the laws of reflection of light in your construction, and state what you know concerning the image. [L.U.

12. Explain the action of a convex mirror used on a motor vehicle for obtaining a rear view.

Compare this with the action of a plane mirror used for the same purpose. [L.U.

13. A spherical mirror is placed 25 cm. from an illuminated object and the real image produced is found to be ¼ the size of the object. What kind of mirror is it, and what is the focal length? Where must an object be placed so that the image is real and three times as big as the object? [L.U.

14. Describe any *one* method of determining the radius of curvature of a concave mirror.

Show diagrammatically that the image formed by a convex mirror is always virtual and diminished.

Name and explain *one* practical use of a convex mirror, giving the reason for its employment. [J.M.B.

15. State the laws of reflection of light.

Draw diagrams showing how an image is formed by reflection (*a*) in a plane mirror, (*b*) in a convex mirror of 20 cm. radius of curvature, the object in each case being 15 cm. from the mirror. In each case carefully describe the image and state the magnification. [J.M.B.

CHAPTER XXVI

REFRACTION

Place a thick block of glass on a page of your book. Look downwards on it and note that the print under the glass appears to be raised nearer to the eye. Look obliquely through the block at vertical window bars. The part of a bar seen through the block will appear to be displaced slightly to one side. Note also the apparent displacements of objects when viewed through a three-cornered block (triangular prism) of glass. Hold a stick in a slanting position dipping into water and note that it appears to be bent at the surface of the water.

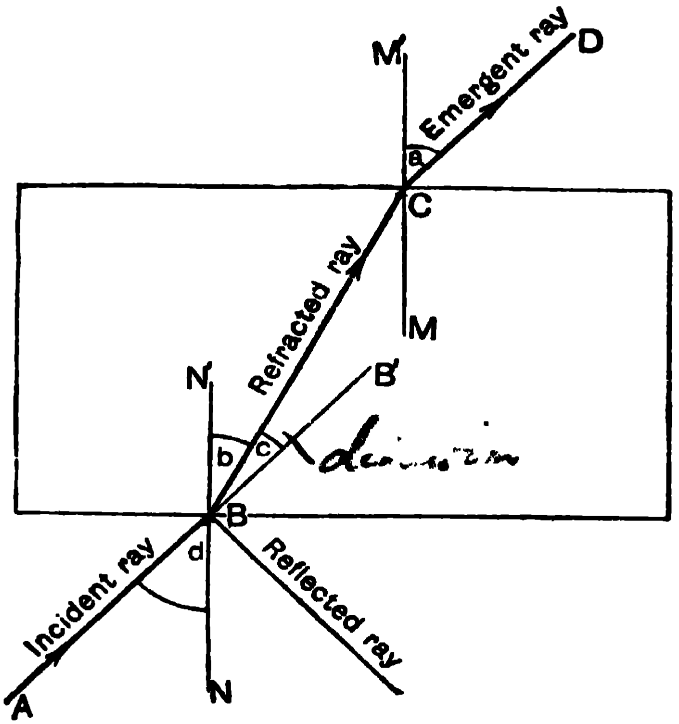


FIG. 234.

∠ a = ∠ of emergence. ∠ c = ∠ of deviation.
∠ b = ∠ of refraction. ∠ d = ∠ of incidence.

All these observations suggest that there is a change in the direction in which light travels when it passes from one transparent substance to another. This change of direction is known as refraction.

Ray Box Experiments on Refraction

When the sheet of clear glass was used in the experiment on page 280, no change of direction in the light passing through it was observed owing to the thinness of the sheet. If, however, a wide block of glass is used instead of the thin sheet, the path of the beam will be found to be as indicated in Fig. 234. Note that part of the incident beam is reflected according to the laws of reflection at the first face. Also note that *the beam turns towards the normal at the point of incidence on entering the block and away from the normal at the point of incidence on leaving it.* Further, the final direction of the beam in the air is parallel to its original direction but there has been a lateral displacement,—that is, a displacement to one side. Verify this for various angles of incidence on the first face. Note also that there is no refraction if the beam meets the first face normally.

Fig. 234 also illustrates various terms which are used in connexion with refraction, the meanings of which should be carefully noted.

Similar results may be obtained by using a thin walled rectangular glass trough filled with water instead of the glass block.

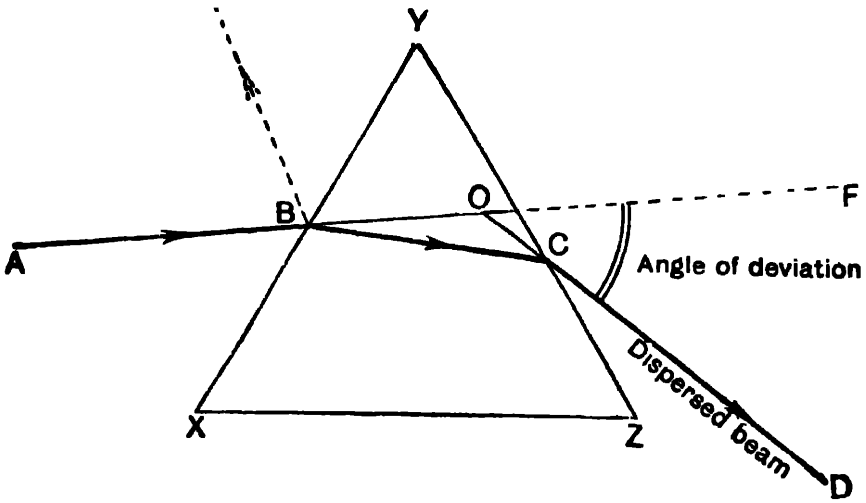


FIG. 235.

If a triangular prism of glass is used instead of the block, results as indicated in Fig. 235 will be obtained. Once more, the turning is towards the normal at the point of incidence on entering the glass and away from the normal on leaving it, but, owing to the inclination of the two faces, these turnings are not in this case equal and in opposite directions, so that the emergent beam is not brought back to a direction parallel to the incident beam. In this case the beam is said to be **deviated**, and the angle FOD between the original and final directions measures the amount of deviation.

It will also be noted that the emergent beam is no longer a single white beam but is made up of a number of overlapping coloured beams. The light is said to have been dispersed. This colour effect will be dealt with in Chapter XXIX.

The results of these experiments may be summarised as follows:—

(1) Beams of light passing from a given substance into a denser substance are usually refracted towards the normal at the point of incidence.

(2) On passing from a dense substance to a less dense one they are usually refracted away from the normal.

(3) If the two surfaces through which a beam passes are parallel, the emergent beam is parallel to the incident beam but laterally displaced.

(4) If the two faces are not parallel the beam is deviated.

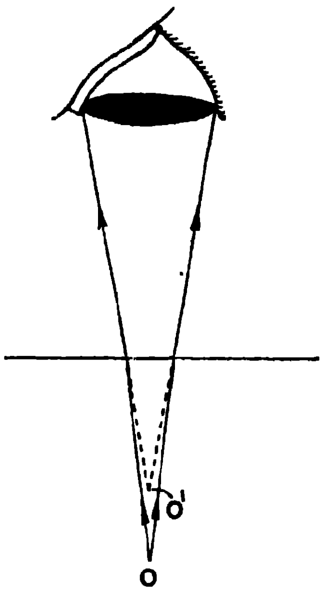


Fig. 236.

Simple Applications of Refraction

Fig. 236 shows why print appears nearer to the eye when a glass block is placed over it. Beams of light travelling upwards from the point O at the bottom of the glass bend away from the normals at the upper surface and so enter the eye as though they came from O′ so that an image of O is formed at O′. For a similar reason, water viewed from above appears less deep than it really is, as you may have observed at the swimming baths.

Fig. 237 similarly explains the apparent bending of a stick when dipped obliquely in water. The refraction of the beams from the lower end of the stick when they enter the air results in an image of the part B being formed at A.

The apparent break in the window bar when viewed obliquely through the glass block is explained in Fig. 238. The parts of the bar seen above or below the block are seen by means of beams of light which travel directly to the eye and so appear in their true positions. The part seen through the block is seen by beams which follow paths such as OABE and so it appears to be displaced to O′.

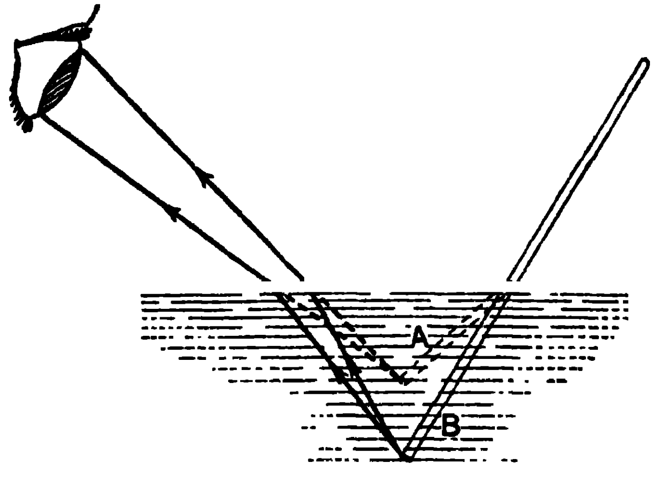


Fig. 237.

A = Apparent position of stick
B = Actual position of stick.

The apparent displacement of objects viewed through triangular prisms is illustrated in Fig. 239. Note that the deviation of the light is away from the edge, A, between the two faces through which the light passes. The apparent displacement of the object is towards that edge. This edge is known as the refracting edge of the prism.

Laws of Refraction

Pass a beam of light from a ray box through a block of glass on a sheet of paper. Mark round the block and make marks on corresponding edges of the incident and emergent beams so that by joining those marks the paths of the beams may be drawn. Draw in the normal at the point of incidence on the first face and measure the angle of incidence and angle of refraction. Repeat a number of times, varying the angle of incidence. Look up the sines of the angles measured from mathematical tables and draw up a table as follows:—

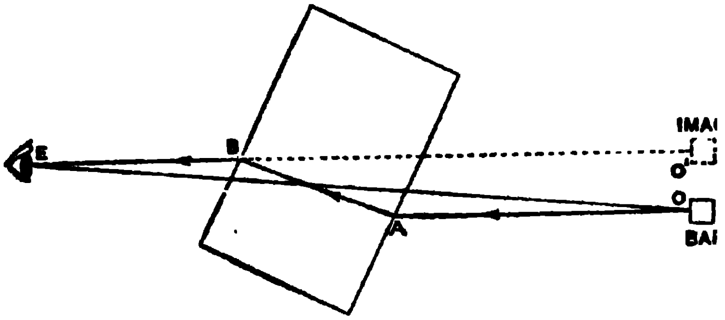


Fig. 238.

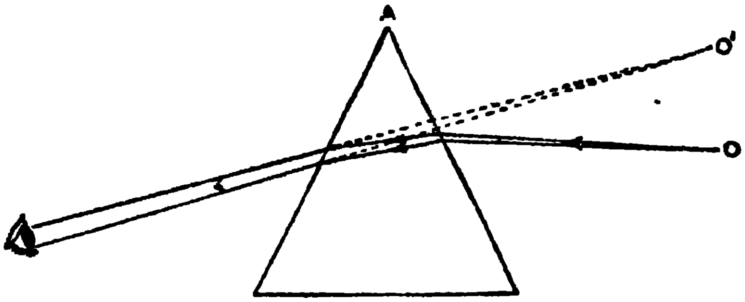


Fig. 239.

∠ of Incidence (i)	∠ of Refraction (r)	Sin i	Sin r	$\frac{\text{Sin } i}{\text{Sin } r}$

An approximately constant value for $\frac{\sin i}{\sin r}$ will be obtained. Thus we may state the following law. **The ratio of the sine of the angle of incidence to the sine of the angle of refraction is constant for all beams passing from one given substance to another.**

The ratio mentioned in the law is called the **index of refraction** from the first substance to the second. When the first substance is air, the ratio is simply called the index of refraction of the second substance. For example, for light passing from air to glass, the constant ratio = $\frac{3}{2}$, so the index of refraction of glass is $\frac{3}{2}$.

The second law of refraction is similar to that of reflection, namely, **the incident beam, the normal at the point of incidence, and the refracted beam are all in the same place.** It may be verified as in the case of reflection.

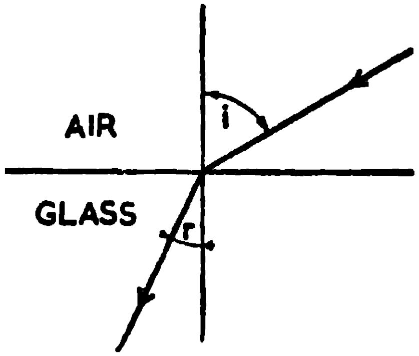


FIG. 240.

$\frac{\sin i}{\sin r}$ = index of refraction.

SOME INDICES OF REFRACTION

Water	1·33	Ice	1·31 ⑦	Crown glass	1·50 ④	Flint glass	1·62
Glycerine	1·47 ⑤	Plate glass	1·52 ③	Turpentine	1·47	Diamond	2·60 ①

The approximate values $\frac{4}{3}$ for water and $\frac{3}{2}$ for glass should be remembered.

Geometrical Construction of Refracted Beams

Suppose it is required to find the path in glass of a beam which meets the glass surface at an angle of incidence of 70°, given that the index of refraction of glass is $\frac{3}{2}$. Let XY (Fig. 241) be the surface of the glass. Draw AB at the required angle to represent the incident beam. Below AB mark off from B along XY three equal lengths giving the point E. At E draw a perpendicular to XY cutting the incident beam at A. With centre B and radius BA describe a circle. On the opposite side of B to E mark off along BX a distance equal to two of the three divisions in BE, giving the point F. At F draw a perpendicular to XY cutting the circle at G. Then BG produced is the required path.

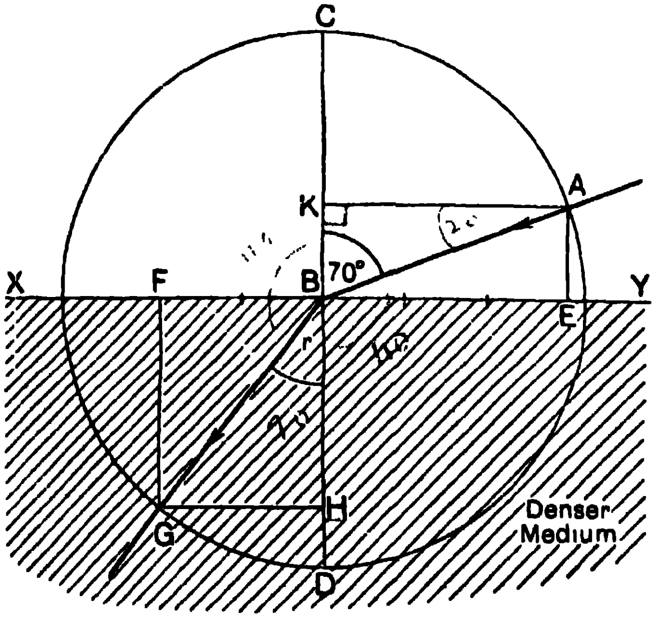


FIG. 241.

To prove that the correct path has been obtained, draw perpendiculars AK and GH respectively from A and G to CD.

$$\text{The path is correct if } \frac{\sin 70^\circ}{\sin r} = \frac{3}{2}.$$

$$\text{Now } \sin 70^\circ = \frac{AK}{AB} \quad \text{and} \quad \sin r = \frac{GH}{BG};$$

$$\therefore \frac{\sin 70^\circ}{\sin r} = \frac{AK}{AB} \Big/ \frac{GH}{BG}. \quad \text{But } AB = BG \text{ (radii of circle);}$$

$$\therefore \frac{\sin 70^\circ}{\sin r} = \frac{AK}{GH} = \frac{BE}{BF} = \frac{3}{2} \text{ (construction);} \quad \therefore \text{ the path is correct.}$$

From the principle of reversibility of beams, it follows that a beam travelling along GB would be refracted along BA when it emerged into the air. Hence the construction can be used for beams from glass to air by taking $\frac{3}{2}$ as the required index of refraction. Thus the index of refraction from a substance to air is the reciprocal of the index of refraction of the substance.

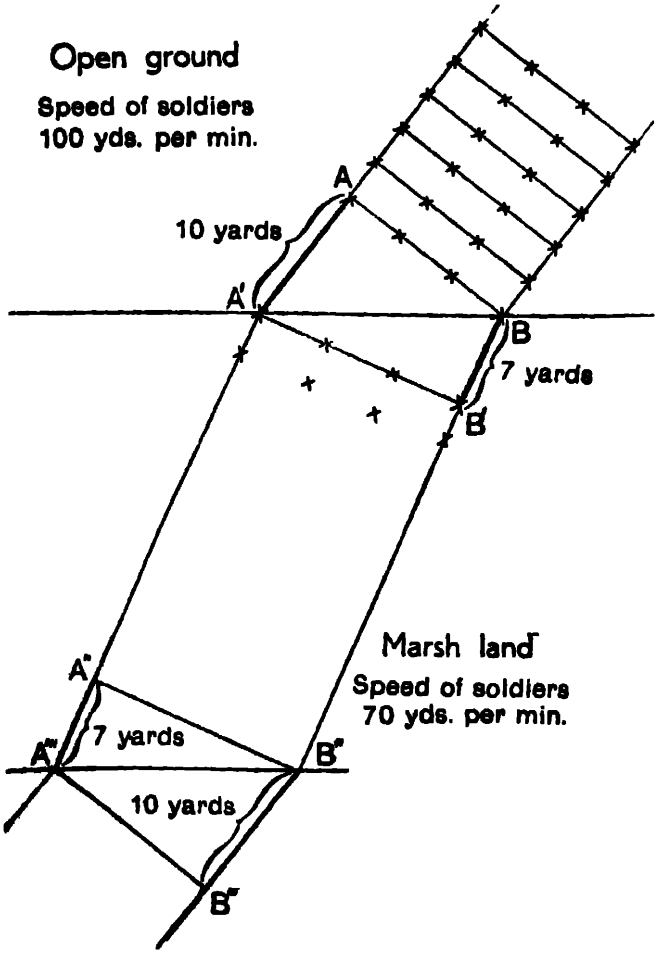


FIG. 242.

Cause of Refraction

Fig. 242 shows what would happen if a column of soldiers on the march approached obliquely the line separating firm ground from marshy ground on which they would have to march more slowly.

At the moment when the left-hand man of the front rank has reached B, the right-hand man has reached A and has still 10 yards to go to reach the marshy ground. While he is marching this 10 yards the man at B only marches 7 yards, so the front rank comes into the position A′B′. As the men still march at right angles to the line of the front rank, they cross the marshy ground in the direction A′A‴. On reaching the firm ground on the other side the process would be reversed and the line of march would become parallel to the original direction. Note that the column is "refracted" towards the normal on entering the marshy ground and away from the normal on reaching firm ground again.

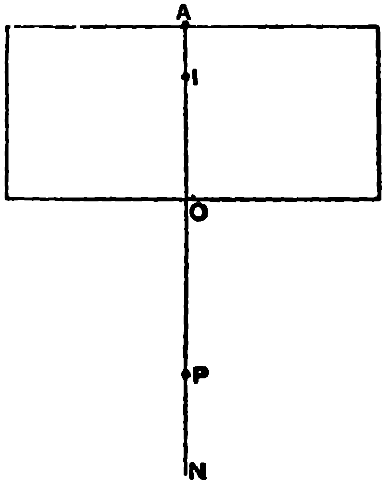


FIG. 243.

It has been shown by experiment that light has a smaller speed in dense substances than in air, and by analogy with the above example the refraction of light when passing into and out of a dense substance might be expected to result from this change of velocity. Moreover, it can be shown geometrically from Fig. 242 that the "index of refraction" for the marching column is equal to

$$\frac{\text{Velocity on firm ground}}{\text{Velocity on marshy ground}}$$

and it is actually found for light that:—

$$\text{Index of refraction of a substance} = \frac{\text{Velocity of light in air}}{\text{Velocity of light in substance}}.$$

Apparent Thickness or Depth

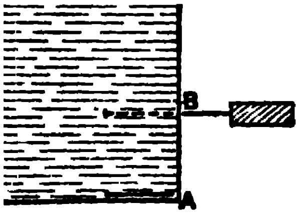


FIG. 244.

Place a block of glass on a sheet of paper and trace round its outline. Draw a line, NOA (Fig. 243), perpendicular to two opposite sides of the trace. Replace the block and stand up a pin as near as possible to the block at A. On looking at this pin through the block it will appear to be at a point I, so that IO is the apparent thickness of the block.

To find the position of I, stand another pin, P, on the line ON. An image of this pin formed by reflection at the front face of the block will be formed somewhere along OA. Move P backwards and forwards until the image of A formed by refraction and the image of P formed by reflection coincide (Parallax test). Then the rule about the position of an image formed by reflection at a plane surface will tell us that I is as far behind O as P is in front of it. Hence OP is equal to the apparent thickness of the block. Measure OP and also the real thickness AO. Find the index of refraction of the glass by the method on page 310. Verify that:—

$$\frac{\textbf{Real thickness}}{\textbf{Apparent thickness}} = \textbf{Index of refraction.}$$

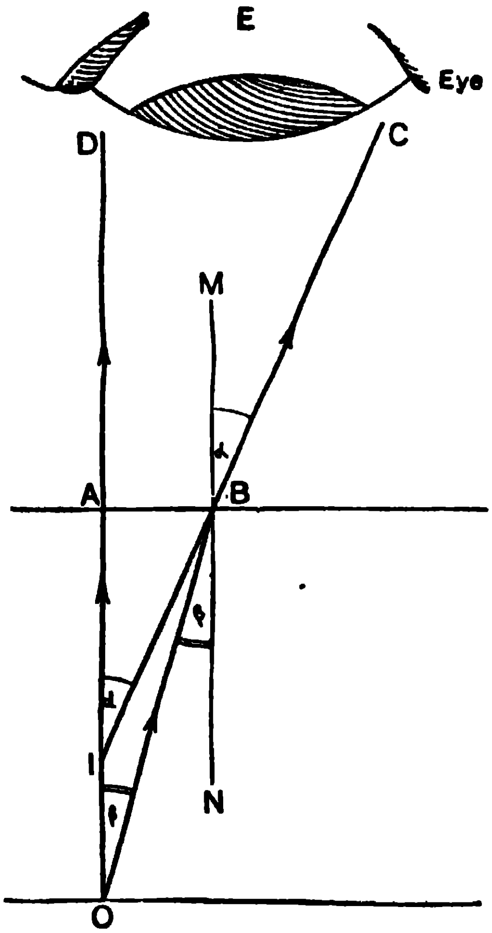


Fig. 245.

The relation between the real and apparent depth of water may be found in a similar way. Use a deep glass vessel to hold the water (Fig. 244). Place a pin on the bottom of the vessel with its point touching the side. Support another pin in a burette stand at the side of the vessel. Look down on the edge of the vessel and adjust the height of the latter pin until it appears to be on the same level as the one in the water. Then OA is the real depth of the water and OB is its apparent depth. The relation—

$$\frac{\text{Real depth}}{\text{Apparent depth}} = \text{Index of refraction}$$

will be found to apply in this case also.

Assuming that relation these methods of measuring apparent thickness or depth may be used for finding indices of refraction of transparent solids or liquids.

Examples.—(1) *The apparent depth of a quantity of liquid was 13·5 cm. when its real depth was 20 cm. What is its index of refraction?*

$$\text{Index of refraction} = \frac{\text{Real depth}}{\text{Apparent depth}} = \frac{20}{13{\cdot}5};$$

$$\therefore \text{ Index of refraction} = 1{\cdot}48.$$

(2) *What is the real depth of a swimming bath where the water appears to be 5 ft. deep? Index of refraction of water =* $\frac{4}{3}$.

$$\frac{\text{Real depth}}{5\text{ ft.}} = \frac{4}{3};$$

$$\therefore \text{ Real depth} = \tfrac{4}{3} \times 5 \text{ ft.} = 6{\cdot}67 \text{ ft.} = 6 \text{ ft. } 8 \text{ in. (approx.).}$$

Proof of Apparent Depth Formula

Let O, Fig. 245, be a point on the bottom of a bowl containing water. From O, a beam of light, OA, meets the water surface normally and passes into the air without refraction. Another beam, OB, from O meets the surface obliquely at B and is refracted away from the normal MN on entering the air. To an eye receiving these two beams O would appear to be at I. Now:—

$$\frac{\sin \angle \text{CBM}}{\sin \angle \text{OBN}} = \text{the index of refraction of water.}$$

Let this be μ.

$$\angle \text{CBM} = \angle \text{NBI} = \angle \text{BIA} \quad \text{and} \quad \angle \text{OBN} = \angle \text{BOA};$$

$$\therefore \frac{\sin \angle \text{BIA}}{\sin \angle \text{BOA}} = \mu.$$

$$\sin \angle \text{BIA} = \frac{\text{AB}}{\text{IB}} \quad \text{and} \quad \sin \angle \text{BOA} = \frac{\text{AB}}{\text{OB}};$$

$$\therefore \mu = \frac{\text{AB}}{\text{IB}} \div \frac{\text{AB}}{\text{OB}} = \frac{\text{OB}}{\text{IB}}.$$

If B is very near A, as it must be for both beams to enter the eye, OB is approx. = OA and IB approx. = IA;

$$\therefore \frac{\text{OA}}{\text{IA}} = \mu, \quad \textit{or} \quad \frac{\text{Real thickness}}{\text{Apparent thickness}} = \text{Index of refraction.}$$

Refraction through a Prism

This was mentioned on page 308, and the meanings of deviation and angle of deviation were noted.

Arrange a prism so that a beam from a ray box meets one face nearly along a normal [Fig. 246 (*a*)]. Now rotate the prism so that the first angle of incidence gradually increases. Note that the deviation becomes smaller, that is, ∠ EOD becomes smaller as ∠ ABN increases [Fig. 246 (*b*)]. A position is reached, however, when further increase in ∠ ABN causes ∠ EOD to increase instead of continuing to decrease. Thus there is one particular angle of incidence for which the deviation produced by the prism is a minimum.

If the prism is placed so that it is just in this position of minimum deviation, it will be found that the beam makes equal angles with the two faces as shown in Fig. 247. It will then be found that turning the prism either way results in an increased deviation of the beam.

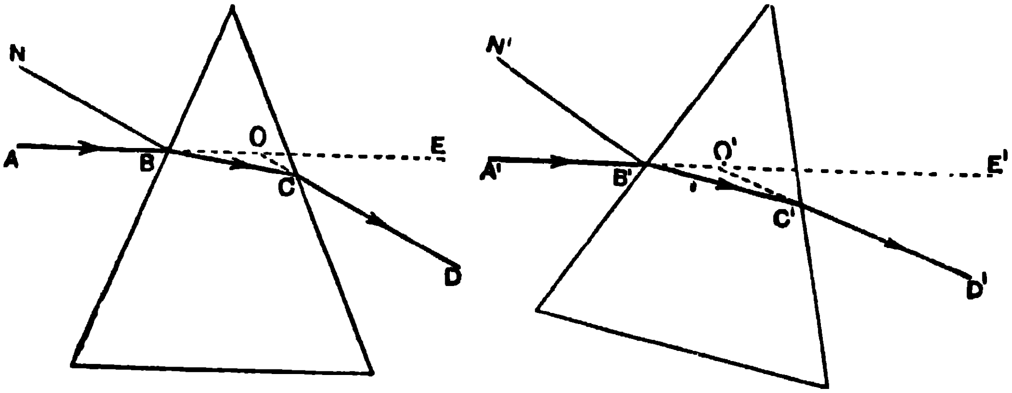


FIG. 246.

Experiments with different prisms of the same substance will show that the greater the angle of the prism, the greater is the minimum deviation it will produce. If a thin walled triangular bottle filled with carbon disulphide, which has an index of refraction of 1·63, is used, it will be found to produce a greater minimum deviation than a prism of crown glass of equal angle, thus showing the effect of index of refraction.

Internal Reflection. Total Reflection

Reference has already been made to the faint image in front of the main image which is often seen in a thick glass mirror and which is due to some of the light being reflected from the front surface of the glass. Frequently other faint images, further back than the main image, may

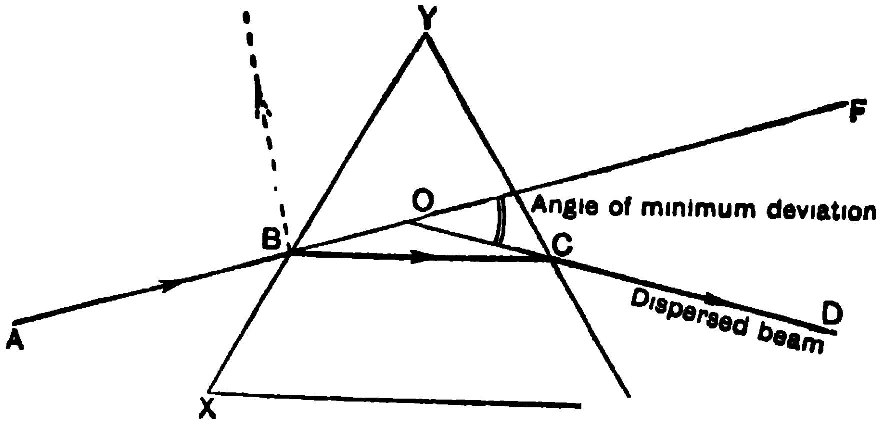


FIG. 247.

be seen as shown in Fig. 248. This is due to part of the beam which has been reflected from the back being reflected into the glass again when it reaches the front surface. Fig. 249 illustrates this. The dark line shows the path followed by most of the light in the beam IO, giving rise to the main image at I_2. A little of the beam is reflected at O instead of entering the glass, and gives rise to the image I_1 in front of I_2. Some of the light is reflected at P instead of leaving the glass, and after reflection at the back surface once more, reaches Q. There part of it leaves the glass giving rise to a faint image at I_3, but part is reflected, and so on.

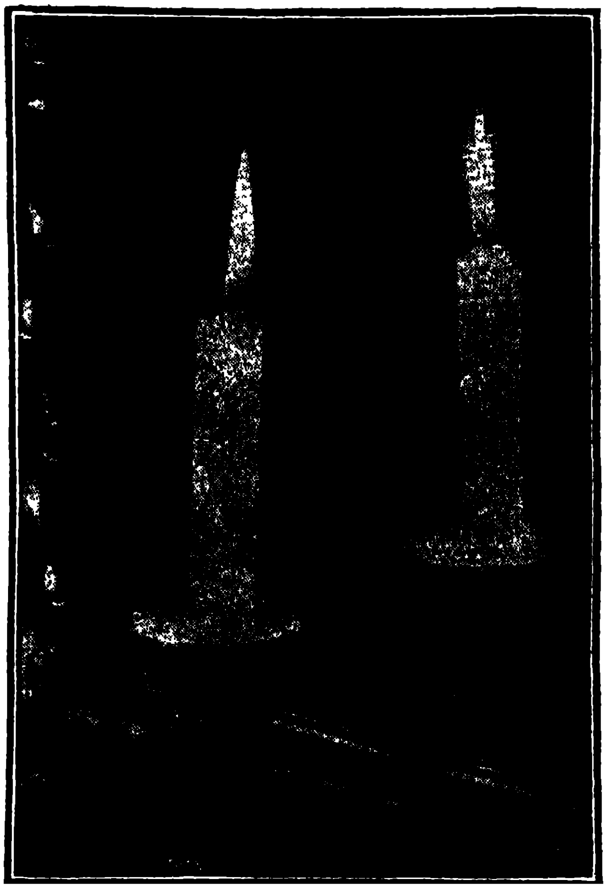


FIG. 248. CANDLE IMPERFECTLY REFLECTED IN A MIRROR. (NOTE MULTIPLE IMAGES.)

This internal reflection may readily be observed if a semicircular slab of glass mounted on a circular scale, as shown in Fig. 250, is used. Arrange the ray box to throw a beam on to the curved surface, its direction being along one of the radial lines from the mid-point of the straight surface. This arrangement enables the beam to enter the block without being refracted.

When the angle of incidence on the straight face is about 35°, while most of the light will leave the slab and be refracted, a clear reflected beam will be seen. When the angle of incidence is just over 40°, the refracted beam is nearly parallel to the flat surface. If the angle of incidence is then increased by a few degrees, it will be found that none of the light escapes at O, but that the whole of the beam is reflected (COC′). In this case the beam is said to be **totally reflected.** This can happen only to light passing through a dense medium and meeting the surface of a less dense one in which case, owing to the refraction being away from the normal, the angle of refraction is always larger than the angle of incidence. Thus there will be a certain angle of incidence which will make the angle of refraction = 90° [Fig. 251 (*b*)]. The particular angle of incidence which causes this is called the **critical angle between the two media.** It is not possible for an angle of refraction to be greater than 90°. Hence, if the angle of incidence is greater than the critical angle, none of the beam can leave the dense medium and be refracted, so it is totally reflected.

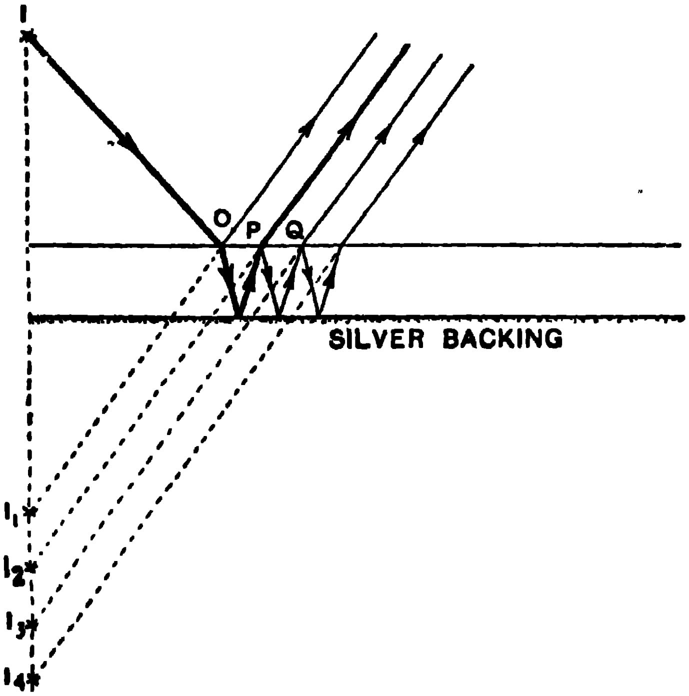


FIG. 249

Any beam passing through the glass and meeting the air surface with an angle of incidence less than the critical angle will be partly refracted into the air and partly reflected. Any beam whose angle of incidence on the air surface is greater than the critical angle will be totally reflected. If, in the experiment with the semicircular glass block, the angle of incidence is gradually increased until total reflection just takes place, the critical angle for glass may be read from the circular scale. It will be found to be just above 42°. For water the critical angle is about 48·5°.

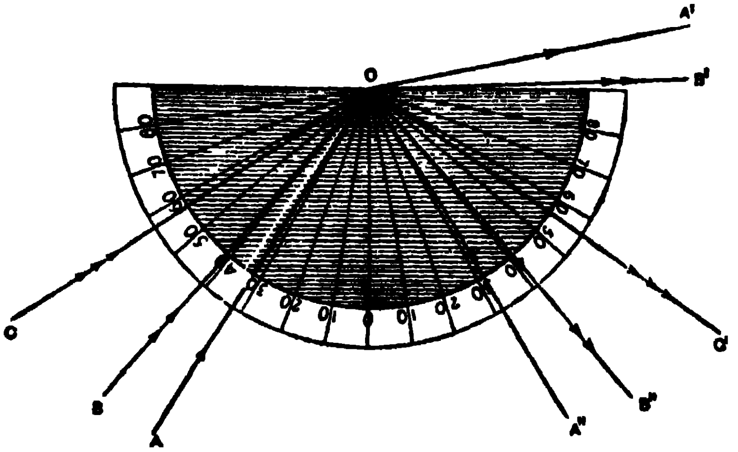
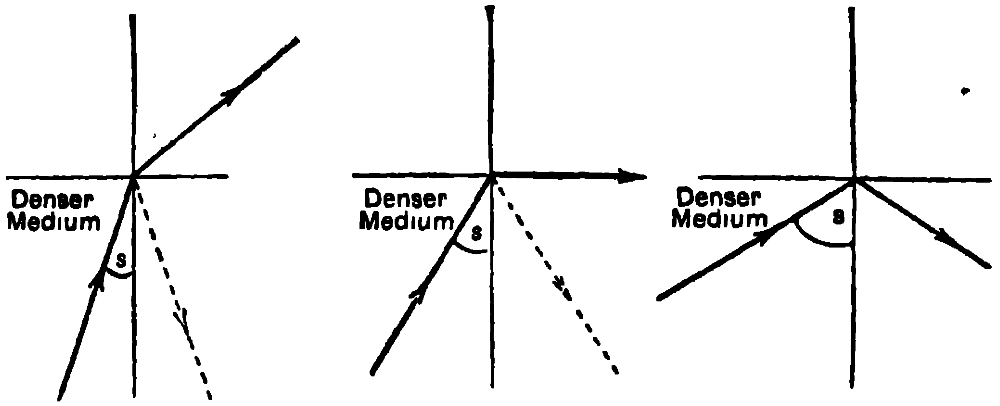


FIG. 250.

Reflection by Prisms

Try the effect of passing a beam from the ray box into an isosceles right-angled glass prism, in which each of the acute angles will be 45°.

If the beam enters one of the short faces at right angles it will be totally reflected from the hypotenuse and will leave the prism at right angles to its original path, as shown in Fig. 252 (*a*). There is no refraction at the first face as the incident beam is normal to it. Thus the beam meets the hypotenuse with an angle of incidence of 45° which is greater than the critical angle of glass. Hence it is totally reflected, and the angle of reflection will also be 45°, so that the beam is turned through a right angle and meets the third face normally, passing out of it without further change of direction.



(*a*) ∠ s < Critical angle. (*b*) ∠ s = Critical angle. (*c*) ∠ s > Critical angle.

FIG. 251. REFRACTION OF LIGHT FROM A DENSE TO A LESS DENSE MEDIUM.

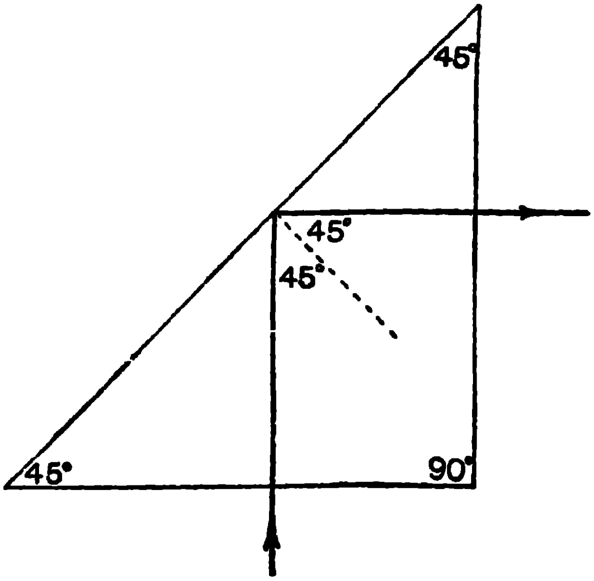


FIG. 252 (*a*).

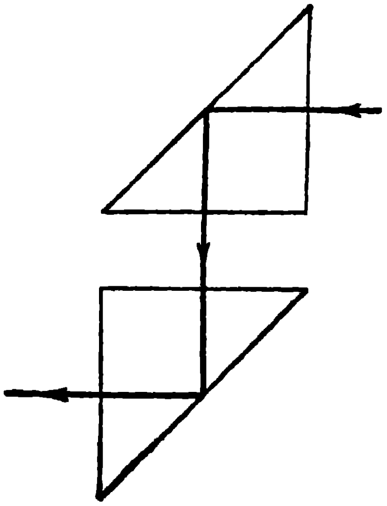


FIG. 252 (*b*).

By placing two such prisms in the relative positions shown in Fig. 252 (*b*) a periscope may be constructed. Such prisms are better than mirrors for instruments of that kind since all the light is reflected in one beam without the formation of multiple images, as on page 317. Also there is no metal backing to become tarnished or damaged.

Such a prism, placed like the lower one in Fig. 252 (*b*), is often used in the view-finder of a camera. The light from the view enters the vertical face and is reflected through the horizontal face to the eye.

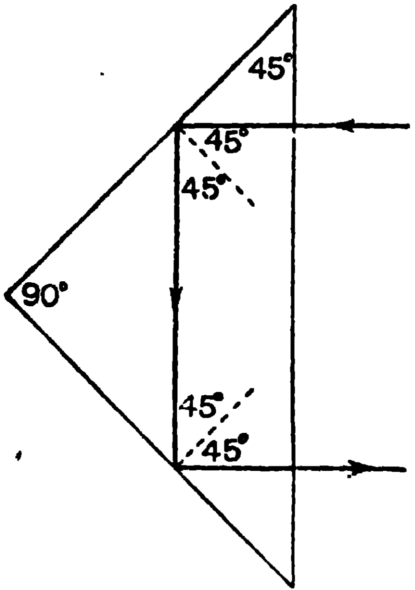


FIG. 253.

Fig. 253 shows how such a prism can be made to turn a beam through 180° and make it travel in the opposite direction to the incident beam. This should be tested by means of the ray box.

Prisms are used in this way in binoculars. To obtain a powerful telescope a considerable distance between the objective lens and the eyepiece is necessary. By using two isosceles prisms, as shown in Fig. 254, the light may be made to traverse the barrel of the telescope three times so that a short instrument may have the same effective length as a telescope about three times its length. The "optical length" as we word it is three times the actual length: hence a long focus object glass can be used and high magnification obtained. They are usually in pairs, one for each eye.

Some Effects of Total Reflection

Look upwards at the surface of some water in a tumbler from a position just below its level. It will appear silvery and mirror-like. This is because light travelling upwards from the other side of the tumbler is totally reflected from the surface to the eye as shown in Fig. 255. An image of the part of the spoon just below the surface will be seen apparently above the surface.

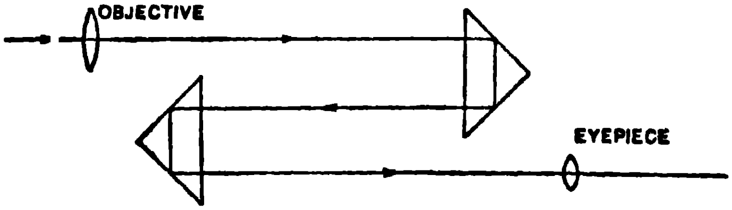


FIG. 254.

A test-tube placed slantingly in water and viewed from above, as shown in Fig. 256, will appear silvery for the same reason. If the tube is filled with water it no longer has that appearance.

If you look obliquely through a cracked pane of glass the crack appears silvery owing to total reflection from the air layer between the two glass surfaces.

The mirage is caused by total reflection at the surface separating two layers of air of different densities. In the desert the sand becomes very hot during the daytime and heats the layer of air in contact with it causing expansion and a fall in density. Thus successive layers of air going upwards are denser than the layers below them. A beam of light travelling downwards from the top of a tree will be refracted away from the normal on entering a new and less dense layer. Thus the angle of incidence at which successive layers are met increases and finally one surface of separation is met with an angle of incidence greater than the critical angle between the two layers and total reflection takes place, directing the beam upwards. If this reflected beam enters the eye an inverted image of the tree is seen, and the appearance is that which would be seen if there was a pool of water at the foot of the tree.

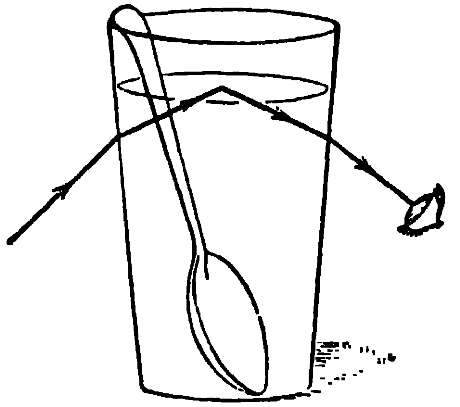


FIG. 255.

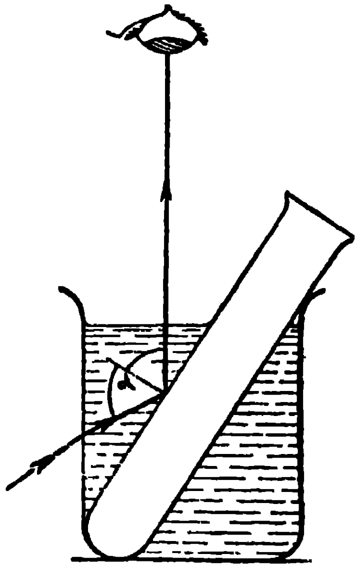


FIG. 256.

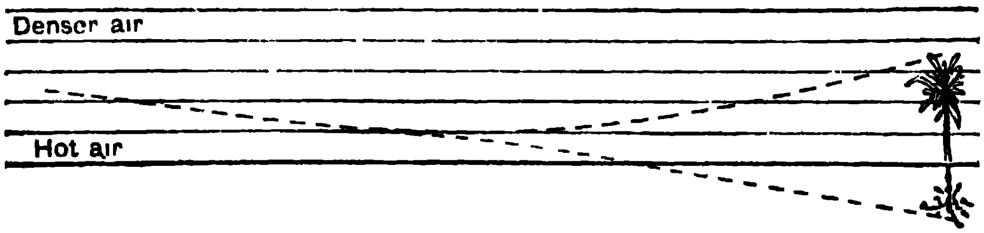


FIG. 257. A MIRAGE.

Similar reflections are often observed on tarred roads on hot days.

In cold regions an inverted mirage is often seen. Here the layer of air near the ground is colder and denser than layers above it, so that beams of light travelling upwards from objects may be totally reflected downwards and produce inverted images in the air.

QUESTIONS ON CHAPTER XXVI

1. State what happens when a beam of light passes through (*a*) a parallel sided block of glass, (*b*) a triangular glass prism. Draw diagrams to illustrate your answers.

From the diagrams explain what is meant by the terms *refraction* and *deviation*.

2. Explain why (*a*) a stick appears to be bent when dipped obliquely into water, (*b*) a pond appears to be less deep than it really is, (*c*) an object viewed through a triangular glass prism with its vertex pointing upwards appears to be displaced upwards.

3. State the laws of refraction and describe an experiment to verify them.

Hence explain what is meant by the *index of refraction* of a substance.

4. O is a small object 6 in. above the surface of the water in a pond. Construct on a scale diagram the paths of two beams diverging from O, one perpendicular to the surface of the water and the other having an angle of incidence of 30° at the water surface. Index of refraction of water $= \frac{4}{3}$.

Hence determine the position at which O would appear to be to a fish in the water vertically below O.

5. Describe how you would determine the apparent thickness of a block of glass.

If a block has a real width of 6·5 cm. and an apparent width of 4·5 cm., what is the refractive index of the glass?

If it is 2·5 cm. deep, by how much will print over which it is placed appear to be raised?

6. Describe an experiment to illustrate what is meant by *minimum deviation* produced by a prism. What relation is there between minimum deviation and (*a*) the angle of the prism, (*b*) the refractive index of the substance of the prism?

7. The index of refraction of glass being $\frac{3}{2}$ construct the path of a beam passing through a glass prism with an angle of 45° in the position of minimum deviation.

8. Describe an experiment to illustrate the meanings of the terms *total internal reflection* and *critical angle.*

Explain why (*a*) beams of light in air incident on a glass surface cannot be totally reflected, (*b*) a water surface viewed from below may act as a mirror.

9. Explain how, by use of prisms, you can (*a*) turn a beam of light through a right angle, (*b*) make a beam travel in a direction directly opposite to its original path.

Describe one practical application of each case.

10. Explain what is meant by saying that the refractive index of glass is 1·5 and its critical angle 42°.

If you were provided with a semicircular slab of glass and some pins, describe how you would proceed to measure these quantities. [L.U.

11. A ray of light falls upon the surface of water in a tank, making an angle of 60° with the surface.

Make a diagram full size, showing the path of the ray if the water is 3 in. deep. Measure the angle of refraction. [Refractive index of water, $\frac{4}{3}$.]

Using your diagram, explain the meaning of the term *total internal reflection.* [L.U.

12. ABC is a glass prism, AB = BC = 3 in. and $\angle$ ABC = 90°. Construct the path of a ray of light which passes through the prism, being incident on the face AB at a point 1 in. from A and parallel to AC. [Refractive index of glass = $\frac{3}{2}$.] Give a brief explanation of your construction and make your diagram full size. [L.U.

13. When looking perpendicularly into a vessel of water 8 in. deep, a point on the bottom of the vessel appears to be only 6 in. below the surface of the water. Explain this, and draw a diagram showing the passage of a cone of rays from this point to an eye placed to one side of, but near to, the normal drawn from the point to the surface of the water.

Explain how you would determine the exact position of the image experimentally. [L.U.

14. What is meant by saying that the *refractive index* of water is $\frac{4}{3}$?

A point source of light is at the bottom of a large tank containing water which is 3 in. deep. Draw the complete paths of rays which pass up through the water making angles of 0°, 30°, and 60° respectively with the vertical through the source of light. Give such explanation as you consider necessary. [L.U.

CHAPTER XXVII

LENSES

In many instruments, such as cameras, telescopes, and microscopes, use is made of lenses which are portions of transparent substances between two faces, one or both of which are curved. If the lens is thicker at its centre than at its edges, it is a **convex** or **converging** lens. If it is thicker at the edges than at the centre it is **concave** or **diverging**. There are various types of lenses in each class, these being shown in Fig. 258. The curved faces of the lenses are usually parts of spheres.

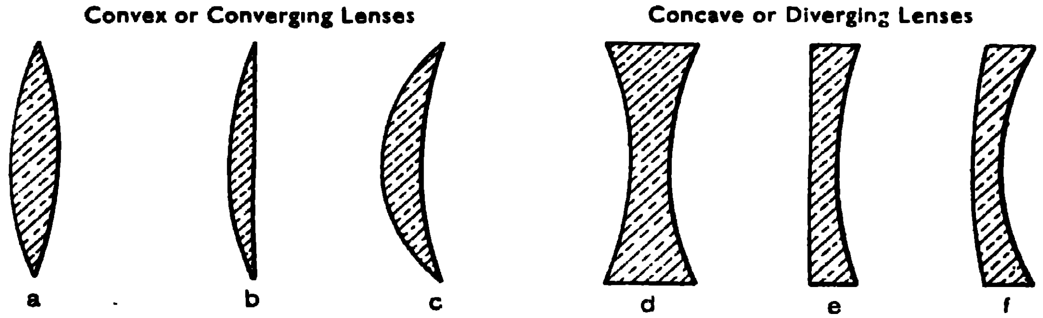


FIG. 258.

(*a*) Double convex.
(*b*) Plano-convex.
(*c*) Concavo-convex.
(*d*) Double concave.
(*e*) Plano-concave.
(*f*) Convexo-concave.

If both faces are curved, each will have a **centre of curvature.** The line through the two centres is the **principal axis** of the lens. One point on the principal axis is called the **optical centre** of the lens. If the two faces are of equal radii, the optical centre is midway between them. A section of the lens in a plane passing through the two centres of curvature is a *principal section.* As in the case of spherical mirrors, all principal sections of a given lens are alike, so diagrams showing what happens in one principal section may be taken to apply to all.

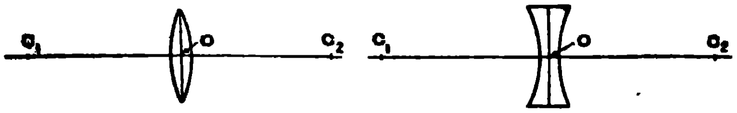


FIG. 259.

C=CENTRES OF CURVATURE. O=OPTICAL CENTRES.

Ray Box Experiments with Lenses

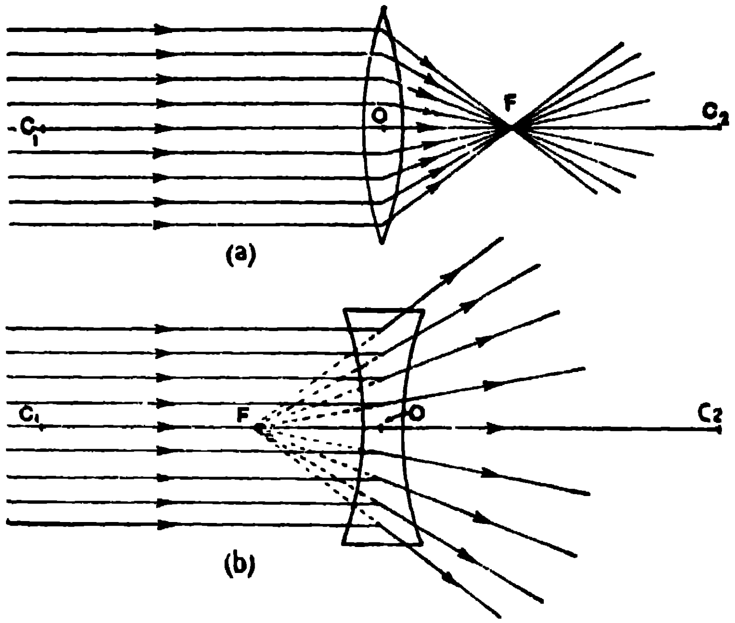


FIG. 260.

For ray box experiments half-lenses, which will stand on their flat surfaces, can be used. The part of the lens in contact with the bench will be a principal section.

Fit the ray box to give a set of parallel beams. Place a half convex lens across them. They will all be found to pass through one point on the principal axis on the opposite side of the lens, as in Fig. 260 (*a*). If a concave lens is used the beams will be found to spread out as though coming from one point on the principal axis after they have passed through the lens [Fig. 260 (*b*)]. These results indicate why the terms "converging" and "diverging" are sometimes used instead of "convex" and "concave" in naming lenses.

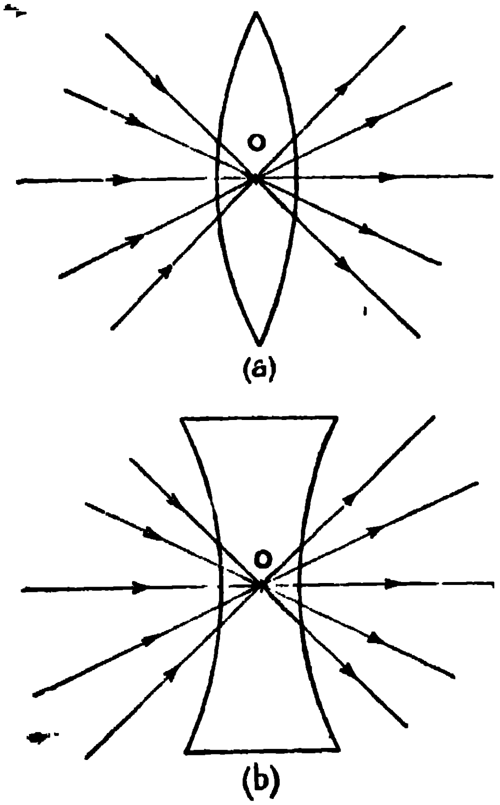


FIG. 261.

The point on the principal axis to which beams originally parallel and near to the principal axis converge after passing through a convex lens, or from which they appear to diverge after passing through a concave lens, is called the **principal focus** of the lens, and its distance from the optical centre is called the **focal length** of the lens.

If lenses of large aperture are used it will be found that, as with mirrors, there is spherical aberration. The beams passing through the outer parts of the lens cut the principal axis between the principal focus and the optical centre.

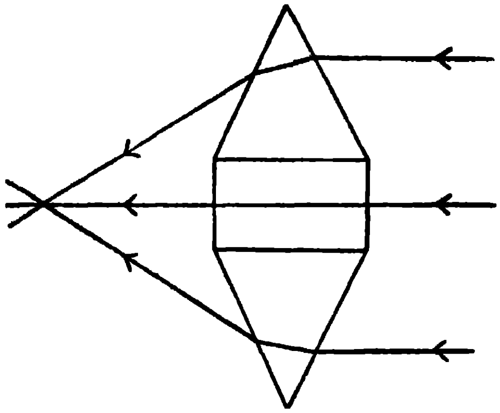


FIG. 262.

Arrange the box to give beams converging to a point. Place a half-lens of either type so that its optical centre is at the point of convergence. Note that the beams do not suffer deviation but pass straight through the lens (Fig. 261). A beam directed towards the lens in any direction other than towards the optical centre will undergo deviation. Thus **the optical centre of a lens may be defined as the point on the principal axis through which all beams which are not deviated by the lens pass.**

The results of these experiments may be summarised as—

(1) Beams parallel and near to the principal axis are converged to the principal focus on the other side of the lens by a convex lens or diverged from the principal focus on the same side of the lens by a concave lens.

(2) Beams directed towards the optical centre pass through the lens without deviation.

Also the principle of reversibility will give from (1)—

(3) Beams diverging from the principal focus on the incident side of a convex lens or directed towards the principal focus on the other side of a concave lens became parallel to the principal axis after passing through the lens.

Explanation of Lens Action

If a plate of glass is placed between two prisms as indicated in Fig. 262, beams passing through the prisms will be deviated away from their edges, that is, inwards. Thus three parallel beams might be converged to one point as shown in the figure. Other beams parallel to those shown would not be deviated through the same point.

A principal section of a convex lens may be regarded approximately

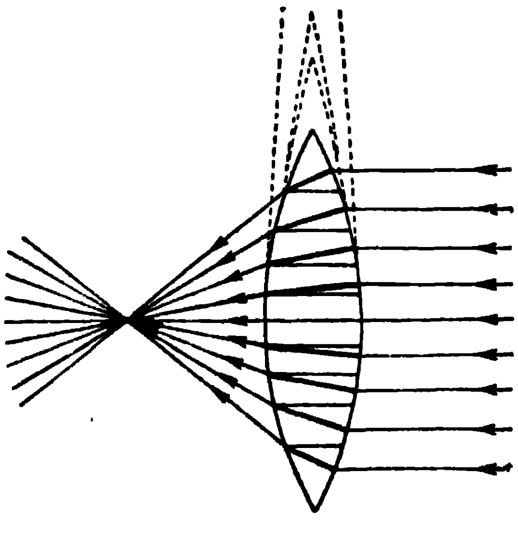


FIG. 263.

as made up of a number of thin slices taken from various prisms (Fig. 263), those near the edge being parts of prisms with large angles, whilst those near the centre are from prisms of small angles. Thus the outer prism sections would cause greater deviation than the inner ones, and a large number of parallel beams may be focussed to one point.

Owing to the curvature of the faces of the lens, any two beams, however near to one another they may be, may be considered to pass through different prisms, and so all beams parallel to the principal axis may be focused to one point.

The action of a concave lens may be similarly explained from an arrangement of prisms placed point to point.

The two faces of a lens will be almost parallel at its central portion which may therefore be regarded as a parallel sided plate. It has been shown that such plates do not deviate beams but do displace them laterally. If the plate is thin the lateral displacement is small, and the beam appears to pass straight through the plate. This explains the action of a lens on beams passing through the optical centre, provided that the lens is thin.

Image Formation by Lenses

(A) CONVEX LENS

Fit up an illuminated object as for the experiments with concave mirrors on page 297. Stand a convex lens in front of it and place a screen on the other side of the lens.

Place the object at a considerable distance from the lens and move the screen until a clear image of the object is seen on it. It will be evident that this is a real image. Gradually move the object nearer to the lens and find the image positions for various object positions. Verify the following:—

(1) Object at a considerable distance from lens. Image near the lens, real, diminished, and inverted.

(2) As object moves nearer to lens, image moves further from it and becomes larger but still diminished, real, and inverted.

(3) A position is reached where object and image are at equal distances from the lens and are equal in size. Image real and inverted.

(4) Object distance less than in (3). Image distance greater than object distance. Image real, inverted, and magnified.

(5) Object distance less than half that in (3). No real image can be obtained. By looking through the lens from the side opposite to the object an upright, magnified image on the same side of the lens as the object will be seen. Since it is behind the object, it is obviously a virtual image.

In a number of cases where real images are produced measure object distance (u) and image distance (v), and also height of object and height of image. Verify that:—

$$\frac{1}{v} + \frac{1}{u} = \frac{1}{f}, \text{ where } f \text{ is a constant, and that}$$

$$\frac{\text{Height of image}}{\text{Height of object}} = \frac{v}{u}.$$

(B) CONCAVE LENS

(1) Substitute a concave lens for the convex one. It will be found that, however the object position is varied, no real image can be formed. For all object positions images which are upright and diminished and on the same side of the lens as the object will be observed by looking through from the other side. Clearly all these images are virtual.

Construction of Images

The three types of beams mentioned on page 326 are used in constructions for finding the positions and nature of images formed by lenses. Figs. 264 and 265 show such constructions for convex and concave lenses and also indicate the drawing of beams by which such images are seen. Reference to Fig. 260 will show why the principal focus must be taken on the opposite side of the lens to the object in

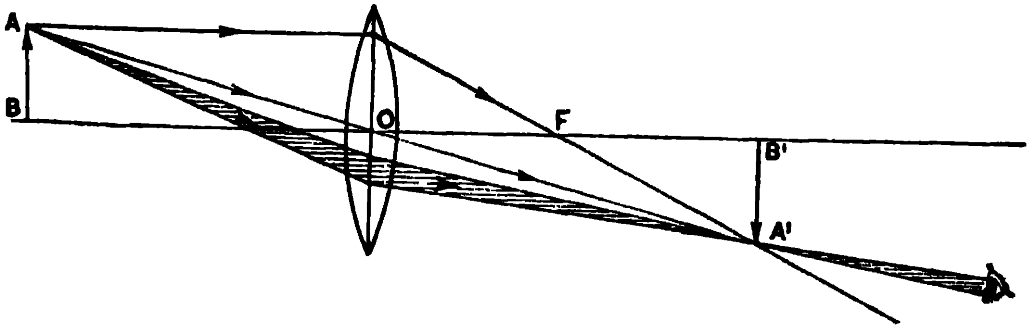


FIG. 264.

the case of a convex lens and on the same side in the case of a concave lens.

By means of such constructions the following cases should be verified and compared with the results given on page 327.

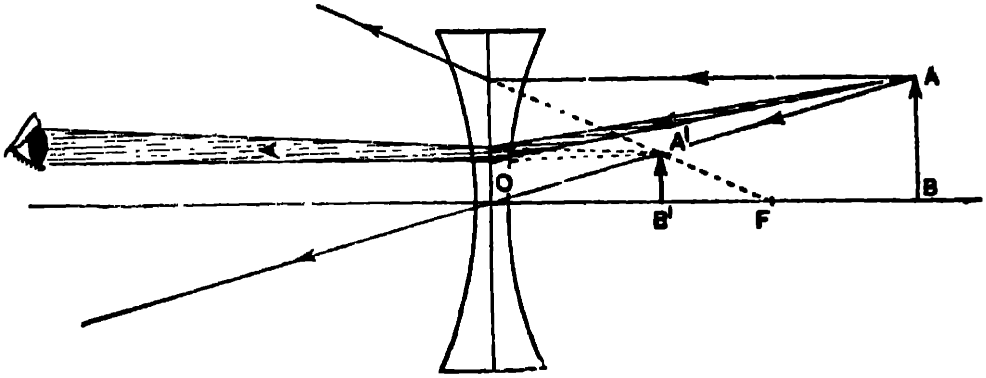


Fig. 265.

(1) Convex Lens (focal length $= f$).—(*a*) Object at distance greater than $2f$. Image at distance less than $2f$ but greater than f on opposite side of lens. Real, inverted, diminished.

(*b*) Object at distance equal to $2f$. Image at distance equal to $2f$ on opposite side. Real, inverted, same size as object.

(*c*) Object at distance less than $2f$ but greater than f. Image at distance greater than $2f$ on opposite side. Real, inverted, magnified.

(*d*) Object at distance equal to f. Image at infinity. (Beams diverging from same point on object are parallel to one another after passing through lens.)

(*e*) Object at distance less than f. Image on same side as object at distance greater than that of object. Virtual, erect, magnified.

(2) Concave Lens.—All object positions. Image on same side as object at distance less than that of object. Virtual, erect, diminished.

Measurement of Focal Lengths

(A) Convex Lens

(1) Parallel incident beams are brought to a focus at the principal focus of the lens. Hence the image of a very distant object will be formed at a distance from the lens which is approximately equal to the focal length. (Compare case 1, page 303.) Hence the focal length may be found by standing the lens facing the sun, adjusting a screen

on the opposite side of it until the sharpest possible image is obtained, and then measuring the distance from the image to the optical centre of the lens.

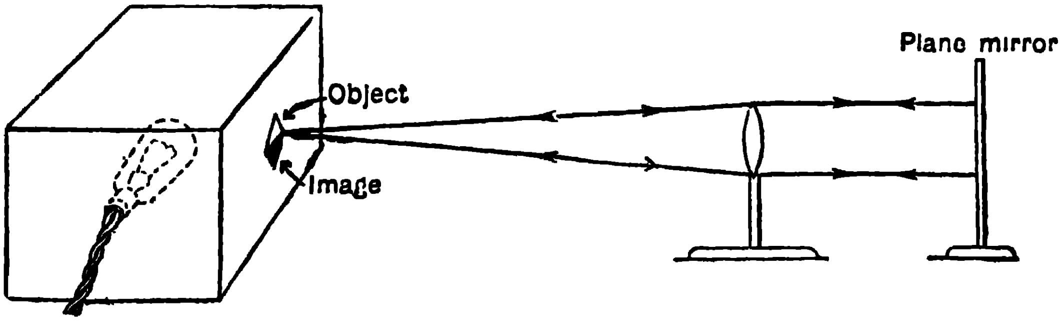


FIG. 266.

(2) Light diverging from the principal focus emerges from the lens as parallel beams. If these beams are allowed to fall on a plane mirror placed parallel to the lens, they will be normal to it and so will be reflected back along their former paths. The lens will then bring them to a focus again at the principal focus. Fig. 266 shows how this case is used for measuring the focal length. The distance of the lens from the illuminated object is adjusted until a sharply focused image is formed just by the object. The distance from the object to the optical centre is then the focal length.

(3) Directions were given on page 328 for verifying that $\frac{1}{v} + \frac{1}{u}$ is constant for a given lens. If f is determined for the same lens by either method 1 or 2, it will be found that $\frac{1}{v} + \frac{1}{u} = \frac{1}{f}$. This relation may be used for finding focal lengths of other convex lenses. Measure a number of corresponding values of v and u as on page 328. From each set calculate f by means of the above equation and average the results.

(B) CONCAVE LENS

The equation $\frac{1}{v} + \frac{1}{u} = \frac{1}{f}$ can be shown to apply to all cases of image formation by lenses provided that a sign convention similar to that for mirrors is adopted. That is, **all distances are measured from the optical centre. Real images, foci, and objects have positive distances and virtual images and foci have negative distances.** Note that the focal length of a convex lens will be positive and that of a concave lens will be negative.

Since a concave lens gives only virtual images, methods such as those used for convex lenses are not available. The position of the virtual image formed may, however, be found by a parallax method as in the case of the convex mirror.

Stand a tall object, such as a white pencil stuck into a cork, in front of the lens. Looking from the other side of the lens a virtual image can be seen. Move a pin supported in a retort stand so that it can be seen above the lens backwards and forwards until it appears to be at the same place as the image of the pencil. Measure distance of the pencil from the lens for u and that of the pin from the lens for v. Calculate f from the equation, remembering that a negative value must be substituted for v because the image is virtual.

Examples on Lens Calculations

(1) *An object 2 cm. high is placed (a) 50 cm., and (b) 15 cm. from a convex lens of 20 cm. focal length. Find in each case the position, size, and nature of the image.*

(*a*)

$$\frac{1}{v} + \frac{1}{u} = \frac{1}{f}; \quad \therefore \frac{1}{v} + \frac{1}{50} = \frac{1}{20};$$

$$\therefore \frac{1}{v} = \frac{1}{20} - \frac{1}{50} = \frac{3}{100}; \quad \therefore v = \frac{100}{3} = 33{\cdot}3.$$

$$\frac{\text{Ht. of image}}{\text{Ht. of object}} = \frac{v}{u};$$

$$\therefore \frac{\text{Ht. of image}}{2\text{ cm.}} = \frac{\frac{100}{3}}{50} = \frac{2}{3}; \quad \therefore \text{Ht. of image} = \tfrac{2}{3} \times 2 = 1{\cdot}33\text{ cm.}$$

Since v is positive the image will be real and so it will be inverted. Its position will be 33·3 cm. from the lens on the side opposite to the object.

(*b*)

$$\frac{1}{v} + \frac{1}{u} = \frac{1}{f}; \quad \therefore \frac{1}{v} + \frac{1}{15} = \frac{1}{20};$$

$$\therefore \frac{1}{v} = \frac{1}{20} - \frac{1}{15} = -\frac{1}{60}; \quad \therefore v = -60.$$

$$\frac{\text{Ht. of image}}{\text{Ht. of object}} = \frac{v}{u};$$

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$$\therefore \frac{\text{Ht. of image}}{2 \text{ cm.}} = \frac{60}{15} = 4; \quad \therefore \text{ Ht. of image} = 2 \times 4 = 8 \text{ cm.}$$

Since v is negative, the image is virtual and so will be upright. Its position will be 60 cm. from the lens on the same side as the object.

(2) *Take distances as in (a) above but for a concave lens.*

$$\frac{1}{v} + \frac{1}{u} = \frac{1}{f}; \quad \therefore \frac{1}{v} + \frac{1}{50} = \frac{1}{-20} \text{ (Note negative value for } f\text{);}$$

$$\frac{1}{v} = -\frac{1}{20} - \frac{1}{50} = -\frac{7}{100}; \quad \therefore v = -\frac{100}{7} = -14{\cdot}3.$$

$$\frac{\text{Ht. of image}}{\text{Ht. of object}} = \frac{v}{u};$$

$$\therefore \frac{\text{Ht. of image}}{2 \text{ cm.}} = \frac{\frac{100}{7}}{50} = \frac{2}{7}; \quad \therefore \text{ Ht. of image} = \frac{4}{7} = 0{\cdot}57 \text{ cm.}$$

Since v is negative the image is virtual and so is erect. Its position will be 14·3 cm. from the lens on the same side as the object.

(3) *A lens* 20 *in. from an object produces a virtual image* $\frac{2}{3}$ *the size of the object. Find the position of the image, the kind of lens, and its focal length.*

$$\frac{\text{Ht. of image}}{\text{Ht. of object}} = \frac{v}{u}; \quad \therefore \frac{2}{3} = \frac{v}{20 \text{ in.}};$$

$$\therefore 3v = 40 \text{ in.}; \quad \therefore v = 13{\cdot}3 \text{ in.}$$

Since the image is virtual its position is 13·3 in. from the lens on the same side as the object.

$$\frac{1}{v} + \frac{1}{u} = \frac{1}{f}; \quad \therefore \frac{1}{-\frac{40}{3}} + \frac{1}{20} = \frac{1}{f} \text{ (Note negative value for } v\text{);}$$

$$\therefore -\frac{3}{40} + \frac{1}{20} = \frac{1}{f}; \quad \therefore \frac{1}{f} = -\frac{1}{40}; \quad \therefore f = -40.$$

The focal length of the lens is — 40 in.

Since the focal length is negative, the lens is *concave*.

Geometrical Proof of Lens Formulae

Triangles A′B′C and ABC (Fig. 267) are similar; hence:—

$$\frac{A'B'}{AB} = \frac{A'C}{AC} = \frac{v}{u} \text{..................................} (1)$$

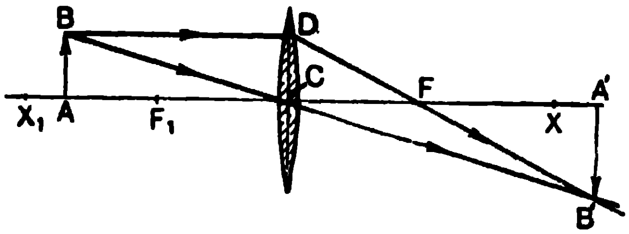


FIG. 267. IMAGE BY CONVEX LENS.

Also triangles A′B′F and DCF are similar;

$$\therefore \frac{A'B'}{CD} = \frac{A'F}{CF} = \frac{v-f}{f};$$

$$\therefore \frac{A'B'}{AB} = \frac{v-f}{f} \qquad (AB = CD) \text{ ....} (2)$$

$$\therefore \text{ From (1) and (2), } \frac{v}{u} = \frac{v-f}{f}; \quad \therefore vf = uv - uf.$$

$$\text{Divide throughout by } uvf\text{:— } \frac{1}{u} = \frac{1}{f} - \frac{1}{v}; \quad \therefore \frac{1}{f} = \frac{1}{u} + \frac{1}{v}.$$

$$\text{Note that (1) gives } \frac{\text{Ht. of image}}{\text{Ht. of object}} = \frac{v}{u}.$$

Similar proofs may be given for the case where a convex lens produces a virtual image and for concave lenses if due attention is paid to the signs of v and f when substituting.

QUESTIONS ON CHAPTER XXVII

1. What is a lens? Distinguish between convex and concave lenses. Explain, with diagrams, why the former are sometimes called converging and the latter diverging lenses.

2. Draw diagrams to show the meanings of the terms *centres of curvature*, *principal focus*, and *optical centre* in connexion with both convex and concave lenses.

3. Explain by reference to prisms the action of convex and concave lenses on parallel beams of light.

4. Describe experiments to show the various types and sizes of images which can be produced by a convex lens.

How would you verify the relation between object distance, image distance, and focal length for such a lens.

5. Construct diagrams to illustrate the formation of (*a*) a magnified real image by a convex lens, (*b*) a diminished real image by a convex lens, (*c*) a magnified virtual image by a convex lens, (*d*) a diminished virtual image by a concave lens?

6. Calculate the position and size of the image in each of the following cases:—

Convex Lens

	f	*u*	Ht. of Object
(*a*)	10 cm.	25 cm.	5 cm.
(*b*)	12 in.	20 in.	2 in.
(*c*)	15 cm.	10 cm.	4 cm.
(*d*)	10 in.	20 in.	3 in.

Concave Lens

	f	*u*	Ht. of Object
(*e*)	30 cm.	50 cm.	8 cm.
(*f*)	25 in.	50 in.	6 in.
(*g*)	20 cm.	30 cm.	5 cm.

7. Describe two methods of determining the focal length of a convex lens. Give an explanation of the principles involved in each method.

8. Define the focal length of a convex lens.

Describe with full experimental details how you would determine the focal length of a convex lens. [L.U.

9. You are required to form an upright image of an object with (*a*) a convex lens, (*b*) a concave lens.

State the position of the object with respect to the lens in each case. Illustrate the formation of these images with suitable diagrams and point out how these images differ. [L.U.

10. Explain what would be the effect of a convex lens of focal length 10 in. upon: (*a*) a parallel beam of light; (*b*) a beam diverging from a point 20 in. from the lens; (*c*) a beam diverging from a point 5 in. from the lens; (*d*) a beam converging to a point 20 in. behind the lens.

Draw careful diagrams to illustrate your answer, taking in each case the axis of the beam as the principal axis of the lens and considering light of one colour only. [L.U.

11. Describe the action of a convex lens upon a beam of light of one colour.

An object when placed 15 cm. from a convex lens gives a *real* image twice the size of the object. Where must the object be placed relative to the same lens to give a *virtual* image twice the size of the object? Draw scale diagrams to illustrate the formation of the two images. [L.U.

12. Find the position, size, and character of the image produced by a convex lens of focal length 7 in. of an object 2 in. tall placed (*a*) 4 in., (*b*) 10 in. from the lens. Draw a diagram showing the paths of two rays in each case. [L.U.

CHAPTER XXVIII

OPTICAL INSTRUMENTS

The Camera

A camera is a light-tight box at the back of which is placed a plate or film coated with chemicals which are affected by light. In the front is an opening fitted with a convex lens to focus images of objects to be photographed on to the film.

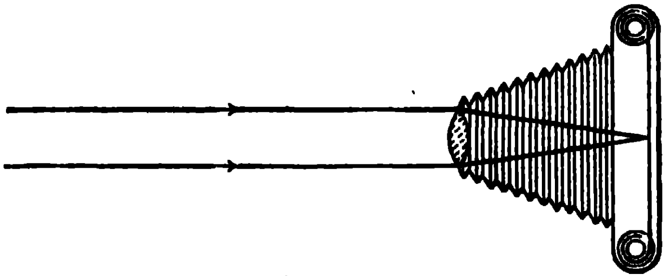


Fig. 268.

A DISTANT OBJECT IS BROUGHT TO A FOCUS ON THE FILM OF THE CAMERA.

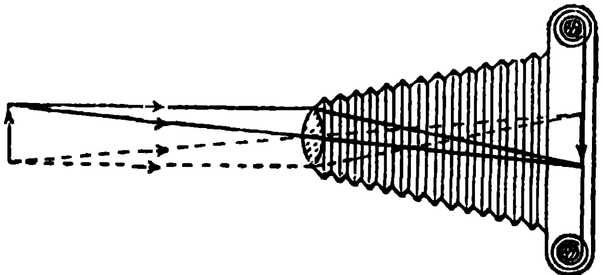


Fig. 269.

A NEAR OBJECT IS BROUGHT TO A FOCUS ON THE FILM BY MOVING THE LENS FORWARD.

In box cameras the distance between the lens and the film is fixed, and consequently there is only one object distance at which a sharply focused image is formed on the film. Better cameras have a bellows body, so that the distance between the lens and the film can be varied. The results of the last chapter will show that, for distant objects, this distance should be equal to the focal length of the lens (Fig. 268). For nearer objects, the image distance will be greater than the focal length, and so the camera should be extended (Fig. 269).

The amount of chemical action at any point on the film depends on the amount of light reaching that point during the time of exposure. Thus, on very bright days a short exposure will produce a sufficiently dark negative, while on duller days a longer exposure is necessary.

The amount of light focused at any point also depends on the diameter of the opening, since with a larger aperture more beams from any given point on the object will enter the camera. Thus, adjustment for different brightnesses can be made by the use of "stops" with which the size of the opening can be varied. This enables the same time of exposure to be used with varying illuminations.

In British cameras the stops are usually marked to show the relation of the diameter of the opening to the focal length of the lens. Thus F/8 means a stop with a diameter one-eighth the focal length of the lens. Common sets of stops are F/8, F/11·3, F/16, F/22·6, F/32. On American cameras these are simply marked 4, 8, 16, 32, 64. The areas of the stops, and therefore the amount of light admitted by them in a fixed time with the same illumination, are proportional to the squares of their diameters. Hence, under the conditions mentioned, the amounts of light admitted by the F/8 and F/11·3 stops respectively would be in the ratio $\frac{11\cdot3^2}{8^2} = \frac{128}{64} = 2$. (approx.)

That is, the F/8 stop would let in twice as much light in a given time as the F/11·3 stop. Hence, with the same illumination, twice as long an exposure is required with the F/11·3 stop as with the F/8. Similar relations hold with regard to other succeeding members of the series. This is indicated by the American system in which a F/4 stop is regarded as requiring 1 unit of time exposure, so that F/8 requires 4 units, and so on.

With a given illumination, a large stop enables the time of exposure to be reduced but, with a large aperture, spherical aberration has a considerable effect, and the image tends to be somewhat blurred.

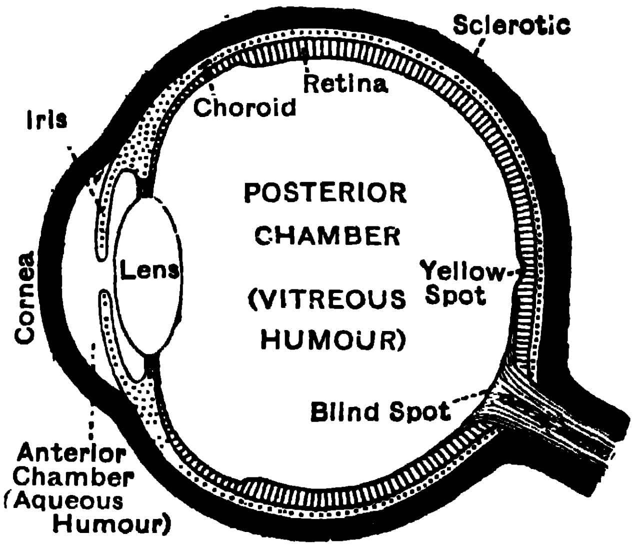


FIG. 270.

The Eye

As shown in Fig. 270, the eye is very similar in construction to a camera. A lens of jelly-like substance in the pupil of the eye focuses images of objects on the retina, which is a sensitive nerve lining of the back of the eye chamber. These images are real and inverted, but the brain interprets them in such a way that the objects are seen as being right way up. In front of the lens there is a filament, the iris or coloured part of the eye, which, by muscular action, can open or close to adjust the aperture of the eye to varying intensities of light. Further details on the structure of the eye may be found in encyclopaedias or books on physiology.

The distance between the lens and the retina is not adjustable for focusing objects at varying distances. This is accomplished by a ring of muscle surrounding the lens which can alter the curvature of the lens surfaces and so change its focal length. When a normal eye is in a state of rest, the focal length of the lens is equal to the distance from it to the retina, so that distant objects are focused on the retina [Fig. 271 (*a*)]. If this were a fixed state, images of nearer objects would be focused behind the retina, and the objects would not be seen. Such objects can, however, be brought into focus by an increase in the curvature of the lens which decreases its focal length and brings the image nearer to the lens [Fig. 271 (*b*)].

This accommodation of the normal eye can continue as nearer and nearer objects are viewed until the object is at a distance of about 10 in. from the eye, but the eye cannot increase its curvature more than is necessary for seeing objects at this **nearest distance of distinct vision,** and images of objects nearer to the eye than 10 in. are formed behind the retina [Fig. 271 (*c*)]. Fig. 271 *d* shows how objects at still smaller distances may be viewed by placing another convex lens in front of the eye (see page 341).

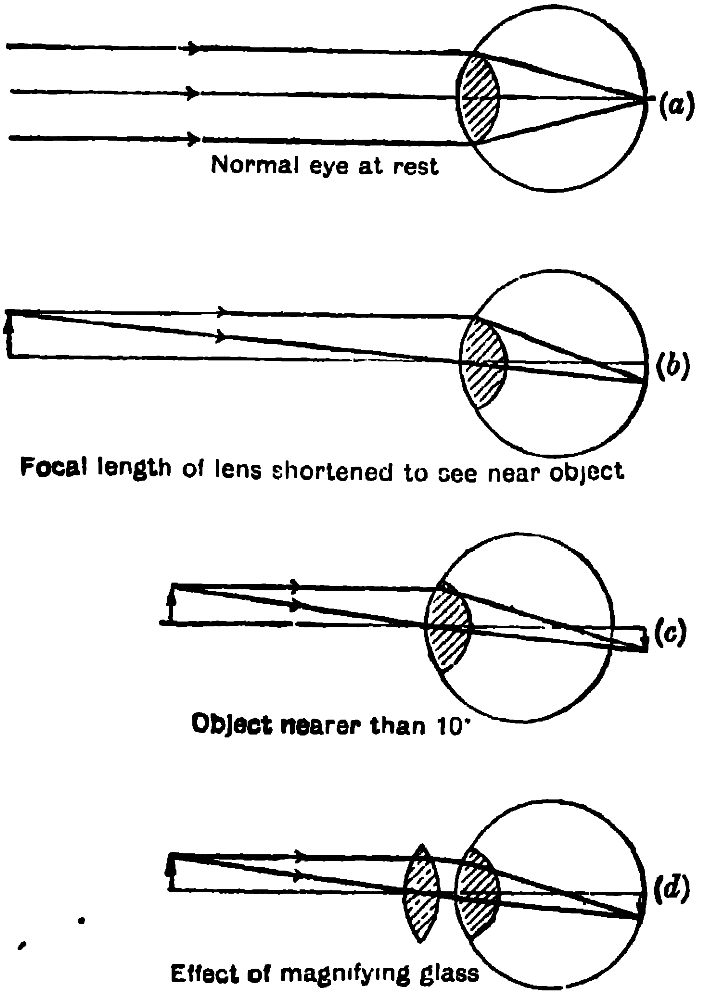

(a)
Normal eye at rest
(b)
Focal length of lens shortened to see near object
(c)
Object nearer than 10"
(d)
Effect of magnifying glass

FIG. 271.

This shows why "eyestrain" is felt after long periods of close work. All the time, the muscles are compressing the lens. Th should be rested from time to time by looking at distant objects.

Short Sight (myopia).—This is due to the lens in its resting state having a focal length less than the distance to the retina, so that images of distant objects are formed nearer to the lens than the retina. As the focal length cannot be *increased* by accommodation, such objects are invisible to the unaided eye. There will be some object distance at which the image distance is equal to the width of the eye, and that distance is the farthest distance of distinct vision for that eye. Nearer objects can be seen by accommodation as with a normal eye. A short sighted eye usually has its nearest point of distinct vision at a distance of less than 10 in.

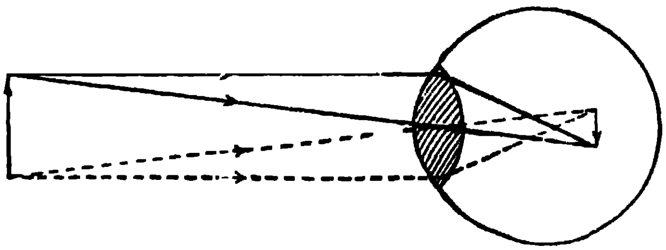


Fig. 272. Short Sight.

Short sight is corrected by using spectacles with concave lenses. These make beams from objects more divergent before entering the eye, so that the image formed by the eye lens is further back than when the spectacles are absent.

Long Sight (hypermetropia).—Long sight is due to the eye lens in its resting state being insufficiently convex, so that its focal length is greater than the width of the eye, and images of distant objects are formed behind the retina (Fig. 274). By accommodation such objects can be brought into proper focus, but since there has to be some accommodation for viewing distant objects, the maximum accommodation of which the eye is capable has to be used for objects at some distance from the eye, and near objects cannot be properly focused.

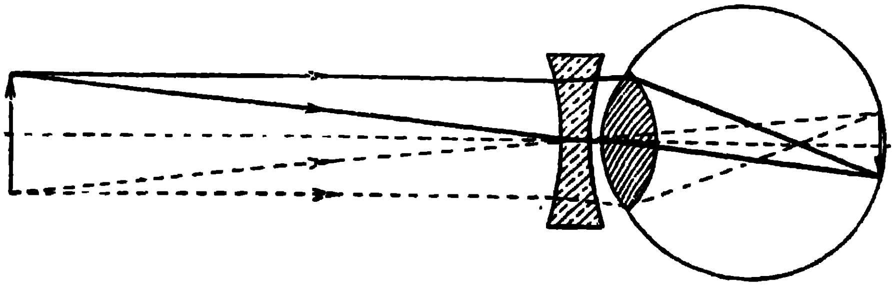


Fig. 273. Short Sight Corrected.

Long sight is corrected by the use of convex lenses (Fig. 275). These make beams from a point on an object less divergent before entering the eye, so that the image formed by the eye lens is nearer to it than when the spectacle lens is absent.

Focal Lengths of Spectacles

In correcting short sight the lens used should be one which gives a virtual image of very distant objects at the point of farthest vision of the unaided eye.

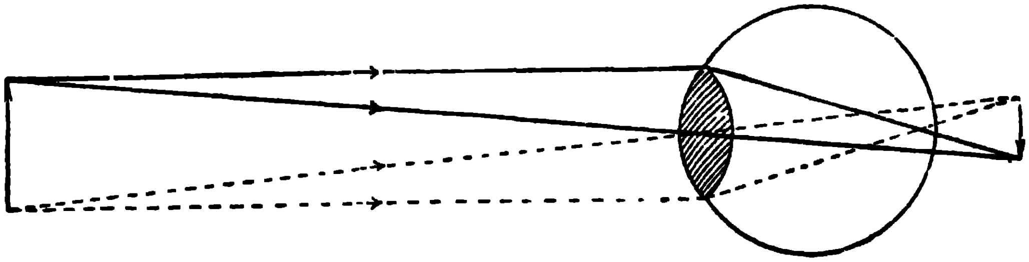


FIG. 274. LONG SIGHT.

EXAMPLE.—*An eye cannot see clearly objects at a distance of more than 200 in. What lens should be used to correct this ?*

A lens is required which gives a virtual image at 200 in. from it of an object at an infinite distance.

$$\frac{1}{v}+\frac{1}{u}=\frac{1}{f};$$

$$\frac{1}{-200}+\frac{1}{\infty}=\frac{1}{f}; \quad \therefore f=-200 \qquad \left(\frac{1}{\infty}=0\right);$$

∴ A concave lens of 200 in. focal length is required.

In correcting long sight a lens should be used which will form a virtual image at the nearest point of distinct vision of the unaided eye, when an object is at the nearest point of distinct vision of a normal eye.

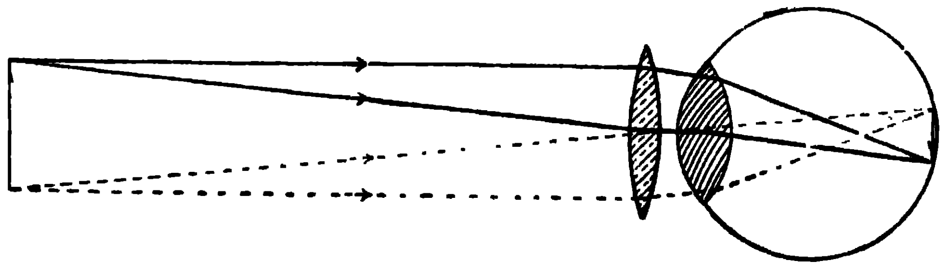


FIG. 275. LONG SIGHT CORRECTED.

EXAMPLE.—*An eye cannot see clearly objects at a distance of less than 30 in. What spectacles are required to enable objects at a distance of 10 in. to be seen ?*

Here, a virtual image at a distance of 30 in. is required when the object is at a distance of 10 in.

$$\frac{1}{v}+\frac{1}{u}=\frac{1}{f};$$

$$\frac{1}{-30}+\frac{1}{10}=\frac{1}{f}; \qquad \frac{1}{f}=\frac{2}{30}; \qquad f=15.$$

∴ A convex lens of 15 in. focal length is required.

ASTIGMATISM.—This is a defect of vision due to the cornea being more convex in one direction than in another, the surface having the shape of the bowl of a teaspoon. As a result, when the eye lens is focused for sharp vision of vertical lines, it may not give clear images of horizontal lines at the same distance.

There are very few eyes that do not suffer more or less from this defect, horizontal lines being usually brought to a focus *in front of* the focus of vertical lines. It is a defect moreover which usually changes until the person is about 30 years of age. The remedy for astigmatism is a *cylindrical lens*, but this cannot be dealt with in this book.

LOSS OF ACCOMMODATION (presbyopia).—In aged persons the muscles controlling the eye lens become less flexible, so that very little change can be made in the lens curvature and clear vision is possible only for objects over a short range of distances. In this case different lenses may be required for viewing distant and near objects.

The Magnifying Glass

Small objects, the details of which are invisible when they are at the nearest point of distinct vision, may be examined by using a convex lens to produce magnified virtual images of them (Fig. 276). The lens is placed near the eye and the object is placed at such a distance that the image is formed at the nearest point of distinct vision. To obtain the virtual magnified image, the distance of the object from the lens must be less than the focal length of the lens (see page 329). The magnification produced depends on the focal length of the lens; lenses of short focal length giving greater magnification than those of long focal length. For this reason opticians speak of the **power** of a lens rather than of focal length, the power being equal to the reciprocal of the focal length. If the focal length is measured in metres the power is said to be measured in **dioptres.** For example, a lens with a focal length of 30 cm., = 0·3 m., would have a power of 1/0·3 = 3·3 dioptres.

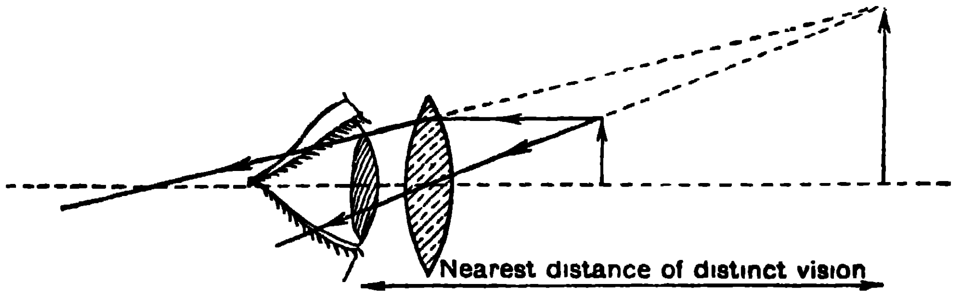


FIG. 276. THE MAGNIFYING GLASS.

The influence of focal length on magnification may be illustrated as follows:—Suppose a lens of 5 in. focal length is being used, the point of nearest distinct vision being 10 in., that is, a virtual image is to be formed at 10 in. from the lens.

$$\frac{1}{v}+\frac{1}{u}=\frac{1}{f}; \qquad \frac{1}{-10}+\frac{1}{u}=\frac{1}{5},$$

$$i.e.\ \frac{1}{u}=\frac{1}{5}+\frac{1}{10}=\frac{3}{10}; \qquad u=\frac{10}{3}.$$

The object should be $\frac{10}{3}$ in. from the lens.

$$\text{Magnification}\cdot\ \frac{\text{Image}}{\text{Object}}=\frac{v}{u}=\frac{10}{\frac{10}{3}}=3.$$

The image is 3 times as large as the object.

For a lens of 2·5 in. focal length the calculation would be

$$\frac{1}{v}+\frac{1}{u}=\frac{1}{f}; \qquad \frac{1}{-10}+\frac{1}{u}=\frac{1}{2{\cdot}5},$$

$$\frac{1}{u}=\frac{1}{2{\cdot}5}+\frac{1}{10}=\frac{5}{10}=\frac{1}{2};$$

$$\text{Magnification}=\frac{v}{u}=\frac{10}{2}=5.$$

An image 5 times as large as the object would be obtained.

Visual Angle

Magnification may also be considered from the point of view of visual angle. The apparent size of an object will depend on the size of the image formed on the retina. The nearer the object is to the eye the greater this image will be, and as shown in Fig. 277, this is connected with the fact that, when the object is brought nearer to the eye, the visual angle, that is the angle between beams entering the eye from extreme points on the object, becomes larger. When a magnifying glass is used an image is produced at the nearest point of distinct vision which has a greater visual angle than the object would have if placed at that point.

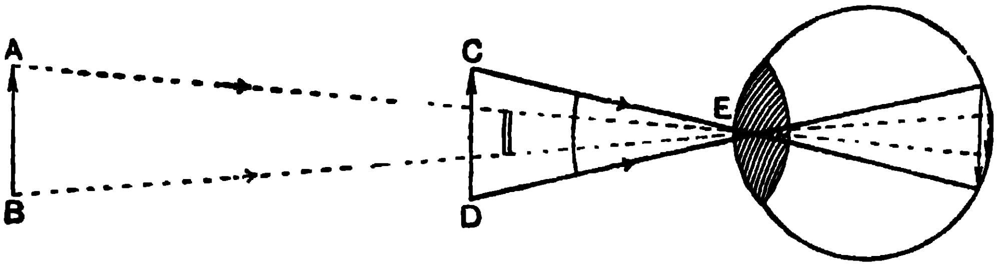


FIG. 277.

The Compound Microscope

Very high magnification can be obtained by using a combination of lenses. Set up a small illuminated object, O (Fig. 278), in front of a convex lens, A, of short focal length. Make the distance OA a little greater than f_1, the focal length of A. A real magnified image of O will be formed at I_1, its distance from A being greater than $2f_1$. Adjust a ground glass screen until this image is clearly focused on it. Now place another convex lens, B, of long focal length on the other side of the screen, its distance from I_1 being less than its focal length, f_2. A virtual magnified image of I_1 will be seen at I_2 on looking through B from the right. Note how much greater than the visual angle of O is the visual angle of I_2 at B. The diagram illustrates how the image positions may be found by construction and also the path through the two lenses of a beam of light from a point on O.

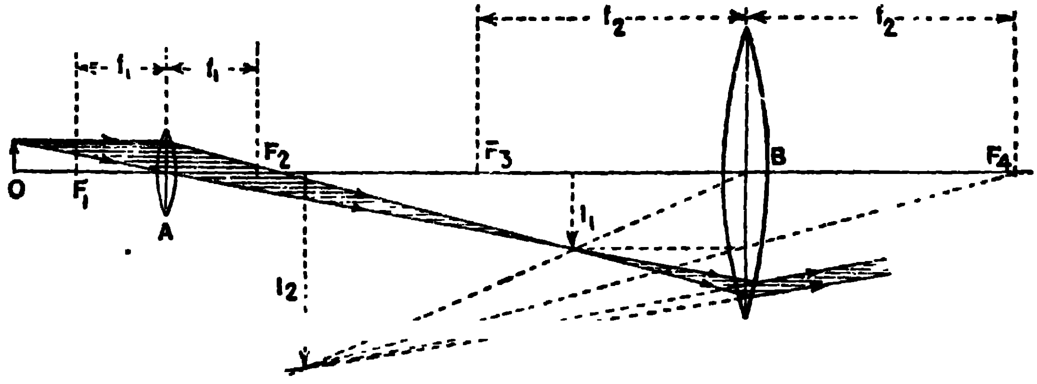


FIG. 278. ACTION OF COMPOUND MICROSCOPE.

The beams of light would follow these paths through the lenses even if the ground glass screen at I_1 were absent, so that the greatly magnified image I_2 would still be visible on looking through B. A compound microscope consists of two lenses such as A and B mounted in a tube. A is called the **objective** glass and B the **eyepiece.** In focusing the microscope, the objective is brought to such a distance, greater than f_1, from A that the final image I_2 is at the least distance of distinct vision from the eye at B.

The Telescope

Telescopes are designed to enable distant objects to be examined. The image of a distant object formed by a telescope is smaller than the actual object, but because it is so much nearer the eye it has a greater visual angle, so that the image formed on the retina is larger than it would be if the object were viewed directly. The ratio

$$\frac{\text{Visual angle of image}}{\text{Visual angle of object}}$$

is said to measure the **magnification** produced by the telescope as it gives the ratio of the sizes of the images formed on the retina with and without the telescope.

The Astronomical Telescope

A telescope can be formed by reversing the arrangement for a microscope. The objective is a convex lens of long focus which will form a diminished real image of a distant object at a point a little beyond its principal focus. The eyepiece is a convex lens of short focal length. By drawing out or pushing in the sliding part of the tube the eyepiece is brought to a position where its distance from the first image is a little less than its focal length. By looking through it a virtual magnified image of the first image is then seen. By a method similar to that used in the case of the microscope a pair of lenses may be set up to illustrate this action of the telescope. It will be observed that the image is inverted, since inversion occurs in the formation of the first image, but there is no re-inversion in forming the second one. The distance apart of the lenses is approximately the sum of their focal lengths. It can be shown that the magnification as defined above is approximately given by:—

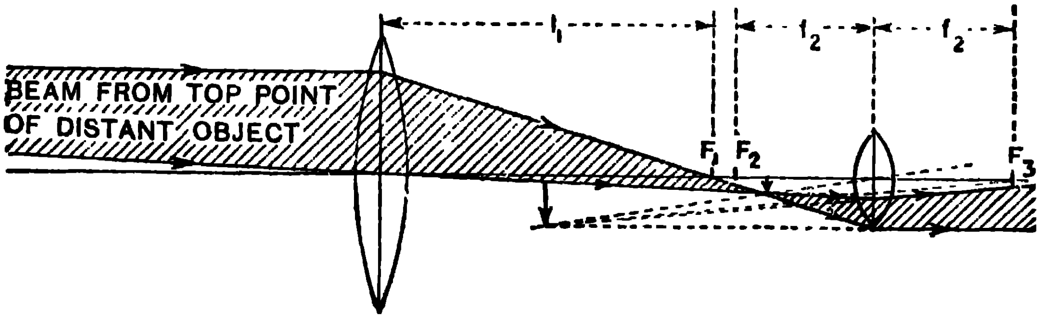


FIG. 279. ACTION OF ASTRONOMICAL TELESCOPE

$$\text{Magnification} = \frac{\text{Focal length of objective}}{\text{Focal length of eyepiece}}.$$

The Terrestrial Telescope

Inversion of the image does not matter when astronomical observations are being made, but when distant objects on the earth are being examined, we wish to see them right way up. The astronomical telescope can be adapted to terrestrial use by using a third lens as an **erecting lens.** This is a moderately short focus convex lens and it is placed at such a position that it is at a distance equal to twice its focal length from the small inverted image formed by the objective. It will then form a re-inverted real image equal to the first image and at a distance of twice its focal length on the other side of it (see page 329). This is then magnified by a short focus eyepiece as in the astronomical telescope.

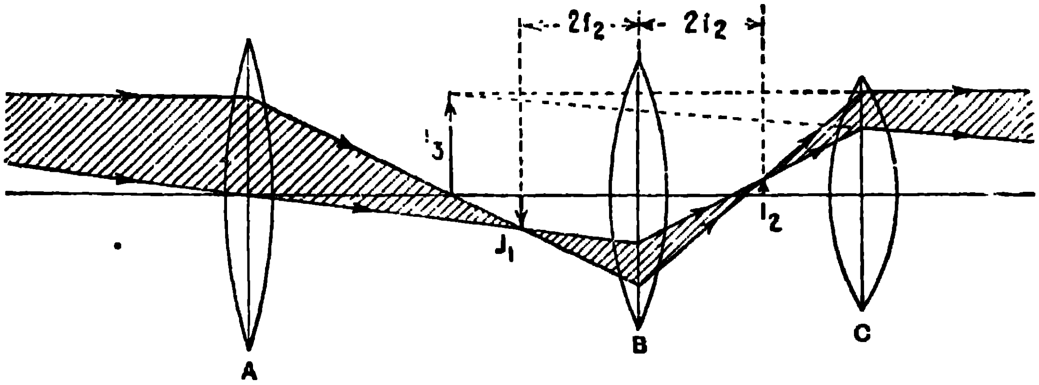


FIG. 280. ACTION OF TERRESTRIAL TELESCOPE.

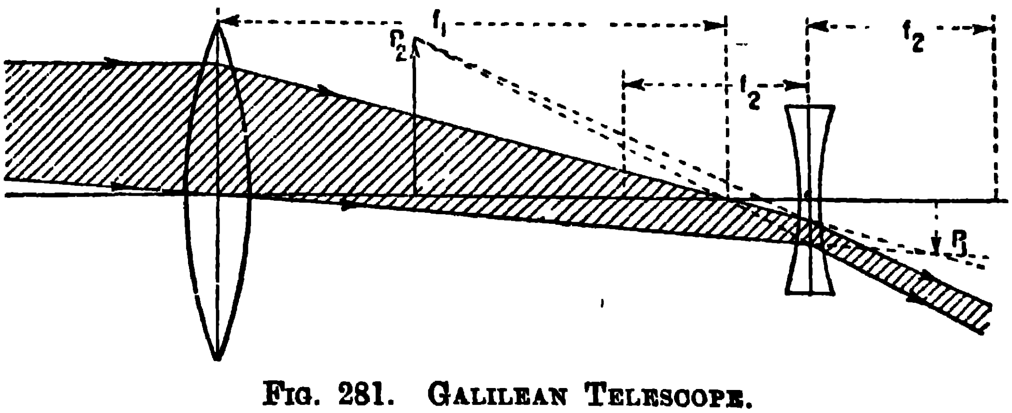


FIG. 281. GALILEAN TELESCOPE.

The Galilean Telescope

Galileo devised a telescope in which a concave lens was used as an eyepiece. As shown in Fig. 281 a long focus convex lens is used as objective. The eyepiece is of short focus and is in a position between the objective and the point where the primary image would be formed. Its distance from that point is slightly less than its focal length.

In the absence of the eyepiece beams from the top point of a distant object would be brought to a focus at P_1. But the eyepiece causes these beams to diverge so that to an eye applied to it they seem to come from P_2, and thus an erect image with a large visual angle is seen.

Galileo's telescope has the advantage of needing no separate erecting lens. Also the distance between the lenses is short. In practice it is approximately the difference between their focal lengths, since the primary image would actually be formed much nearer the principal focus of the objective than shown in the figure. For this reason the Galilean design is often used in opera glasses.

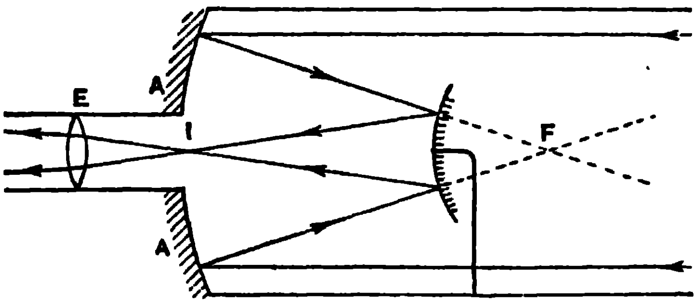


FIG. 282. REFLECTING TELESCOPE.

The Reflecting Telescope

Since chromatic aberration (see page 353) becomes very troublesome when large lenses are used, telescopes in which a concave mirror takes the place of the object glass in forming the primary image have been designed. There are several types of these. Fig. 282 illustrates the principle of one designed by Cassegrain. A large concave mirror, A, is fitted to the end of a wide tube. Facing it at a distance less than its focal length is a smaller convex mirror B.

As indicated in the diagram, parallel beams entering the tube would be reflected towards the principal focus, F, of A. The convex mirror intercepts them and reflects them through I. Thus a real image of a distant object would be formed near I. This image is examined through an eyepiece, E, fitted to an opening in the concave mirror. The mirrors are silvered on their front surfaces to prevent multiple image formation (see page 317).

Although lenses which do not give chromatic aberration can now be constructed, it is easier to make large mirrors than to make large lenses, so many of the largest astronomical telescopes are of the reflecting type.

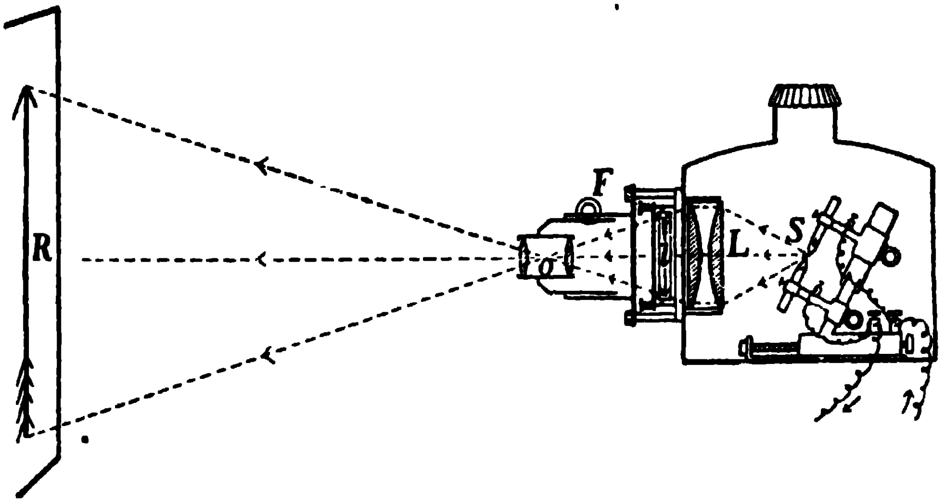
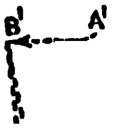


FIG. 283 THE OPTICAL LANTERN.

The Projection Lantern

A lantern for showing enlarged images of transparent slides has two sets of lenses: L is known as the condenser and O is the objective (Fig. 283). Each set may be regarded as having the action of a convex lens. The source of light S is at the principal focus of the first lens of L, so that light passes out of it as a parallel beam. The second lens of the condenser then concentrates this light on the slide *l*. Thus light spreading in a wide cone from S is concentrated on the slide, illuminating it very brightly. The objective is at a distance from *l* greater than its focal length and less than twice its focal length, so that it forms a real magnified image of *l*. The distance O*l* is adjustable, so that the image may be sharply focused on the screen. Since the image will be inverted, the slide must be placed upside down in the lantern.



The Episcope

For projecting images of opaque objects an episcope is used. The object AB is placed on the floor of the lantern. A condensing lens, L, concentrates the light from a powerful lamp, S, on it. Light diverging from

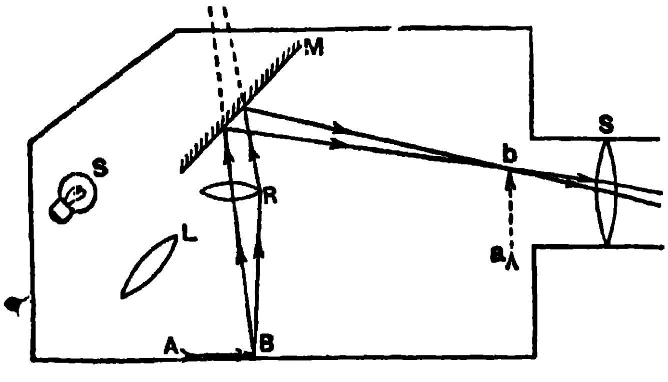


FIG. 284. EPISCOPE.

AB passes through another lens R which would produce an enlarged real image at A′B′. The light then meets a mirror, M, placed at 45° to the horizontal, so that the converging beams are reflected and actually form the image at *ab*. The beams then pass through an objective lens, S, which projects a further magnified image of *ab* on the screen.

QUESTIONS ON CHAPTER XXVIII

1. Describe the main features of a camera. Compare the focusing arrangements of a camera with those of the human eye.

2. Explain the action of a converging lens.

Give a simple account of the human eye considered as an optical instrument. [L.U.

3. Explain the defects known as *long sight* and *short sight* in the human eye, and describe how each may be corrected.

4. A person cannot see objects at a greater distance than 20 yd. nor at a nearer distance than 6 in. State what kind of lens you would provide to enable him to see distant objects and calculate what its focal length should be. What would be his shortest distance of distinct vision when using the lens?

5. Draw a careful diagram to illustrate the use of a convex lens as a magnifying glass.

Explain the use of the term *power* in connexion with such a lens and state what is the power of a lens of 10 cm. focal length.

6. What do you understand by the term *focal length* applied to (*a*) a convex lens, (*b*) a concave lens?

Draw careful diagrams to show how one and the *same* lens may be used (*c*) as a magnifying glass, (*d*) in a camera.

State the nature of the image produced in each case. [L.U.

7. Draw two diagrams to show how one and the same converging lens used in a camera forms images for two different distances of the object.

What do you infer from your diagrams concerning the focusing of a camera? [L.U.

8. Explain why a spherical thin glass globe filled with water can concentrate a beam of sunlight approximately to a point focus, and draw a diagram to show the passage of two rays of light.

Give an explanation also of the following observations: (*a*) the focus is nearer the globe when carbon bisulphide is used instead of water; (*b*) with both water and carbon bisulphide the image on the screen shows a coloured fringe round the central spot. [J.M.B.

9. Explain with the aid of diagrams the use of a convex lens (*a*) as a burning glass, (*b*) as a simple magnifying glass.

In case (*b*) show where to place an object relative to such a lens, of focal length $2\frac{1}{2}$ cm., in order that the size of image produced may be 5 times that of the object. [J.M.B.

10. Describe how you would set up arrangements of lenses to illustrate the principle of each of the following: (*a*) a compound microscope; (*b*) an astronomical telescope; (*c*) a terrestrial telescope; (*d*) a Galilean telescope.

In each case refer carefully to the focal lengths of the lenses used, and explain how you would ensure that they were placed in correct relation to one another.

11. What kind of lenses would you select if you wished to make the simplest form of compound microscope? Draw a diagram showing the arrangement of the lenses and draw the paths of two rays through the lenses, illustrating the formation of the final image of an object. [L.U.

12. Explain the action of the astronomical telescope, giving a careful diagram showing the passage of rays of light through it. [L.U.

13. On a diagram, drawn to a scale of 1 in 3, show how two converging lenses of focal lengths 3 in. and 15 in. respectively should be arranged for use as a telescope. Explain the function of each lens, using the diagram to illustrate your answer.

Give *one* reason why such a telescope would not be used for viewing the stage in a theatre and *one* reason why it would be more suitable for astronomical observations. [J.M.B.

14. Describe arrangements for projecting large images of (*a*) transparent slides, (*b*) opaque objects.

CHAPTER XXIX

COLOUR

On page 308 the band of colours formed by passing a beam of light through a prism was noticed. You will probably also have noticed colour effects in connexion with images formed by lenses, and may have noted coloured borders to images seen when using cheap microscopes and telescopes, and you will certainly have observed the formation of the coloured rainbow when sunlight passes through raindrops. It may, too, have struck you as being strange that by reflection of white sunlight from various objects we obtain all the varied colour effects of nature.

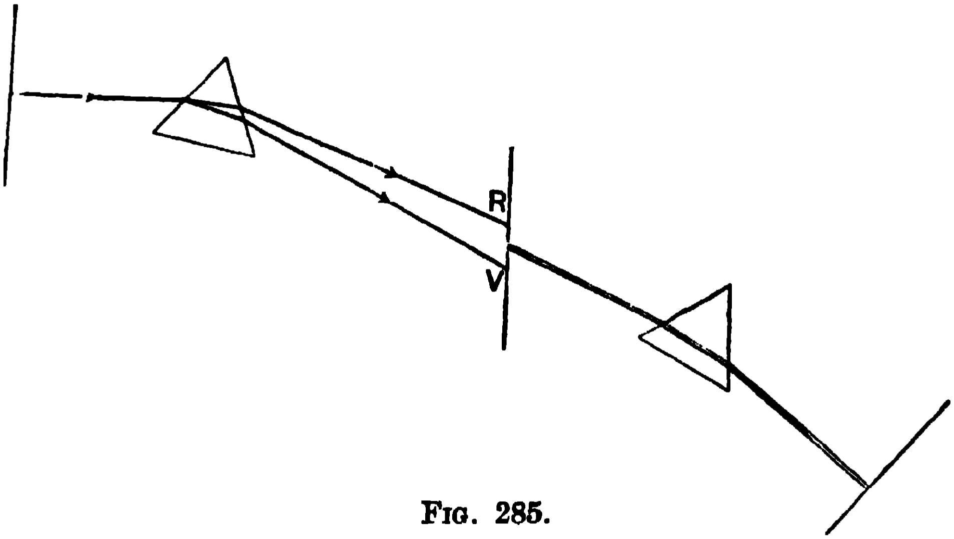


FIG. 285.

The Nature of White Light

Newton was the first to investigate the coloured bands formed by passing white light through a prism. He placed a prism in the path of a narrow beam of sunlight admitted to a dark room through a small hole in the window shutter. On a screen placed on the other side of the prism he observed a band of coloured patches, showing the colours of the rainbow in the same order as in the rainbow. A band of colours formed in this way is called a **spectrum**. Newton gave the list of spectrum colours, in order, as *red, orange, yellow, green, blue, indigo,* *violet*. These colours merge into one another, and most people cannot distinguish indigo as a distinct colour between blue and violet, so it is often left out of the list nowadays.

To test whether the prism gave the colour to the light or whether it merely separated colours already present in the white light, Newton made a small slit in the screen on which the spectrum was formed thus allowing a beam of a certain colour to pass through it. A second prism was placed in the path of this beam which, while it was deviated, underwent no further colour change. Newton therefore concluded that white light was a mixture of lights of the various colours, and that the prisms merely separated these colours by deviating the red light least and the violet light most, as indicated in Fig. 285, other colours being deviated to varying intermediate extents.

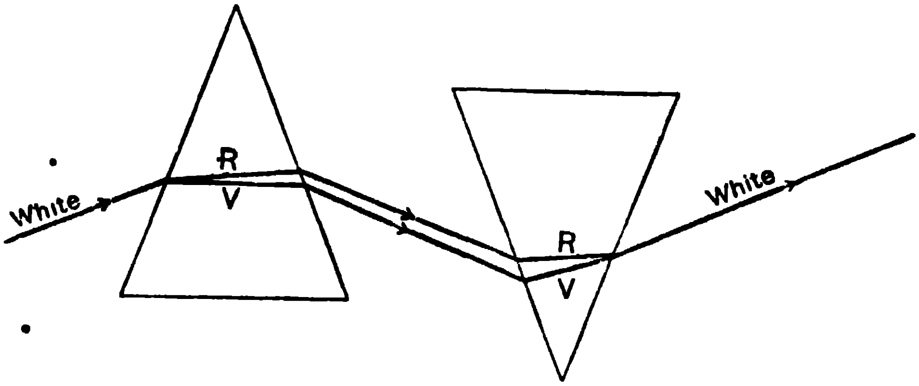


Fig. 286.

This view of the nature of white light was confirmed by Newton by the experiment illustrated in Fig. 286. White light, having been **dispersed** into its component colours by one prism, was passed through a second prism exactly like the first one but placed in an exactly reversed position. This, by deviating the coloured beams to unequal extents in the reverse direction brought them into the same path as one another, and a beam of white light emerged.

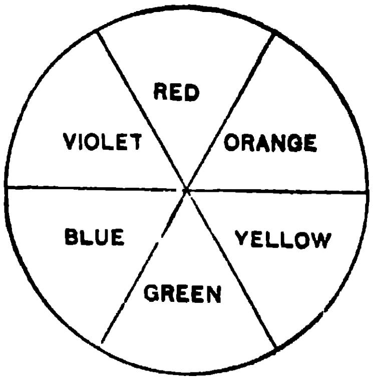


Fig. 287.

This composition of white light from light of the spectrum colours may be shown by dividing a disc of cardboard into six sectors and painting them as indicated in Fig. 287. If the card is rapidly rotated it will appear to be almost white.

Pure Spectrum

In the spectrum formed as described on page 350, the bands of colour overlap. They can be obtained more clearly defined, forming a pure spectrum, by using a lens as indicated in Fig. 288. A narrow beam from a source of white light is allowed to pass through a **narrow** slit in a screen. The **convex lens** is placed so that it forms a sharp image of the slit on a screen at A. The prism is then placed so that it is in the **position of minimum deviation** for the mean direction of the beam. The screen is then moved to position B and a sharp image of the slit in each colour will be formed on it. Provided that the slit is very narrow these images will be sufficiently separated not to overlap.

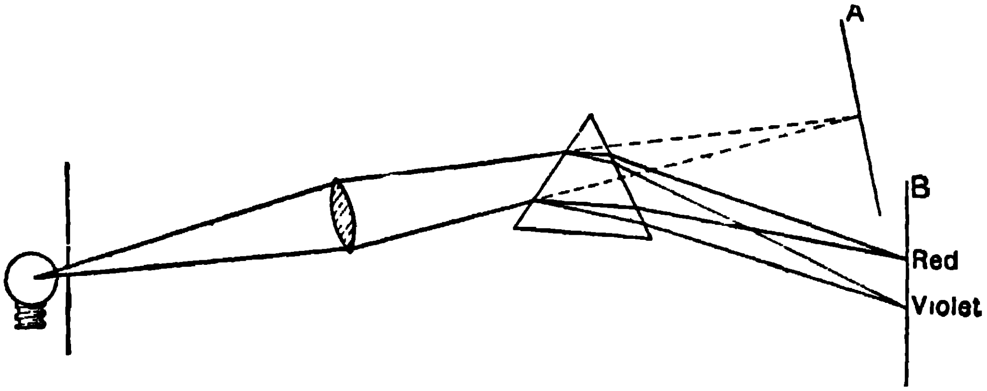


FIG. 288. FORMATION OF A REAL PURE SPECTRUM.

Colour and Index of Refraction

It will be clear from the preceding paragraphs that the dispersion of light by a prism is due to the fact that the amount of refraction which takes place when light passes from one substance to another depends on the colour of the light, violet light being refracted more than red light under the same conditions. It follows that a substance has a number of indices of refraction—one for each colour of light. The indices generally used are mean indices, that is, average indices for light of all colours.

A parallel-sided plate does not cause dispersion, for the refraction at the second face is just equal and opposite to that at the first (see page 308), so that all the coloured beams are brought into one path as in the experiment illustrated in Fig. 287, where the two prisms would form a parallel-sided plate if moved until their faces were in contact.

Dispersion by Lenses

Since the action of a lens depends on deviation produced by refraction at its two faces, a lens will cause dispersion of white light. Thus, if light diverging from a white source passes through a convex lens, the violet component, being most *refrangible*, will be brought to a focus nearest to the lens, whilst the red, being the least refrangible, will come to a focus further away, other colours being focused to intermediate points. Thus, if a screen is placed at A (Fig. 289) an image of the slit with a violet centre and red edges will be obtained; but if the screen is placed at B the image will have a red centre and violet edges.

This formation of coloured fringes in lens images is called **chromatic aberration**, and frequently causes lack of sharpness of images formed by instruments in which lenses are employed. By suitable combinations of lenses made of different kinds of glass chromatic aberration can be prevented, and the better cameras, microscopes, telescopes, etc., have such **achromatic** combinations of lenses instead of simple lenses.

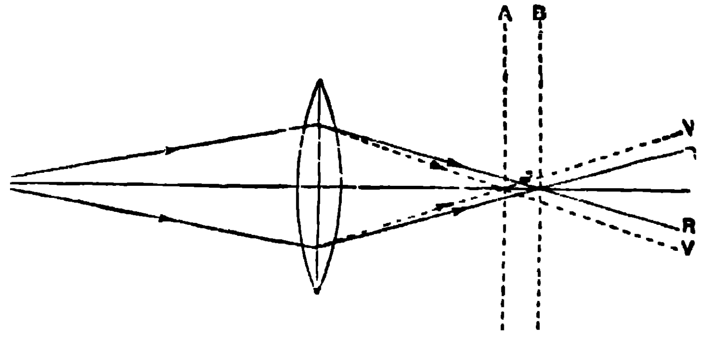


Fig. 289.

Complementary and Primary Colours

If the red sector is cut out of the disc shown in Fig. 287 and the remainder is rotated, it will appear of a greenish-blue colour known as peacock blue. Thus it may be inferred that peacock blue and red light when mixed form white light. Any pair of colours which do this are said to be **complementary**. Other complementary pairs may be obtained by rotating discs with other missing sectors.

Complementary Pairs of Colours

Red and peacock blue
Green and magenta
Blue and orange.

These complementary properties may also be demonstrated by the use of colour filters, which are sheets of gelatine so treated that each lets through a particular type of light, absorbing other types. Thus.

if a green filter is put in the end of a ray box it will let out green light only, and if this light falls on a white screen the illuminated patch will appear to be green. Similarly a magenta screen placed in another box will produce a magenta patch on the screen. If the two patches are allowed to overlap, the part where they overlap will appear to be white.

The effect of mixing colours which are not complementary may also be investigated in the same way. Thus by allowing blue and red patches to overlap magenta may be produced.

A colour which cannot be obtained by mixing other colours is said to be a primary colour. Others are mixed colours. The artist speaks of red, yellow, and blue as the primary colours because he cannot make pigments of those colours by mixing the other pigments in his box. Experiments on mixing coloured light, however, show that yellow can be obtained by mixing red and green, and that violet rather than blue is the primary colour. Also green cannot be formed by mixing light of other colours. Thus to the physicist the primary colours are red, green, and violet. The following table gives the results of some colour mixings:— Very important.

Colours Mixed	Result
Blue + red	Magenta
Blue + green	Peacock blue
Red + green	Yellow
Magenta + green	White
Blue + red + green	White
Peacock blue + red	White
Yellow + blue	White

Note that 4, 5, 6, and 7 are all really the same, since magenta is blue + red, peacock blue is blue + green, and yellow is green + red.

Colour by Subtraction

If a blue and a red filter are put together in the end of a ray box, no light will come out and the filters will appear black. Suppose the light meets the blue filter first. Only blue light passes through it. This blue light then meets the red filter through which only red light can pass, and so no light gets through both filters.

If a peacock-blue and a yellow filter are put together green light will come out and illuminate a green patch on a white screen. The peacock blue filter transmits a mixture of green and blue light. Since yellow is a mixture of green and red, the yellow filter will transmit the green but stop the blue, and so only green light emerges.

An interesting result can be obtained by mounting overlapping magenta, peacock-blue, and yellow filters on a lantern slide as shown in Fig. 290. When the slide is placed in the lantern the image on the screen shows segments of colours as indicated in the diagram. Note that the segment in the middle is black because no light of any kind can pass through it.

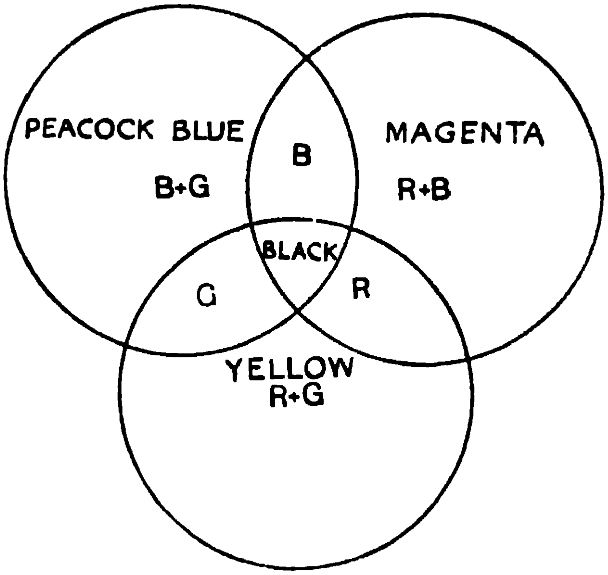


FIG. 290.
SUBTRACTION OF COLOURS BY FILTERS.

Colours of Objects

The fact that most objects are seen by means of light which they reflect has been mentioned a number of times.

Produce a pure spectrum and move a piece of white paper about in it. It will always appear to be the colour of the part of the spectrum in which it is placed. This is because it can reflect any kind of light, so that, for example, when it is placed in red light it reflects red light to the eye and appears red.

Now move a bright red poppy petal about in the spectrum. It will appear to be red while in the red part, but will appear black in any other part. Red light is the only kind of light it can reflect. When put in light of another colour it absorbs that light, so that no light is reflected to the eye.

Test pieces of other coloured materials in the same way. Some may show colour in more than one part of the spectrum. For example a yellow body may appear yellow in the yellow part, red in the red part, and green in the green part since it is capable of reflecting red and green as well as yellow light. The colour of such a body is not a true spectrum colour.

It will be seen from these results that *the colour of a body depends partly on the kinds of light which it absorbs and reflects, and partly on the kind of light falling on it.* A pure green body reflects green light and absorbs light of all other colours so it will appear green in any kind of light containing a green component but black in any light from which green is absent. Interesting effects can be obtained by applying this principle. For instance, make a drawing on white paper, some features being drawn with red crayon and others with green. When it is illuminated with red light the red features will be indistinguishable from the background which will reflect red light, and the green features only will show up as black on a red background. In green light the red features will show up black on a green background. In white light, of course, the red and green features are seen on a white background. Use is sometimes made of this in connexion with theatrical scenery. By changing the light with which the scenery is illuminated a totally different scene can be produced.

Few bodies have pure colours, but usually can reflect light of more than one colour. As a body is usually seen in white light, the mixed colour produced by all the components it will reflect is regarded as its natural colour. Artificial light is seldom pure white. It may be deficient in certain colours or may have an excess of some particular colour. Thus, objects often appear to have a colour different from their natural colour when viewed in artificial light. This is very noticeable in the light from mercury vapour or sodium vapour lamps now used to light many main roads. Light from a mercury vapour lamp has very strong green, yellow, and blue components, whilst that from a sodium vapour lamp is strongly yellow.

It is well known that colours matched in artificial light often appear to differ considerably in daylight. Thus, in a light which contained an excess of red and a deficiency of blue, objects coloured red, white, and magenta might all appear to match since all would reflect red light.

PIGMENTS.—The mixing of pigments should not be confused with the mixing of colour. The artist regards green as a mixed colour because he produces green pigment by mixing blue and yellow. The explanation is that neither his blue nor his yellow pigment has a pure colour. The blue pigment will reflect green light as well as blue, as may be shown by its green appearance in green light. The yellow pigment reflects green and red light. When they are mixed the blue pigment absorbs the red light and the yellow one the blue light so that the only colour which is not absorbed by one or the other and can be reflected by the mixture is green (see Fig. 291).

COLOUR OF TRANSPARENT SUBSTANCES.—A stained glass window which appears richly coloured when seen from inside a church during the daytime usually appears dull and uninteresting when viewed from outside or when viewed from inside by artificial light at night time. In the latter case the colours show up when the window is viewed from the outside. Thus the apparent colour of the glass depends on whether the eye receives light which has passed through it or light reflected from it.

A coloured transparent substance usually reflects light of certain colours, absorbs light of other colours, and transmits certain colours. Thus a piece of glass might reflect red light, absorb orange, yellow, green, and violet, and transmit blue light. If light which had passed through it entered the eye it would appear to be blue, but on looking at it with one's back to the light so that it was seen by reflected light it would appear to be red. This effect is well shown by a solution of fluorescein in alkali which appears red when held up to the light but green when viewed by reflected light. Such a body is generally said to have the colour of the light it transmits.

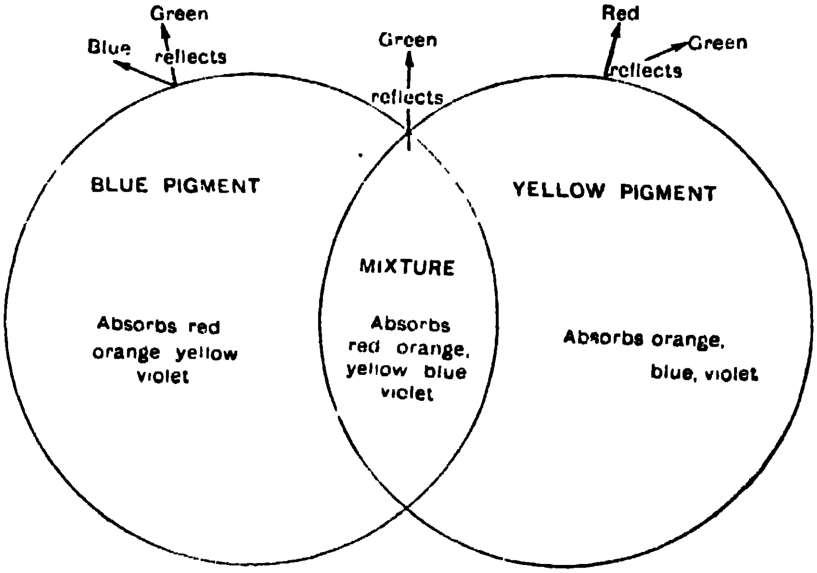

Green
Blue
reflects
Green
reflects
Red
Green
reflects
BLUE PIGMENT
YELLOW PIGMENT
MIXTURE
Absorbs red orange yellow violet
Absorbs red orange, yellow blue violet
Absorbs orange, blue, violet

FIG. 291.

Colour and the Eye

The retina contains a number of nerve ends which are sensitive to light falling on them. Some are specially sensitive to red light, some to green light, and some to violet light. When light enters the eye, the three sets are stimulated to different degrees according to the proportion of red, green, and violet constituents in the light, and so various colour sensations are produced.

In some persons one set does not work, and **colour blindness** results. The commonest form of colour blindness is one in which either the red or the green set is missing, so that reds and greens are often confused and may be matched with one another. In some cases two sets are missing so that differences of colour only give the sensation of different shades, and any colour will be matched against some shade of grey. Many people suffering from colour blindness are unaware of the fact since they can distinguish as different shades those things which other people refer to as being of different colours. Only careful colour matching tests can reveal the defect. The importance of detecting it in people such as engine drivers and motor drivers who must be able to distinguish colour signals is obvious.

QUESTIONS ON CHAPTER XXIX

1. Make a clear drawing, with brief description, of the arrangement of apparatus suitable for showing the composite character of white light.

Mention any common phenomena which lead you to infer that the components of daylight and gas light differ from one another. [L.U.

2. Briefly describe experiments by which Newton demonstrated the composite character of white light.

3. Explain what you mean by complementary colours. How could you demonstrate that green and magenta are complementary colours, and how could you find other complementary pairs.

4. Explain the coloured edges often seen when an object is viewed through a telescope.

5. Why does a piece of paper appear white, a rose appear red, and a leaf appear green in daylight?

How would you expect them to appear when viewed (*a*) in green light, (*b*) in red light, (*c*) through green glass, (*d*) through red glass, (*e*) through blue glass.

6. Describe and illustrate with a diagram the arrangements you would make to project a pure spectrum of white light upon a screen. Describe the appearance of the spectrum and explain why the screen must be white. [L.U.

7. Illustrate by a carefully drawn diagram the formation of a pure spectrum from an arc lamp.

What would be the effect of substituting for the arc a Bunsen flame coloured yellow by a sodium salt? [L.U.

8. How would you produce the colours of the rainbow in the laboratory? Draw a diagram showing the apparatus you would use to ensure the minimum of overlapping of the colours. [L.U.

9. Describe exactly how you would project a spectrum on to a screen. What further experiments could you carry out with the apparatus you describe in order to account for the colour of (*a*) opaque objects, (*b*) transparent objects. [L.U.

10. A narrow parallel beam of white light falls upon a white screen. Give clear diagrams to show how a glass prism whose angles are 90°, 45°, 45° can be used (*a*) to invert the beam without producing colour, (*b*) to obtain a coloured patch of light on the screen.

In the latter case explain what would be the effect on the coloured patch (i) if the white screen were replaced by a red one, (ii) if the prism were then replaced by a similar prism of higher refractive index. [J.M.B.

11. Explain two of the following:—

(*a*) The images observed when a lighted candle is held between two plane mirrors inclined at 90° to each other.

(*b*) The apparent bending of a straight stick partly immersed in water.

(*c*) The formation of a pure spectrum on a screen.

In each case illustrate your answer with a diagram. [L.U.

12. A thick piece of red glass is backed with silver to form a mirror. When the image of a stick of white chalk in front of the mirror is viewed from a position to one side of it, a faint white image is seen in front of a clearer red image. Explain the formation and colour of each image.

CHAPTER XXX

PHOTOMETRY

Power of a Lamp

Electric lamps are frequently stated to have a **power** of 100 watts, 50 watts, and so on. This is a statement of *the rate at which they consume electrical energy* and, therefore, of the rate at which they give out energy. (See Chapter XLVII.) It is not, however, a true statement of their powers as illuminants. Much of the electrical energy supplied to a lamp is converted into heat, and only a fraction of it is converted into light. This fraction may vary for different lamps, and so a 100 watt lamp is not necessarily twice as powerful an illuminant as a 50 watt lamp. It is clear from this that some method of measuring the actual light producing power of a lamp is necessary.

The **illuminating power** of a lamp is defined as *the rate at which it gives out energy in the form of light.* This might be measured by the number of ergs of light energy it emits per second. Such measurements are difficult, and it is more convenient to compare the lamp with a **standard candle.** This is defined by Act of Parliament as a spermaceti candle weighing $\frac{1}{6}$ lb. and burning 120 grains per hour. A lamp which gives out 6 times as much light per second as a standard candle is said to be of 6 candle-power. It is difficult to produce candles which are exactly alike and to keep them burning at exactly the same rate under different conditions, so other standards have been devised for practical use. In Britain the **Vernon Harcourt lamp** is much used. This is a lamp which burns the kind of paraffin known as *pentane,* and if made according to certain specifications, is of 10 candle-power. Standard electric lamps which have a known candle-power when a fixed voltage is applied to them have also been devised.

Most sources of light do not emit light equally in all directions so it is usual to-day to measure the **luminous intensity** in a given direction of a source. This is defined as the rate at which light energy flows from the source in the given direction and is measured by the amount of such energy per unit solid angle emitted per second in that direction.

(NOTE.—A unit solid angle is the angle of a cone which has its vertex at the centre of a sphere of 1 cm. radius and which cuts off 1 sq. cm. on the surface of the sphere, Fig. 291 (*a*). Since the whole surface area of such a sphere would be 4π sq. cm. it follows that the total solid angle surrounding the centre of the sphere, or any other point, is 4π units.)

The rate of flow of the light energy is measured in **lumens,** a lumen being the rate per unit solid angle at which light flows from a source of one candle power giving out light uniformly in all directions. It follows that the total rate of flow in all directions from such a source is 4π lumens. If any source of light emits light energy in any direction at the rate of x lumens per unit solid angle the flow in that direction is x times what it would be from a uniform source of one candle power, so the given source may be said to have a luminous intensity of x candle power in the given direction.

Intensity of Illumination

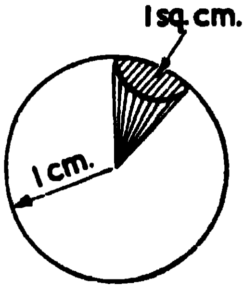


FIG. 291 (*a*).

A page of print can usually be read more easily in some positions than in others in a room, and this is clearly due to more light reaching the page in one position than in another. *The quantity of light energy reaching each unit area of a surface per second* is called the **intensity of illumination** or simply the **illumination** of that surface. Measurement of this quantity is important to lighting engineers so that at each point in a building they may arrange for an intensity of illumination suitable to the work to be carried out there. Again, it is convenient to use a practical unit instead of the usual energy units. The intensity of illumination which would be produced on a surface perpendicular to the incident light by one standard candle one foot from it is called one **foot-candle.** If the surface receives light at twice the rate at which it is received under the above conditions, the intensity of illumination is two foot-candles, and so on. Metre-candles and cm.-candles may be similarly defined.

It is becoming usual to describe the illumination of a surface as being measured by the number of lumens falling on unit area of it. A sphere of one foot radius would have a surface area of 4π sq. ft. Therefore, if a uniform source of one candle power were placed at its centre, light energy would flow on to each square foot of the surface at the rate of one lumen. Thus an illumination of one lumen per sq. ft. is identical with one foot-candle. Similarly, an illumination of one lumen per sq. metre is identical with one metre-candle. This latter illumination may be referred to as one **lux.**

It will be evident that *the intensity of illumination of a surface depends on the power of the source of light illuminating it,* and a little thought will lead to the conclusion that the intensity will be proportional to the power of the source. For example, if two standard candles are placed together objects around them will receive twice as much illumination when both are lighted as when one only is lighted.

The intensity of illumination of a surface will also depend on its distance from the source of light. This follows from the fact that the light spreads out as it travels from the source, and therefore light which would be intercepted by the surface when near the source will pass by it when it is at a distance. (Compare *a* and *b*, Fig. 292.) *It also depends on the angle at which the light is incident on the surface.* (Compare *b* and b_1, Fig. 292.)

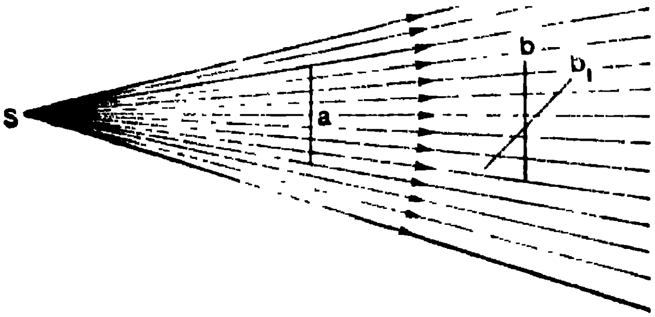


FIG. 292.

Inverse Square Law

Cut a square hole with sides 1 in. long in a sheet of cardboard, and stand the sheet 1 ft. from a small electric lamp. Behind it and parallel with it place another sheet 2 ft. from the lamp. A square patch of the second sheet will be illuminated by the light which passes through the hole in the first screen. Measure the side of the patch and it will be found to be 2 in. Hence the area of the patch is 2^2 sq. in. Hence light which passes through 1 sq. in. of the first screen is spread over 4 sq. in. on the second one, and each sq. in. of the second screen receives only $\frac{1}{4}$ of the light falling on 1 sq. in. of the first one. That is, the intensity of illumination at a distance of 2 ft. from the source is $\frac{1}{4}$ of the intensity at a distance of 1 ft.

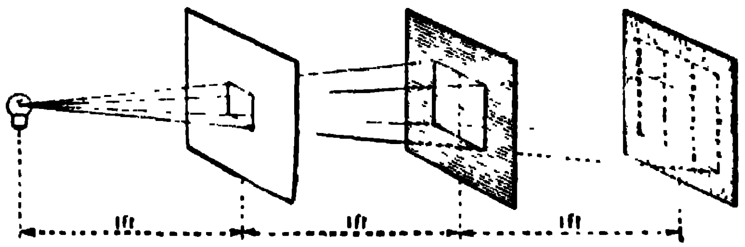


FIG. 293.

If the second screen is moved back until it is 3 ft. from the lamp, the side of the illuminated patch on it will be 3 in., so that its area is 3^2 sq. in., and the intensity of illumination on it is $\frac{1}{3^2}$ of the intensity at a distance of 1 ft. In this way, taking the intensity of illumination at 1 ft. from the lamp as being 1 unit, a table such as the following may be obtained:—

DISTANCE FROM LAMP	SIDE OF SQUARE	AREA OF SQUARE	INTENSITY OF ILLUMINATION
1 ft.	1 in.	1 sq. in.	1
2 ft.	2 in.	2^2 sq. in.	$\frac{1}{2^2}$
3 ft.	3 in.	3^2 sq. in.	$\frac{1}{3^2}$
4 ft.	4 in.	4^2 sq. in.	$\frac{1}{4^2}$
5 ft.	5 in.	5^2 sq. in.	$\frac{1}{5^2}$

From the results of an experiment of this kind we deduce the following important fact:—

The intensity of illumination on a surface is inversely proportional to the square of its distance from the source of light. This statement is known as the law of inverse squares.

General Relation for Intensity of Illumination

Since the intensity of illumination is directly proportional to the power of the source and inversely proportional to the square of the distance of the surface from the source, results such as the following may be obtained:—

POWER OF SOURCE	DISTANCE OF SURFACE	INTENSITY OF ILLUMINATION
1 c.p.	1 ft.	1 ft.-candle (Definition)
2 c.p.	1 ft.	2 ft.-candles
5 c.p.	2 ft.	$\frac{5}{2^2}$ ft.-candles
5 c.p.	3 ft.	$\frac{5}{3^2}$ ft.-candles
10 c.p.	6 ft.	$\frac{10}{6^2}$ ft.-candles
x c.p.	d ft.	$\frac{x}{d^2}$ ft.-candles

It will be seen that the general relation may be written as follows:—

$$\text{Int. of illumination } (\textit{ft.-candles}) = \frac{\text{Power } (\textit{candle power})}{(\text{Distance})^2 \ (\textit{feet})}.$$

EXAMPLE.—*A lamp gives an intensity of illumination of* 5 *ft.-candles on a surface* 2 *ft. from it. What is its power and what intensity of illumination will it give at a distance of* 6 *ft.?*

Let x c.p. be the power of the lamp. Then:—

$$5 = \frac{x}{2^2}; \quad \therefore x = 20.$$

Let y ft.-candles be the intensity produced at a distance of 6 ft. Then:—

$$y = \frac{20}{6^2} = \frac{20}{36} = \frac{5}{9} = \cdot 56 \text{ (approx.)};$$

$\therefore$ Power of lamp is 20 c.p.

Intensity of illumination at 6 ft. is ·56 ft.-candles.

Photometers

For comparing the powers of sources of light instruments known as photometers are used. These are of various forms, but all are designed to enable a point to be found at which the two sources being compared produce equal intensities of illumination.

(1) **THE WAX-BLOCK PHOTOMETER** (Joly's Photometer).—This consists of two rectangular blocks of paraffin-wax made as exactly alike as possible. These are separated by a thin sheet of tin-foil.

The two sources are placed a fixed distance apart, 1 metre is very convenient, in a dark room, and the paraffin block is moved along the line joining them, with the tin-foil perpendicular to that line, until the two halves of the block, viewed from the side, appear equally bright. As no light can pass through the tin-foil, each half is illuminated only by the source facing it. Hence, when the half-blocks are equally bright, the two sources are producing equal intensities of illumination at the tin-foil. The distance of each lamp from the tin-foil is then measured.

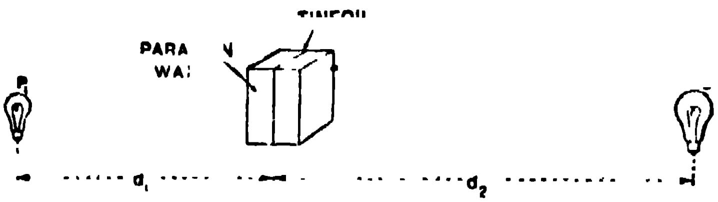


FIG. 294.

Suppose the powers of the two lamps to be P_1 and P_2, and their distances from the tin-foil are d_1 and d_2. Then the intensity at the tin-foil is $\frac{P_1}{d_1^2}$ on one side and $\frac{P_2}{d_2^2}$ on the other. Therefore, when the two intensities are equal

$$\frac{P_1}{d_1^2} = \frac{P_2}{d_2^2}; \quad \therefore \frac{P_1}{P_2} = \frac{d_1^2}{d_2^2}.$$

That is, **the powers of the sources are directly proportional to the squares of their distances from the point where they produce equal intensities of illumination.** In this way the measurements mentioned above enable the ratio of the two powers to be calculated, and if one is a standard lamp of known candle-power, the power of the other can be determined.

The two half-blocks are probably not identically alike, so after taking one set of readings the block should be reversed so that P_1 illuminates the half formerly illuminated by P_2, and a fresh adjustment and measurement should be made. The average distances from the lamps are used in the calculation.

EXAMPLE.—*A standard* 10 *c.p. lamp and a* 500 *watt lamp are placed* 1 *m. apart and a wax photometer is placed between them. The two sides appear equally bright when the tin-foil is* 17·8 *cm. from the standard lamp. On reversing the photometer it has to be moved to a distance of* 18·4 *cm. from the standard for equal brightness. Calculate the candle-power of the* 500 *watt lamp.*

CHAPTER XLI

POTENTIAL AND CAPACITY (CAPACITANCE)

The Electrophorus and the Wimshurst Machine

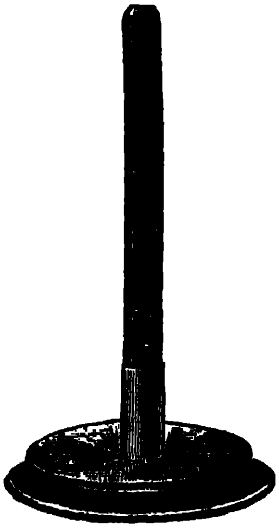
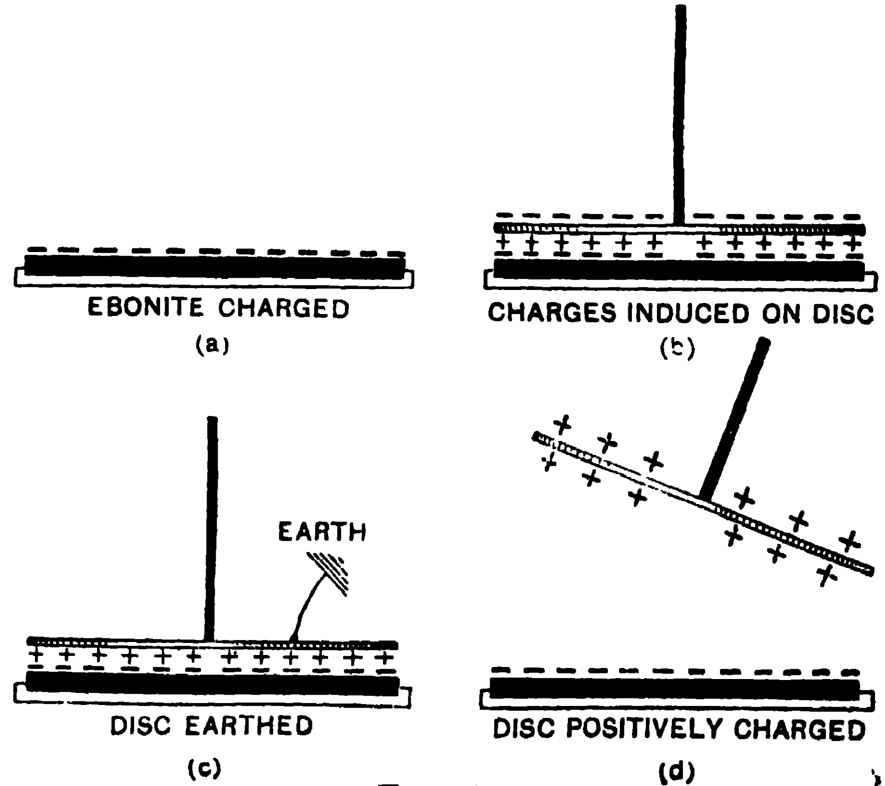


FIG. 404.

A device for securing a large number of similar charges is the **electrophorus.** As shown in Fig. 404 it consists of two parts, a plate of ebonite and a metal disc mounted on an insulating handle. The ebonite is negatively charged by beating with fur. The metal disc is placed on it, momentarily earthed and then removed, holding it by the insulated handle. The disc, if tested, will then be found to be positively charged. Although the disc is placed on the ebonite plate it only makes contact at a few points, and the process of charging it is one of induction as illustrated in Fig. 405. If the disc is now discharged the process may be repeated and theoretically an unlimited number of charges may be obtained in this way. Actually the negative charge gradually leaks from the ebonite disc which requires recharging after a time.



EBONITE CHARGED (a)

CHARGES INDUCED ON DISC (b)

DISC EARTHED (c)

DISC POSITIVELY CHARGED (d)

FIG. 405.

In most cases the ebonite plate has a metal base known as the *sole.* This, by being in contact with the bench, is earthed. As a consequence a positive charge is induced on the surface of the sole in contact with the ebonite, the induced negative charge going to earth (Fig. 406). The attraction exerted by the positive charge on the sole tends to prevent the escape of the negative charge from the ebonite. Frequently a metal pin passes through the ebonite from the sole and just projects above the ebonite surface. This automatically earths the metal disc when it is placed on the ebonite and does away with the need for touching it to earth it.

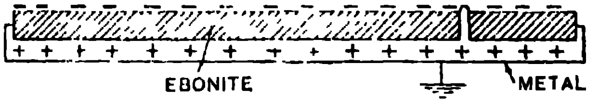


FIG. 406.

The **Wimshurst machine**, which you may have seen in use, is essentially based on the principle of the electrophorus. Full details of its action will not be given here, but if you examine one you will find that it is composed of two insulating discs, face to face, so mounted that they can be rotated in opposite directions. Each disc carries a ring of metal plates on it. Some of the plates on one disc are given a slight charge to begin with. As the charged and uncharged plates come opposite one another inductive effects take place and new charges appear. Two insulated conductors are arranged so that one will collect the positive charges from the plates and the other will collect the negative charges. These conductors may be highly charged by a few turns of the machine.

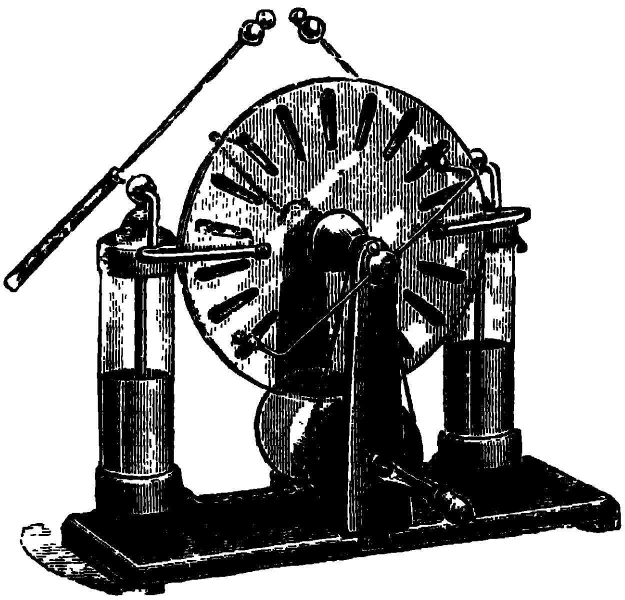


FIG. 407.

Electrical Energy

In collecting a charge a quantity of mechanical work must be done. Imagine a positive charge being built up on an insulated conductor by the gradual addition of small positive charges to it. Each small portion of charge as it is brought up will experience repulsion from the charge already present, and mechanical work will have to be done to bring it up to the conductor against this repulsion. It follows from the law of conservation of energy that the charge must have a store of potential energy equivalent to the quantity of work done in collecting it, and that in discharging it will be able to do an amount of mechanical work equal to that quantity. This quantity of work is said to measure the potential of the charge.

Potential at a Point. Potential Difference

To bring a *unit positive charge* up to any given point in the field due to a positive charge *from a point outside the influence of that field* will require a definite amount of work to be done by some external agency against the repulsion experienced by the unit charge. This amount of work is said to measure the *electrical potential at the point.* If the unit charge is released at the point it will be driven away by the repulsion of the charge to which the field is due until it is once more outside the influence of the field, and an amount of energy equal to the potential of the point will be released during this operation. To bring a positive charge of x units up to the point would require an amount of work equal to x times the potential of the point, and that amount of energy would be released during the removal of the charge from the influence of the field.

If the field is due to a negative charge the work necessary to bring the unit charge to the point will be done by the attraction of the negative charge, and energy will be released from the field. An equivalent amount of work will have to be done by an external agency in removing the unit charge from the influence of the field. In this case the point is said to have a *negative potential.*

POTENTIAL DIFFERENCE.—If two points, A and B, in any field are considered, it will usually require the doing of a certain quantity of work to transfer a unit positive charge from A to B. This quantity is said to measure the potential difference between A and B. If it is done against the forces due to the field B is said to have a higher potential than A, if it is done by the forces due to the field B is said to have a lower potential than A. From this it follows that *positive charges tend to be driven from points of high to points of low potential* when placed in electric fields. Negative charges tend to be driven in the opposite direction, *i.e.* from points of low potential to points of high potential. Note that a potential of — 50 e.s.u. would be lower than a potential of — 10 e.s.u.

Equipotential Surfaces

equipotential

Work is not always done in transferring a charge from one point to another in an electric field. If the journey can be accomplished in such a way that the charge is always moving at right angles to the line of force it meets, it will experience no forces tending to prevent or assist its motion and so no work will be done. In that case there will be no potential difference between the two given points. If a surface is described in an electrical field so that every point on it has the same potential it is an **equipotential surface.** It will be clear from the foregoing that an equipotential surface is everywhere perpendicular to the lines of force meeting it.

A field of force may be mapped by drawing sections of its equipotential surfaces instead of its lines of force. The direction of the electrical force could always be found on such a map as, from a given surface, it would always be towards a surface at lower potential. Also, if the surfaces are drawn so that there is a constant difference between successive pairs, the nearer they are together the greater the intensity of the field, for crowding of the surfaces would indicate the performance of a large amount of work during movement over a short distance which would necessitate a large force being employed In Fig. 408 lines of force are indicated by complete lines and equipotential surfaces by broken lines. Note the line of zero potential passing halfway between the charges, and that the surfaces are most crowded in parts of the field where the lines of force are most crowded.

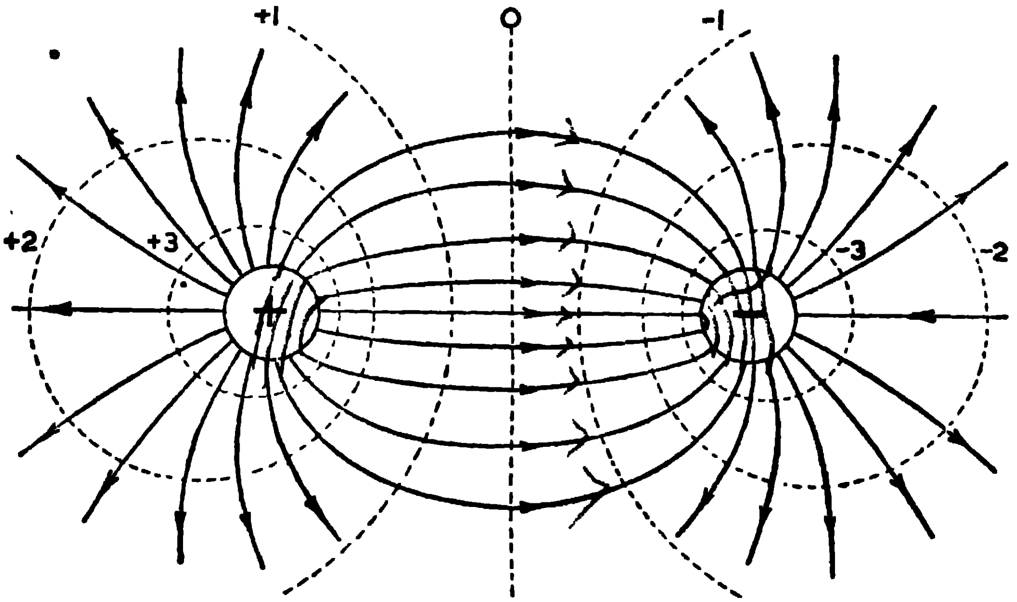


Fig. 408.

Experiments on Potential

Obtain some metal cans of different sizes—cocoa tins, etc., will do. A pair of identically similar gold-leaf electroscopes are also required.

Different cans may be given equal charges by the following method. Charge the ebonite plate of an electrophorus. From it charge by induction a metal disc mounted on an insulated handle, the disc being small enough to go inside the tins. If one of the tins is placed on a slab of wax or ebonite to insulate it and the disc is made to touch the inside of it, it will gain and retain the whole of the charge from the disc. If the process is repeated using another can, the same charge as before will be carried by the disc so the two cans will be equally charged. By discharging the disc a number of successive times inside the same can a charge which is a definite multiple of that carried each time may be given to it.

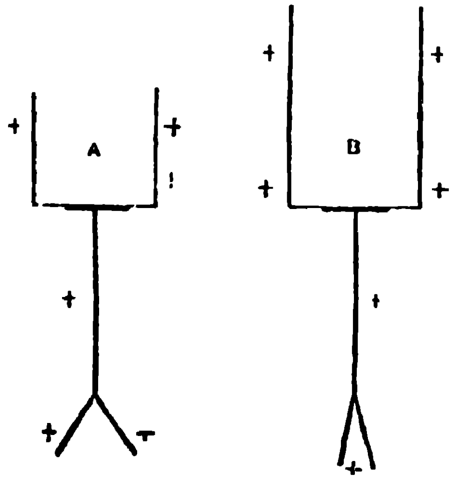


FIG. 409.

Place a small can, A, on the disc of one of the electroscopes, and a larger can, B, on the other. By the method explained above give equal charges to the two cans. It will be found that A causes a greater divergence of the leaves than B (Fig. 409).

Wind the middle of a stout copper wire round an ebonite rod. Lifting the wire by means of the rod place it so that it makes contact with A and B (Fig. 410). The divergence of the leaves below A will decrease and that of the leaves below B will increase, showing that charge is being transferred from A to B. It will also be found that the two electroscopes are now showing equal divergences. Since the charges on A and B were originally equal, it cannot be size of charge alone which determines its flow from one body to another. If we think of each charge as being made up of a number of small units, those units would be more crowded, and hence their mutual repulsions would be greater, on A than on B. Thus we may say that there was an " electrical pressure " tending to drive the charges from A to B. This is what we mean by saying that the charge on A was at a higher potential than that on B. It is clear from this that potential difference is the driving influence which will cause flow of electricity from one conductor to another, and it is to be expected that the flow will cease when the charges on the two conductors are at the same potential.

The experiment also shows that the electroscope is a measurer of potential rather than a measurer of charge. Note that with equal charges there was a wider divergence in the case where the potential was higher (Fig. 409) and with equal potentials the divergences were equal although the charges were unequal.

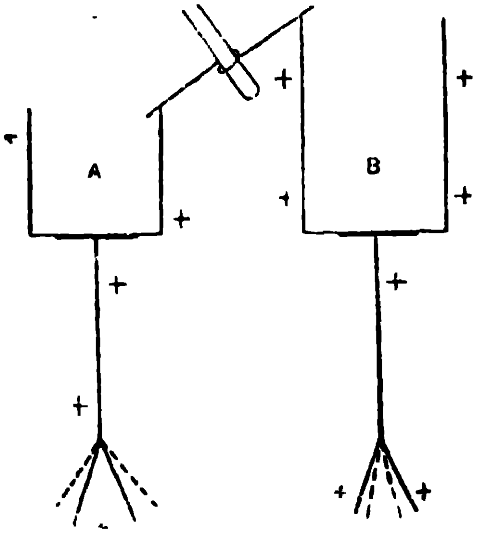


FIG. 410.

Actually, the amount of divergence is a measure of the potential difference between the leaves and the metal case of the electroscope. A positive charge on the leaves will give them a higher potential than the uncharged case, and since positive charges tend to be driven towards points of lower potential, the leaves spread outwards towards the case. If the leaves are negatively charged they will have a lower potential than the case. But negative charges tend to be driven towards points of higher potential, so once more the leaves will spread towards the case.

The dependence of the action of the electroscope on difference of potential may be shown as follows. Charge the electroscope in the usual way, causing a divergence of the leaves (Fig. 411). Stand it on an insulating block and connect the cap to the metal case by means of the wire wound round the ebonite rod. The leaves will collapse and will remain collapsed when the wire is removed. Now earth the metal case and the

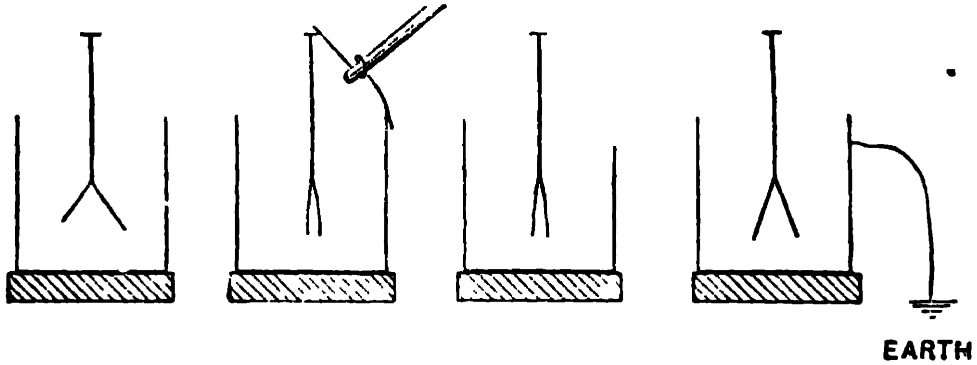


FIG. 411.

leaves again diverge, showing that they still have a charge. When the wire connexion is made, part of the charge from the leaves flows to the case till both are at the same potential. When the wire connexion is removed and the case is earthed, the potential of the case is lowered by loss of charge to the earth, so that it is once more below that of the leaves. Only when the difference of potential exists do the leaves diverge.

Surfaces of Conductors

It may be inferred from the above that **the surface of a conductor is an equipotential surface.** If differences of potential existed on it positive charges would be driven from the points of high potential to those of low potential until the potentials were equalised. Remember this really means that electrons—negative charges—would be driven from the points where potential was low to those where it was high, but it became customary to describe the action as above before scientists had knowledge of electrons and protons.

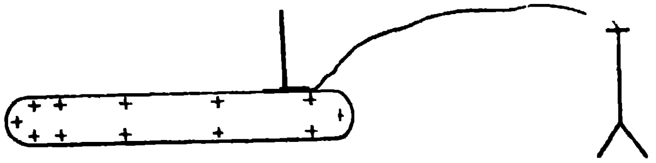


Fig. 412.

This property of the surface of a conductor may be demonstrated by means of the electroscope. Connect the disc of an electroscope to that of a proof plane by means of a flexible copper wire. Touch some part of the surface of a charged conductor with the proof plane and the electroscope leaves will diverge measuring the potential of the point on the conductor touched by the plane. Slide the disc of the proof plane to various points on the surface of the conductor. The divergence of the leaves will remain constant, indicating that there is no change of potential as the disc moves from one part of the surface to another.

Carefully compare this method of using the electroscope to compare potentials with that given on page 479 for comparing charges. The electroscope is really measuring potential when used to compare charges, but if all charges are brought to the same distance from the electroscope disc, a large charge will produce a bigger potential in the electroscope than a small charge and so cause a wider divergence of the leaves.

A still better way of comparing charges carried by a proof plane is to stand a small can on the electroscope disc. Touch the inside of the can with the charged plane so that all charge passes to the can and electroscope, giving them a certain potential and causing the leaves to diverge. Discharge the instrument by earthing and repeat the process after charging the plane again. If the second charge is bigger than the first it will give the instrument a higher potential and so cause a wider divergence of the leaves.

Zero Potential

The earth is a conductor, so its surface is an equipotential surface. Moreover, though it is continually gaining and losing charges, these gains and losses just about balance one another, and the earth is so large that a small gain or loss in its charge has no appreciable effect on its potential which remains practically constant. Hence the potential at the surface of the earth is a convenient standard with which to compare other potentials, and for practical purposes it is taken to be zero. Thus a potential of + 5 e.s.u. means a potential 5 units higher than that of the earth's surface, and one of − 5 e.s.u. means one 5 units lower than that of the earth. Any earth connected conductor will be at zero potential for it and the earth form one conductor. The case of an electroscope is usually earthed, so that it will be at zero potential.

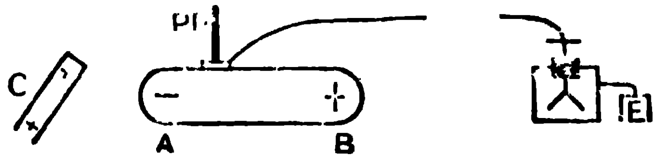


FIG. 413.

Induction and Potential

Induction may be described in terms of potential. If the positively charged body C in Fig. 413 is brought near the insulated conductor AB, A will be at a higher potential than B owing to its shorter distance from C. This will cause electrons to flow from B to A. The negative charge accumulating at A tends to lower the potential there while the positive charge at B raises the potential at B. This proceeds until all points on AB have the same potential once more, although parts of it are differently charged. This may be verified as indicated in the diagram.

Under the conditions shown, AB has a positive potential, so if it is connected to the earth electrons will flow into it from the earth until its potential becomes zero. This means that the positive charge at B is neutralised, and the negative charge on AB just neutralises the positive potential due to C. Under these conditions the electroscope will show no divergence as PP is moved about on AB. If now the earth connexion is broken and C removed, the positive potential due to C disappears, and AB acquires a negative potential due to its own charge. The leaves will now diverge owing to their potential being lower than that of the earthed case.

Capacity (or Capacitance)

The experiment on page 488 shows that different conductors may be raised to different potentials by equal charges. We express this difference by saying that the conductors have different capacities for electricity or have different capacitances. The capacity (or capacitance) of a conductor is measured by **the quantity of electricity which has to be given to it to raise its potential by one unit.** Thus, if 10 e.s.u. of electricity causes a rise of 5 e.s.u. of potential in a conductor, the capacity of the conductor is $\frac{10}{5} = 2$ e.s.u. The general relation is:—

$$\textbf{Capacity} = \frac{\textbf{Quantity}}{\textbf{Potential}};$$

$$\therefore \text{ Potential} = \frac{\text{Quantity}}{\text{Capacity}} \text{ and Quantity} = \text{Capacity} \times \text{Potential.}$$

Sharing of Charges by Conductors

If two conductors at different potentials are placed in contact or connected by a conducting wire there will be a flow of positive charge from the one at higher potential to the other. Also both must come to the same potential and so will finally share the total charge in proportion to their respective capacities.

EXAMPLE.—*A conductor,* A, *of capacity* 20 *e.s.u. with a charge of* + 200 *e.s.u. is placed in contact with a conductor,* B, *of capacity* 10 *e.s.u. and a charge of* − 20 *e.s.u. What charge will each have when they are separated and what will be the potential of each?*

$$\text{Total charge} = +200 - 20 \text{ e.s.u.} = +180 \text{ e.s.u.}$$

Let final charge on A be x e.s.u. Then that on B is $180 - x$ e.s.u.

$$\therefore \text{ Potential of A} = \frac{x}{20} \text{ e.s.u., and Potential of B} = \frac{180 - x}{10} \text{ e.s.u.}$$

But these potentials must be equal;

$$\therefore \frac{x}{20} = \frac{180 - x}{10}; \quad \therefore 10x = 3600 - 20x;$$

$$\therefore 30x = 3600; \quad \therefore x = 120.$$

Hence *Charge on* A = 120 *e.s.u.* *Charge on* B = 60 *e.s.u.*

The common potential $= \frac{120}{20} = 6$ e.s.u.

It is evident that conductors of equal capacities would share a charge equally when placed in contact.

Condensers

By induction give a positive charge to an insulated metal plate which has been connected to the disc of an electroscope. Note the divergence of the leaves. Now place an earthed plate parallel and near to the insulated one. Note that the divergence of the leaves decreases, showing a fall of potential in the charged plate. Since this has occurred without any loss of charge from A, the capacity of A must have been increased by bringing B near it.

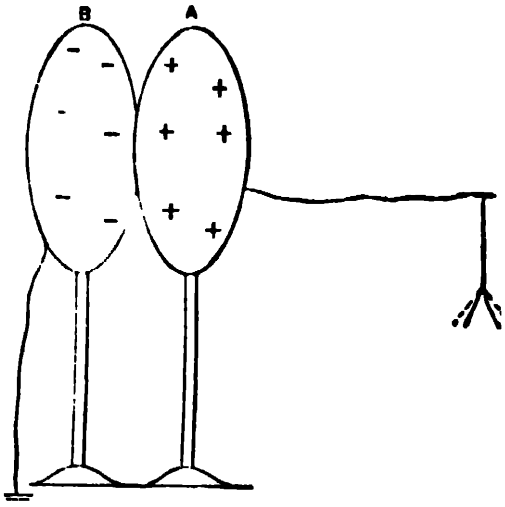


FIG. 414.

The explanation is that the positive charge on A induces a negative charge on B. This tends to set up a field of negative potential around it and so reduces the positive potential at A. Hence more positive charge must be given to A to bring it up to its former potential, so the capacity of A has increased.

Any such arrangement for increasing the capacity of a conductor is called a **condenser**.

Factors Affecting Capacity

Place the plates A and B in Fig. 414 at varying distances apart. Note that the nearer they are together the less the leaves diverge, i.e. *the capacity of* A *increases as the distance between* A *and* B *decreases.*

Slide plates of insulators such as ebonite, glass, or mica between A and B. In each case the introduction of the plate causes the divergence to become less, showing an increased capacity. Thus *the capacity of a condenser depends on the insulator between its plates.* This insulator is generally called the *dielectric* of the condenser, and different dielectrics are said to have different *specific inductive capacities* or *dielectric constants.* That of air is taken to be 1. For any other dielectric it is equal to

$$\frac{\text{Capacity of condenser with given dielectric between plates}}{\text{Capacity of exactly similar condenser with air between plates}}.$$

Most solids have specific inductive capacities or dielectric constants greater than 1.

Stand a tin can on an insulating slab, connect it to the disc of an electroscope, and give it a positive charge. Gradually lower a smaller earthed can into it without allowing the cans to make contact (Fig. 415). Note that the divergence of the leaves decreases as the can is lowered. This indicates that *the capacity of the first can increases as the area of the two surfaces facing one another increases.*

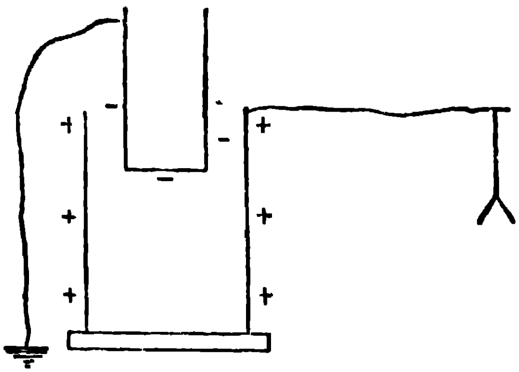


Fig. 415.

From this it is seen that to make a condenser of large capacity we need (1) plates of large area, (2) placed very near one another, and (3) separated by a dielectric of high specific inductive capacity.

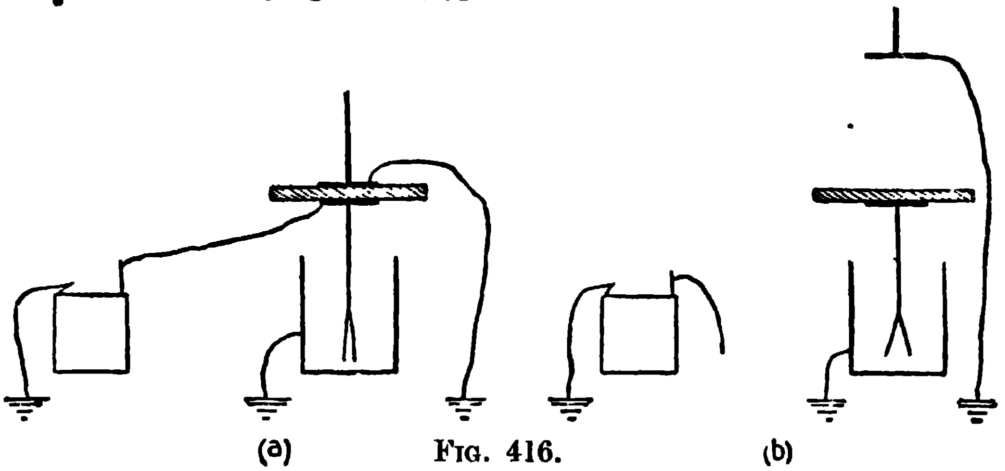
Uses of Condensers

With a condenser of large capacity a very big charge can be accumulated without its producing a high potential. This means that the mechanical work which has to be done to accumulate it is less than would be required to accumulate an equal charge on a conductor of low capacity where it would produce a much higher potential. Thus the collecting conductors of a Wimshurst machine are connected to condensers so that large charges, which will give large momentary flows of electricity on discharging, can be collected on them by means of a small amount of mechanical work.

Condensers are also used in wireless sets where capacity in various parts of the circuits plays a large part in correct "tuning."

Condensing Electroscope

A condenser may be used to enable an electroscope to detect small potential differences. For instance, the potential difference between the poles of a pocket lamp dry battery is too small to affect a gold-leaf electroscope, but it may be made apparent in this way. A thin sheet of ebonite is placed on the disc of the electroscope and an earthed metal plate is placed on the ebonite. The negative pole of the battery is earthed to bring it to the same potential—zero—as the earthed case of the electroscope. The positive pole of the battery is connected by a wire to the electroscope disc which is thus brought to the same potential as the positive pole. The leaves do not diverge visibly as the potential difference between them and the case is too small to cause them to move [Fig. 416 (*a*)].


(a) FIG. 416. (b)

The connexion between the battery and the electroscope is now broken and the earthed plate is raised. This reduces the capacity of the electrometer disc and leaves so that the charge they contain now raises them to a higher potential and the leaves diverge [Fig. 416 (*b*)]. If the earthed plate is always raised to the same height, or, better still, removed to too great a distance to affect the electroscope, the resulting divergencies of the leaves may be used to compare potential differences in different cells.

Types of Condensers

(1) THE LEYDEN JAR.—This is a type of condenser frequently seen in laboratories (Fig. 417). It consists of two metal sheets, one on the inside and the other on the outside of a glass vessel. A metal knob and rod are usually connected with the inner metal coating which is the insulated plate of the condenser. The outer coating is earthed

by its contact with the bench on which it stands. The jar may conveniently be charged by placing the knob in contact with one of the conductors of a Wimshurst machine. For discharging, a pair of metal tongs with an insulating handle are usually provided (Fig. 418). The ends of these tongs are brought into contact with the knob and the outer coating. If one contact is made before the other a spark is usually seen to cross the gap before the second contact is complete. The condensers permanently attached to the conductors of a Wimshurst machine are usually of this type.

An interesting experiment may be carried out by means of a Leyden jar with removable coats. Charge the jar. Lift out the inner coat by means of an ebonite rod. Lift also the glass vessel from the outer coat. Test each of the coatings by taking them near to an electroscope. Neither will be found to carry a charge. Reassemble the jar once more, lifting the inner coating by means of the ebonite rod when replacing it. The jar will still be charged and a spark may be obtained from it. If the jar is very highly charged it may be possible to obtain small sparks by connecting points on the inner and outer surface of the glass with the tongs while the parts are separate.

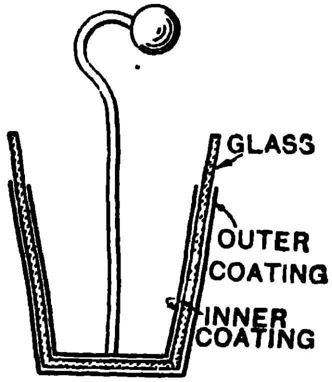


FIG. 417.

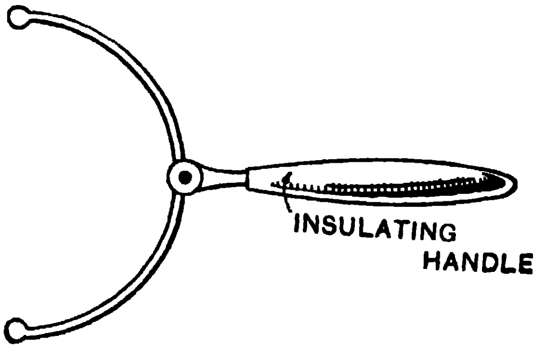


FIG. 418.

These observations show that the two opposite charges which are usually said to exist on the two conductors are really on the surface of the dielectric. The energy given to the condenser in charging it is supposed to be stored in the form of strains in the dielectric due to the attraction of the two charges on its opposite surfaces. If a jar is discharged after being highly charged and then allowed to stand for a time it is frequently possible to get another small spark from it. The dielectric does not lose its strained condition completely during the first discharge, and so retains some of the charge.

(2) Fixed Condensers.—Very compact condensers of high capacity can be formed by building up a pile of alternate layers of tinfoil and thin waxed paper (Fig. 419). The first, third, fifth, etc., sheets of tinfoil are all connected to one terminal, and the second, fourth, sixth, etc., to another terminal. One of these terminals is earthed. Thus the two sets are equivalent to a condenser with very large plates very near together. Condensers of this type are largely used in wireless sets in places where condensers with fixed capacities are required.

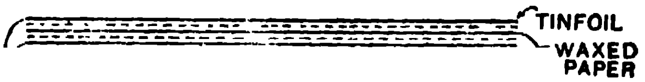


Fig. 419.

(3) Variable Condensers.—A form of variable condenser is shown in Fig. 420. The plates are in two sets which can interlock without touching. One set, all of which are connected to one another, is fixed. The other set, again all connected to one another but insulated from the first set, can be rotated on a spindle. One set is earthed. By turning the spindle the effective area of the plates, that is, the area of one set actually between the plates of the other set, can be varied, and so the capacity of the condenser is variable. Condensers of this type are largely used on the tuning controls of wireless sets.

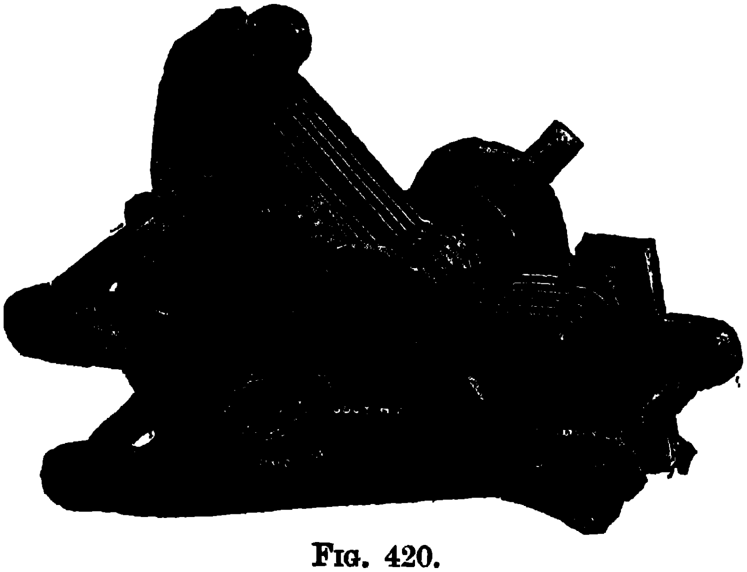


Fig. 420.

Electrical Discharges

Mention has been made of the sparks usually obtained when a condenser is discharged. Sparks of considerable length can be obtained by charging up the condensers of a Wimshurst machine and then bringing the two knobs connected with them towards one another. Still larger sparks may be obtained from an induction coil (page 588). If between two nearby points there is a big potential difference there is evidently a tremendous "pressure" tending to drive charges from one point to another. This may become big enough to drive the firmly-held electrons out of the molecules of the insulating dielectric between the points. The dielectric is then said to "break down" and discharge takes place across the gap, the electrons being driven to the point of high potential, and the positively charged residues of the molecules to that of low potential, so that the potential difference tends to disappear. This releases a very large amount of electrical energy much of which is transformed into heat, light, and sound energy, thus producing the spark and the accompanying crackling sound.

Lightning flashes are discharges of this type on a very large scale caused by the generation of very big potential differences between two neighbouring clouds or between a cloud and the earth beneath it. The quantity of electricity which is discharged in a lightning flash is not as a rule very large, but the potential difference driving it is so great that a tremendous amount of energy is liberated by it enabling it to do serious damage to buildings, etc.

So large a quantity of energy is required to dislodge electrons from atoms of insulating substances that, as a rule, serious material damage does not result from discharges except when they are due to very high potentials. The human body, however, may be seriously damaged by the rapid discharge through it of large quantities of electricity from moderate potentials. The action is not fully understood, but apparently the discharge particularly affects the nerves which control all the organs of the body. Usually a discharge through the body from the ordinary lighting mains gives an unpleasant shock but does not produce serious damage as the body is not a particularly good conductor, and the potential of the mains is not sufficient to cause a heavy discharge through it. If the body is wet, however, a much more serious shock may be experienced since water is a good conductor and causes the contact both with the main and with the earth to be much more complete allowing of more rapid discharge. It is for this reason that any contact with electrical apparatus while one is in the bath is particularly dangerous.

Persons suffering from electric shock should first be removed from contact with the source of electricity, dry insulating material such as rubber being used to handle them while doing this. Artificial respiration should then be applied as in the case of apparent drowning.

Silent Discharge. Action of Points

An insulated conductor charged to a moderate potential will gradually lose any charge given to it. This loss is much more rapid if there are points or sharp edges on the conductor. To show this, take two similar metal balls and solder a piece of pointed wire to one. Mount them on insulating stands and connect them to similar electroscopes. Charge both by contact with the terminal of a Wimshurst machine until there is a considerable divergence of the electroscope leaves. In the case of the plain ball the leaves will slowly collapse, showing the slow loss of charge. The leaves connected with the ball with the point will collapse much more rapidly.

This leakage is due to the charged conductor attracting to its surface particles of dust, moisture, etc., floating in the nearby air. These particles become charged by contact and then are repelled from the surface taking part of the charge with them. Thus the leakage is much more rapid when the air is moist or very dusty than when it is dry and clean.

The process may be illustrated by suspending a gilded pith ball from a silk thread between an insulated and an earthed plate, the thread being long enough for the ball to make contact with each plate as it swings. The insulated plate is charged. The ball is attracted up to it, makes contact and then is repelled, so that it meets the other plate to which it gives up the charge it gained by contact with the first plate. It then swings back and the process is repeated until the insulated plate is discharged.

The rapid discharge when points or sharp edges are present is due to the high surface density of the charge about them. Particles making contact with them are charged up to the same density and so carry away comparatively large charges. It is obvious that conductors on which charges are to be kept should be free from such points and edges, and should be kept free from dust.

The Lightning Conductor

The action of a lightning conductor fixed to a building is based on the action of points. It consists of a stout wire or band of metal connected at the bottom to a metal plate buried in the soil to give good earth connexion and ending in a number of points above the highest point of the building.

If a positively charged cloud should come over the building a negative charge will be induced on the building and the surrounding earth. Negative charge will stream away from the points of the conductor and tend to neutralise the positive charge on the cloud. Thus a lightning flash may be prevented. Even if a flash occurs the conductor provides a better conducting path to earth for it than the material of the building.

QUESTIONS ON CHAPTER XLI

1. Describe an electrophorus and explain its action.

How would you use it to give an insulated conductor a charge of sign opposite to that of the electrophorus conductor ?

Illustrate your answer by diagrams. [J.M.B.

2. Describe and explain fully with good diagrams the action of an electrophorus.

Why can a number of charges be obtained from it after one excitation ? [L.U.

3. If you were provided with an electrophorus, a small copper calorimeter and a gold-leaf electroscope, together with any other necessary apparatus, how would you charge the copper vessel with electricity and investigate the distribution of charge upon it ?

What would you expect to find ? [L.U.

4. Explain what is meant by (*a*) *the potential of a charge*, (*b*) the electric *potential at a point*, (*c*) *potential difference between two points.*

Describe an experiment to show that the same charge may produce different potentials on different conductors.

5. What is meant by an *equipotential surface?* What relations are there between equipotential surfaces and lines of force in an electrical field ?

How would you show by experiment that the surface of a conductor is an equipotential surface ?

6. Explain what is meant by the *capacity of a conductor.*

A conductor of capacity 15 e.s.u. is given a charge of + 45 e.s.u., and one of capacity 20 e.s.u. is given a charge of — 150 e.s.u. What are their respective potentials ?

If they are placed in contact and then separated, what will their common potential and respective charges become ?

7. An electroscope can be used either as a means of comparing quantities of electricity or measuring potentials.

Describe the use of the electroscope for these two purposes, giving one practical example in each case. [L.U.

8. Describe the construction of the electroscope.

Explain how you would use an electroscope (*a*) to determine the sign of an electrostatic charge, (*b*) to compare the magnitude of two charges, (*c*) to determine which of two insulated charged conductors was at the higher potential. [L.U.

9. Explain what is meant by *electric potential.*

Describe how you would decide experimentally whether the potential at a point near to an insulated charged conductor is positive, negative, or zero. [L.U.

10. Define the *capacity* of a conductor.

State the effect upon the capacity of an insulated circular metal plate of (*a*) bringing near to it an earthed metal plate of the same size, (*b*) placing a slab of wax between the two plates.

Describe and explain how you would show these effects experimentally. [L.U.

11. Describe how an ordinary electroscope must be modified so that its leaves may be made to diverge by means of a single voltaic cell.

Explain the principle underlying the action. [L.U.

12. Describe any type of condenser. What is meant by the capacity of a condenser ? What factors determine its capacity ?

Describe fully an experiment which illustrates how the capacity depends on *one* of these factors. [L.U.

13. Describe the Leyden jar, pointing out carefully the factors that affect its capacity.

Explain how you would charge and discharge it. [L.U.

14. Explain why electrostatic apparatus is kept free from sharp edges and points. Describe two experiments in support of your explanation. [L.U.

15. Explain the action of a "lightning conductor," and describe two experiments showing similar electrostatic effects. [L.U.

CHAPTER XLII

ELECTRIC CURRENTS—GENERAL EFFECTS

The Electric Current

Most of the familiar everyday applications of electricity make use of electric currents, that is the flow of electricity along conductors. To produce such a flow there must be a difference in potential between two points in the system of conductors. In cases, already considered, when two conductors at different potentials are connected the flow is only momentary and ceases as soon as there has been a sufficient transfer to bring both conductors to the same potential. To maintain a steady flow there must be some means of counteracting this tendency to equalise potentials and to maintain a potential difference. Voltaic cells and dynamos do this by means which will be studied later. Every such source of current has two poles which the working of the source maintains at different potentials. The one with *higher potential* is called the **positive pole**, and the other is the **negative pole**. To obtain a current the poles are connected by a system of conductors forming the *external circuit*. Electrons will be driven round this external circuit from the negative to the positive pole, but it is more usual to speak of a *positive current* passing round the circuit *from the positive to the negative* pole. The materials in a cell are also conducting substances so that there is an *internal circuit* in the cell, positive current passing from the negative pole to the positive pole in this internal circuit and being raised in potential as it does so.

It is conventional to represent a source of current in a diagram by two parallel lines, a short thick one indicating the negative pole and a longer thin one the positive pole. The direction of the positive current should be represented by arrows on the lines representing the conductors. Fig. 421 (*a*) is a conventional representation of the current through the battery and lamp of a pocket torch. It shows that the battery consists of three cells, each with its positive pole joined to the negative pole of the next one. Fig. 421 (*b*) represents the changes of potential in various parts of the circuit. Note the rise from negative

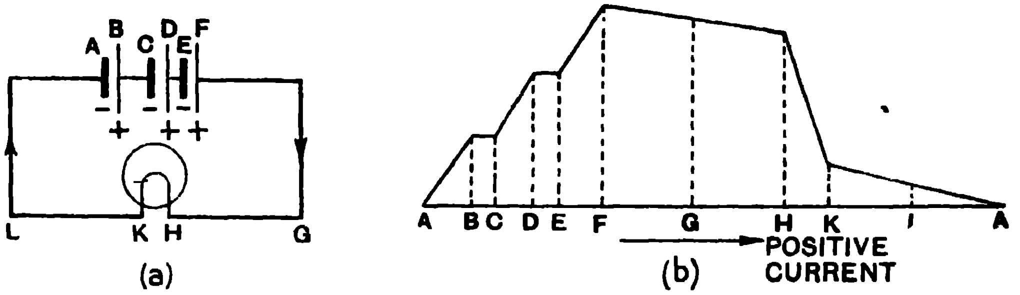


FIG. 421.

pole to positive pole in each cell, the steady fall in the connexions between the cell and the lamp and the much sharper fall in the lamp where most of the electrical energy is released.

Effects of a Current

(1) HEATING AND LIGHTING EFFECTS.—The widespread use of electric lamps, electric stoves, etc., makes it obvious that the electrical energy carried by a current may be converted into heat and light. Heat is always generated when a current passes through a metallic conductor. A piece of thin copper wire—about 30 gauge—joined to the terminals of a 6-volt accumulator by stouter wires soon gets perceptibly hot to the touch. By inserting a coil of varnished constantin wire—about 22 gauge—in a large test-tube full of water and connecting to a 12-volt source of current, the water may be heated, as will be shown by a thermometer inserted into the tube. The varnish insulates the wire from the water.

(2) CHEMICAL EFFECT.—In chemistry lessons you may have seen experiments in which electric currents have been used to decompose chemical compounds. The effect is readily demonstrated by placing two platinum plates so that they dip into dilute sulphuric acid without touching one another. The plates are connected by wires to the terminals of a 6-volt battery and immediately connexion is made bubbles of gas appear on both plates. In such cases electrical energy is transformed into the chemical energy necessary to separate the constituents of the compound.

(3) MAGNETIC EFFECTS.—Support a wire horizontally running in a north-south direction. Below it place a pivoted compass needle which will, of course, set parallel to the wire. Connect one end of the wire to a coil of insulated wire wrapped round a narrow tube. Place the

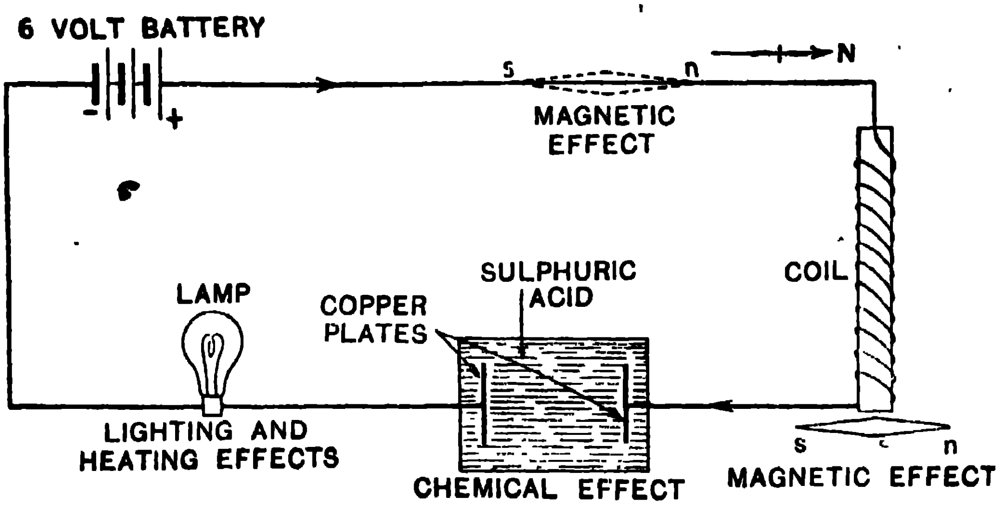


Fig. 422.

coil with its axis in an east-west direction and stand another compass needle opposite one end of it. This needle will set at right angles to the coil. Connect the free ends of the wire and the coil to the terminals of a 2-volt accumulator. As soon as connexion is made both needles will be deflected showing that magnetic fields appear around both the straight wire and the coil immediately a current passes through them.

Each of the above effects of currents will be more fully dealt with in subsequent chapters. Fig. 422 illustrates a circuit which will demonstrate all the above effects.

CHAPTER XLIII

THE CHEMICAL EFFECT OF CURRENTS. ELECTROLYSIS

Many industrial processes, such as silver- and nickel-plating and the manufacture of a large number of chemicals, depend on the fact that a number of substances in solution or in the fused state, particularly those substances known as acids, bases, and salts, conduct electric currents but are themselves decomposed in the process. Such fluids are called **electrolytes,** and the process of decomposing them by the electric current is termed **electrolysis.**

The Voltameter

The vessel in which electrolysis is carried out is called a **voltameter** or an **electrolytic cell.** Two plates of conducting material dip into it and are known as the **electrodes.** They are joined by wires to the poles of a source of current. That which is connected to the positive pole of the source is the **anode,** and that connected to the negative pole is the **cathode** (Fig. 423). Results of experiments, such as that with sulphuric acid mentioned in the last chapter, show that, as a rule, metals or hydrogen are liberated from the electrolyte at the cathode and non-metals or " acid-radicles " at the anode.

Faraday's First Law of Electrolysis

Certain quantitative relations in connexion with electrolysis were investigated in 1833 by Faraday, who established two laws.

First Law.—*The weight of a substance liberated at an electrode is proportional to the quantity of electricity which has passed through the electrolyte.*

Consideration of this law requires an understanding of the terms

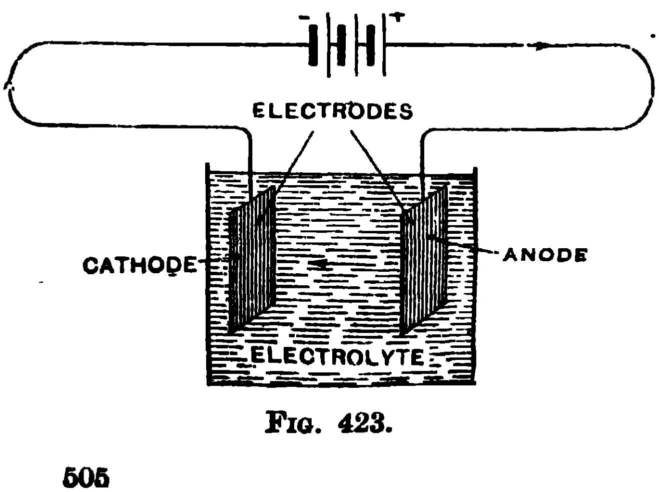


Fig. 423.

" quantity of electricity " and " current " which are not identical. Consider water to be pumped into the vessel A from the pipe X and to be running out through pipe Y (Fig. 424). We should say that a current of water was flowing through the system. The magnitude of the current, that is the rate of flow, would be measured by the quantity of water passing any point in a unit of time, for example the number of pints passing P in one second. If this rate is known the quantity passing in a given time can be calculated from:—

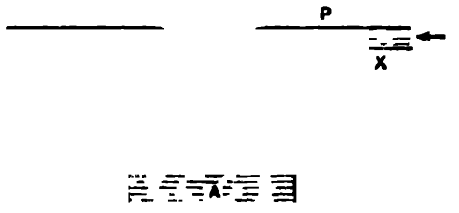


FIG. 424.

Quantity = Current × Time in seconds.

The same considerations may be applied to the flow of electricity round a circuit. When we speak of the magnitude of a current we are speaking of the rate at which electricity passes any given point, for example it might be measured by the number of electrons passing a given point each second. The quantity of electricity passing through any part of a circuit in a given time can then be found from the relation Quantity = Current × Time.

VERIFICATION OF FARADAY'S FIRST LAW.—Set up a **water voltameter** which is illustrated in Fig. 425. The trough contains water made slightly acid by the addition of a little sulphuric acid. This enables it to conduct the current better than pure water. Two long tubes are filled with water and supported inserted in the trough. Into the mouths of the tubes are inserted small pieces of platinum foil to act as electrodes. These are connected to wires which

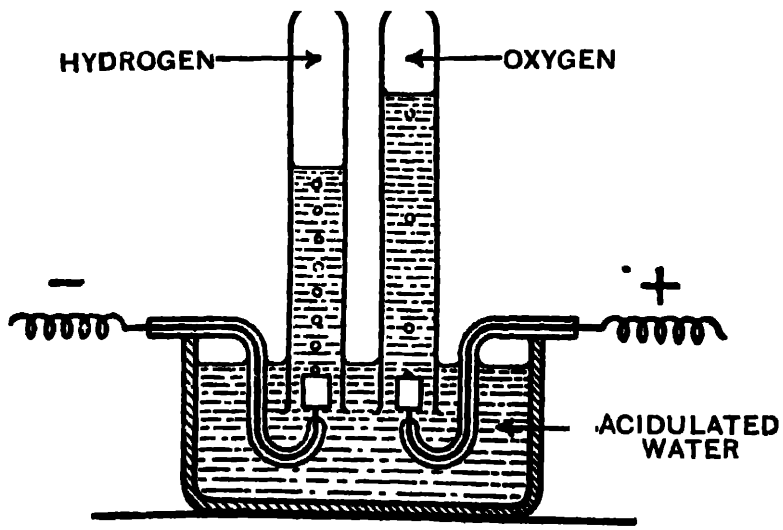


FIG. 425.

have been fused into glass tubes to insulate the parts passing through the water. When the wires are connected to the terminals of a source of direct current hydrogen bubbles will rise from the cathode and oxygen bubbles from the anode. These gases will collect in the upper parts of the tubes. It will be found at the end of a given time that the volume of hydrogen which has collected is double the volume of oxygen liberated.

For this experiment use a source of about 8 volts and include an ammeter and variable resistance in the circuit as shown in Fig. 426. The ammeter measures current and the variable resistance enables it to be controlled. Their action will be explained in later chapters.

Switch on the current, adjust the resistance until the ammeter registers a flow of about 1 amp., and note the time. If necessary adjust the resistance from time to time to keep the current constant. After half an hour mark the points to which the tubes have been filled with gas. Continue for a second half-hour and again mark the points to which the tubes have been filled.

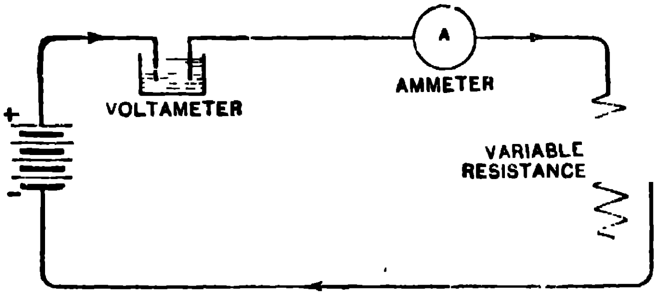


FIG. 426.

Empty the tubes, invert them, and by running in water up to the marks from a burette, measure the volumes of hydrogen and oxygen respectively which were liberated (*a*) in half an hour, (*b*) in an hour. In each case the second quantity will be found to be double the first one. Since the current was constant the quantity of electricity passing in an hour would be double that passing in half an hour. Hence it may be said that the quantities of the substances liberated were proportional to the quantities of electricity passing through the voltameter.

A copper voltameter may be used instead of the water voltameter. In this the electrolyte is a solution of copper sulphate, and the electrodes are plates of copper. When current passes copper is deposited on the cathode. First adjust the resistance so that the current is about one amp. Switch off the current, remove the cathode and wash, dry, and weigh it. The drying should be done by waving it in the warm air some distance above a Bunsen flame. Replace the cathode, switch on, and note the time. Keep the current constant for half an hour and then switch off and wash, dry, and weigh the cathode again. The increase in weight gives the weight of copper which has been deposited.

Repeat, keeping the current at the same value as before for another half-hour. It will be found that the total gain of weight is twice that which occurred during the first half-hour, which shows that the weight of copper deposited is proportional to the quantity of electricity passed.

The experiments may be varied by passing a current of 1 amp. for one half-hour and a current of 2 amps for another half-hour. Twice as much electricity will pass in the second half-hour as in the first, and twice as much substance will be liberated.

Units of Quantity and of Current

In practice it is inconvenient to give measurements of quantity and current in terms of numbers of electrons. Faraday's First Law enables practical units to be defined and provides a means of measuring quantity of electricity and current. It follows from the law that the deposit of a fixed weight of a given substance always corresponds to the passage of a fixed quantity of electricity. Thus by choosing a standard substance and a fixed weight of it we can define a constant quantity of electricity which can be used as a unit. The Board of Trade is responsible for defining units in connexion with the commercial supply of electricity, and it defines the **unit quantity of electricity as that quantity which will deposit 0·001118 grm. of silver from a solution of silver nitrate.** This unit is called a **coulomb.**

A **unit current** is defined as **the steady current which carries one coulomb per second past any given point in its circuit,** or as the current which will deposit 0·001118 grm. of silver in one second from a solution of silver nitrate. This unit is called an **ampere.** It follows from these definitions that:—

$$\frac{\textbf{Coulombs}}{\textbf{Seconds}} = \textbf{Amperes}$$

$$\text{or } \textbf{Coulombs} = \textbf{Amperes} \times \textbf{Seconds.}$$

It should be noted that these units were originally defined from magnetic effects of currents, and the amount of chemical action they would produce was then determined. That is why such an unusual number as 0·001118 occurs in the definition. The advantage of the chemical definitions is that they are more easily understood and applied than the original magnetic ones.

It should also be noted that the coulomb is a much larger unit than the unit of charge used in electrostatics. One coulomb is equal to about 3000 million e.s.u.

Measuring Current by the Silver Voltameter

The silver voltameter, illustrated in Fig. 427, is similar in principle to the copper voltameter. The vessel is a platinum crucible which itself is connected to the negative pole of the source so that it becomes the cathode. The electrolyte is a solution of silver nitrate. In it a silver anode is suspended horizontally from platinum wires. The vessel is cleaned, dried, and weighed. The solution is then poured in, the anode placed in position, and the current passed for a measured time. Silver is deposited on the crucible and its weight is found by washing, drying, and weighing the crucible again. The strength of the current may then be calculated as follows:—

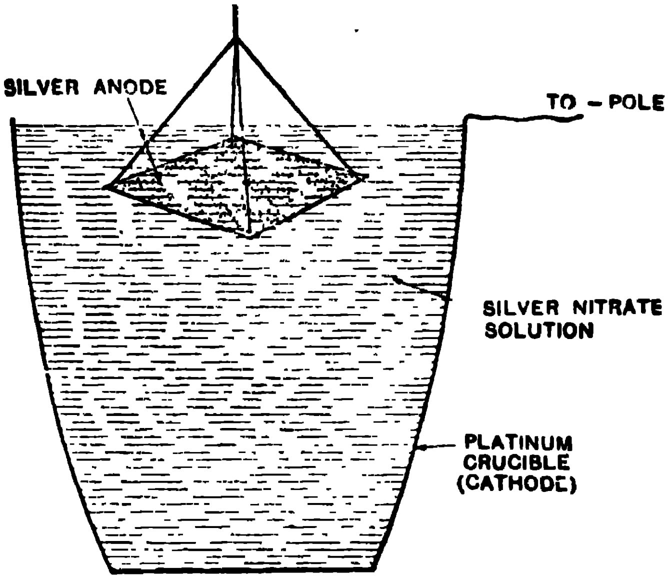


FIG. 427.

Time, 30 min. Increase in wt. of crucible, 1·7 grm.;

$$\therefore \text{ Quantity of electricity passed} = \frac{1{\cdot}7}{{\cdot}001118} \text{ coulombs;}$$

$$\text{Current} = \frac{1{\cdot}7}{30 \times 60 \times {\cdot}001118} \text{ amperes (coulombs per sec.),}$$

$$\textit{i.e.} \text{ Current} = \frac{1{\cdot}7}{2{\cdot}0124} = {\cdot}845 \text{ amps.}$$

Similar measurements may be made with a copper voltameter. The weight of copper deposited by 1 coulomb is 0·0003295 grm.

Faraday's Second Law of Electrolysis

We come now to the second of Faraday's laws referred to on page 505.

SECOND LAW.—*The weights of different substances liberated at electrodes by the same quantity of electricity are in the ratio of their chemical equivalents.*

To illustrate this law set up a silver voltameter and a copper voltameter in series as illustrated in Fig. 428. Since the same current passes through both the same quantity of electricity must pass through both. Find the weights of silver and copper deposited in a given time, and calculate the value of the fraction

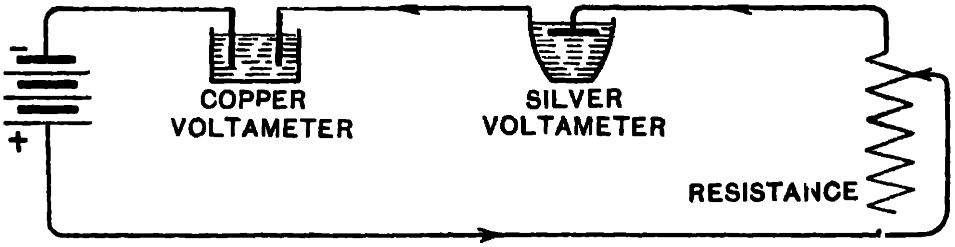


FIG. 428.

$$\frac{\text{Wt. of silver deposited}}{\text{Wt. of copper deposited}}.$$

The chemical equivalent of silver is 108 and that of copper is 31·5, so that

$$\frac{\text{Chemical equivalent of silver}}{\text{Chemical equivalent of copper}} = \frac{108}{31{\cdot}5} \quad 3{\cdot}43.$$

The same value should be found for the fraction determined in the experiment, so—

$$\frac{\text{Wt. of silver deposited}}{\text{Wt. of copper deposited}} = \frac{\text{Chemical equivalent of silver}}{\text{Chemical equivalent of copper}}.$$

Because of this relation the weight of any substance deposited by one coulomb of electricity is called its **electro-chemical equivalent.** It follows from the definition of the coulomb that the electro-chemical equivalent of silver is 0·001118 grm. per coulomb.

The second law may also be illustrated by connecting a copper voltameter and a water voltameter in series. In that case the weights of hydrogen and oxygen deposited would have to be calculated from their volumes, the pressure and temperature at which they were collected being noted. (See Example 2, page 512.) In this case it would be found that:—

Wt. of hydrogen : Wt. of oxygen : Wt. of copper = Chem. equivalent of hydrogen (1) : Chem. equivalent of oxygen (8) : Chem. equivalent of copper (31·5).

Finding Electro-chemical Equivalents

Exact arrangements will depend on the particular case, but the general principle will be to fit up a voltameter from which the required substance will be liberated in a form in which it can be weighed—or measured in the case of a gas.

The voltameter may be connected in series with a silver voltameter and the electro chemical equivalent calculated from the relation (writing E.C.E. for electro-chemical equivalent):—

$$\frac{\text{E.C.E. of substance}}{\text{E.C.E. of silver } (\cdot 001118)} = \frac{\text{Wt. of substance deposited}}{\text{Wt. of silver deposited}}.$$

A copper voltameter may be used instead of a silver one. Electro-chemical equivalent of copper = 0·0003295 grm. per coulomb.

Alternatively, an ammeter may be placed in series with the voltameter and a direct reading taken of the current strength which should be kept constant by means of an adjustable resistance in the circuit. The time for which the current passes is noted. Then:—

$$\text{Electro-chem. equiv. of substance} = \frac{\text{Wt. of substance deposited}}{\text{Quantity of electricity passed}},$$

$$\textit{i.e.}\ \text{E.C.E. of substance} = \frac{\text{Wt. of substance deposited}}{\text{Current} \times \text{Time (in seconds)}}.$$

EXAMPLES.—(1) *When current was passed for a time through copper and silver voltameters in series,* 0·48 *grm. of copper and* 1·62 *grm. of silver were deposited. Calculate the electro-chemical and chemical equivalents of copper.* [*The chemical equivalent of silver is* 108.]

$$\frac{\text{E.C.E. of copper}}{\text{E.C.E. of silver}} = \frac{\cdot 48}{1\cdot 62}; \quad \therefore \frac{\text{E.C.E. of copper}}{\cdot 001118} = \frac{\cdot 48}{1\cdot 62};$$

$$\therefore \text{E.C.E. of copper} = \frac{\cdot 48}{1\cdot 62} \times \cdot 001118 \text{ grm. per coulomb,}$$

$$\textit{i.e.}\ \text{E.C.E. of copper} = \cdot 00033 \text{ grm. per coulomb.}$$

$$\frac{\text{Chem. equiv. of copper}}{\text{Chem. equiv. of silver}} = \frac{\text{E.C.E. copper}}{\text{E.C.E. silver}},$$

$$\therefore \frac{\text{Chem. equiv. of copper}}{108} = \frac{\cdot 00033}{\cdot 001118};$$

$$\therefore \text{Chem. equiv. of copper} = \frac{\cdot 00033}{\cdot 001118} \times 108 = 31\cdot 9.$$

(2) *A current of 2 amperes passing for half an hour through a water voltameter liberated 423 c.cm. of hydrogen measured at 13° C. and 80 cm. pressure. Calculate the electro-chemical equivalent of hydrogen.* [*1 litre of hydrogen at S.T.P. weighs* 0·089 *grm.*]

$$\text{Vol. of hydrogen corrected to S.T.P.} = \frac{423 \times 273 \times 80}{286 \times 76} \text{ c.cm.};$$

$$\therefore \text{ Wt. of hydrogen} = \frac{423 \times 273 \times 80 \times \cdot 089}{286 \times 76 \times 1000} \text{ grm.}$$

$$\text{Quantity of electricity passed} = 2 \times 60 \times 30 = 3600 \text{ coulombs};$$

$$\therefore \text{ E.C.E. of hydrogen} = \frac{423 \times 273 \times 80 \times \cdot 089}{286 \times 76 \times 1000 \times 3600} \text{ grm. per coulomb,}$$

$$\textit{i.e.} \text{ E.C.E. of hydrogen} = \cdot 0000105 \text{ grm. per coulomb.}$$

(3) *For what time must a current of* 2·5 *amps. pass through a solution of zinc sulphate to deposit* 1 *grm. of zinc?* [*E.C.E. of zinc* = 0·0003387 *grm. per coulomb.*]

$$\text{Quantity of electricity required} = \frac{1}{\cdot 0003387} \text{ coulombs};$$

$$\text{Time} = \frac{1}{\cdot 0003387 \times 2{\cdot}5} = 1180 \text{ sec.} = 19 \text{ min. } 40 \text{ sec.}$$

Example 1 should be noted as giving a method by which chemical equivalents of metals may be determined.

Theories of Electrolysis

In Chapter XL. some account was given of the theory according to which atoms are built up of positively charged nuclei and electrons, and electrical charges arise from atoms gaining and losing electrons. This theory can also be used to explain the facts of electrolysis. When an electrolyte is dissolved a certain proportion of its molecules dissociate, that is, each breaks up into two or more parts. These parts are not ordinary atoms or groups of atoms, but bodies which have become positively or negatively charged by losing or gaining electrons. A molecule of common salt contains one atom of sodium and one of chlorine. When these separate in solution the chlorine atom retains one of the electrons from the sodium atom. The dissociation may be represented by

$$\text{NaCl} = (\text{Na} - e) + (\text{Cl} + e),$$

where *e* represents one electron. Hence the sodium particle has a positive charge and the chlorine particle a negative charge. $NaCl = Na^+ + Cl^-$.

These charged particles are called **ions.** Acids, alkalis, and salts all tend to undergo this ionic dissociation in aqueous solution. Probably all the molecules of strong acids and alkalis and of most salts, and not merely a proportion of them, are dissociated in this way when dissolved in water. Metal and hydrogen ions are usually positively charged, while acid radicles and hydroxyl groups (OH) are negatively charged. For example:—

$$HCl = H^+ + Cl^-$$
$$KOH = K^+ + OH^-$$
$$CuSO_4 = Cu^{++} + SO_4^{--}.$$

The number of electrons which may be released by a metal atom corresponds to its valency. Thus the divalent copper atom loses two electrons and the copper ion has twice the charge of a hydrogen or potassium ion. Similarly, the divalent SO_4 group will attract to itself two extra electrons and the SO_4 ion will carry twice the charge of a chlorine or hydroxyl ion.

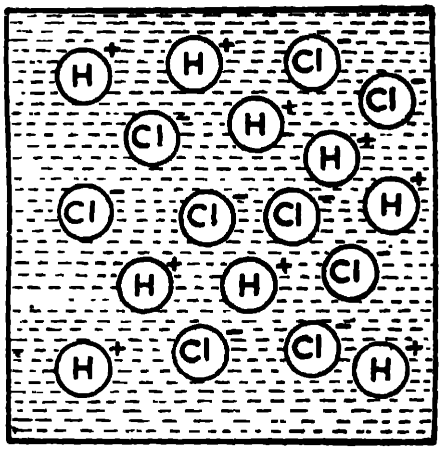


FIG. 429.

In a solution of hydrochloric acid we shall have the conditions indicated in Fig. 429. Instead of molecules there are a number of positively charged hydrogen ions and an equal number of negatively charged chlorine ions. The equality of the numbers of positive and negative charges will make the solution as a whole electrically neutral. The ions are, of course, far too small to be seen and there will be a very large number of them in even a small drop of the solution.

Now a source of current maintains its positive and negative poles at high and low potentials respectively. When electrodes dipping into the solution are connected to those poles, the anode, connected with the positive pole, will have a high potential, and the cathode, connected to the negative pole, will have a low potential. Thus positively charged ions will be driven from the anode towards the cathode, and negatively charged ones will be driven in the opposite direction, as indicated in Fig. 430.

The high potential of the anode is due to electrons having been withdrawn from it by the source of current; the low potential of the cathode is due to extra electrons having been driven to it by the source. When a H^+ ion reaches the cathode it will take one of the excess electrons and become a hydrogen atom. The hydrogen atoms so formed join up in pairs to form molecules and hydrogen gas bubbles from the cathode. When a Cl^- ion reaches the anode it gives up its excess electron and becomes a chlorine atom. The chlorine atoms also join up in pairs to form molecules and chlorine gas bubbles from the anode. At the same time the source withdraws more electrons from the anode and sends more to the cathode. Thus the process continues, and the current is carried through the electrolyte by the movements of the ions.

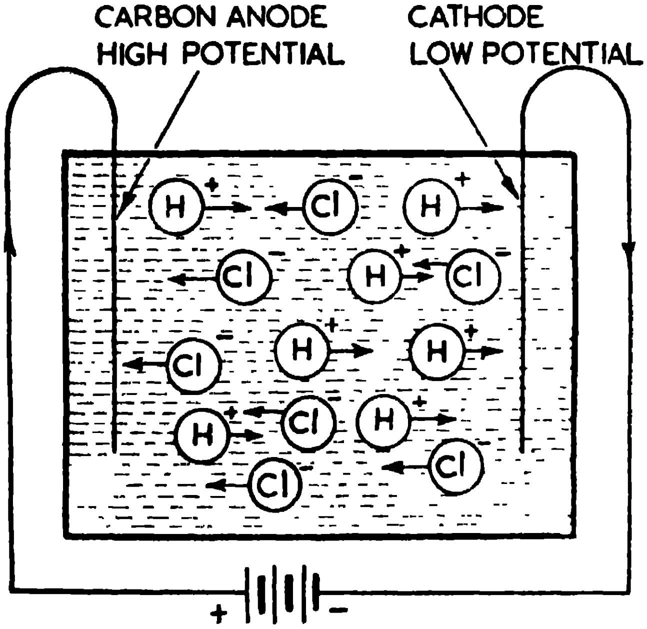


FIG. 430.

Actions at the Electrodes. Secondary Actions

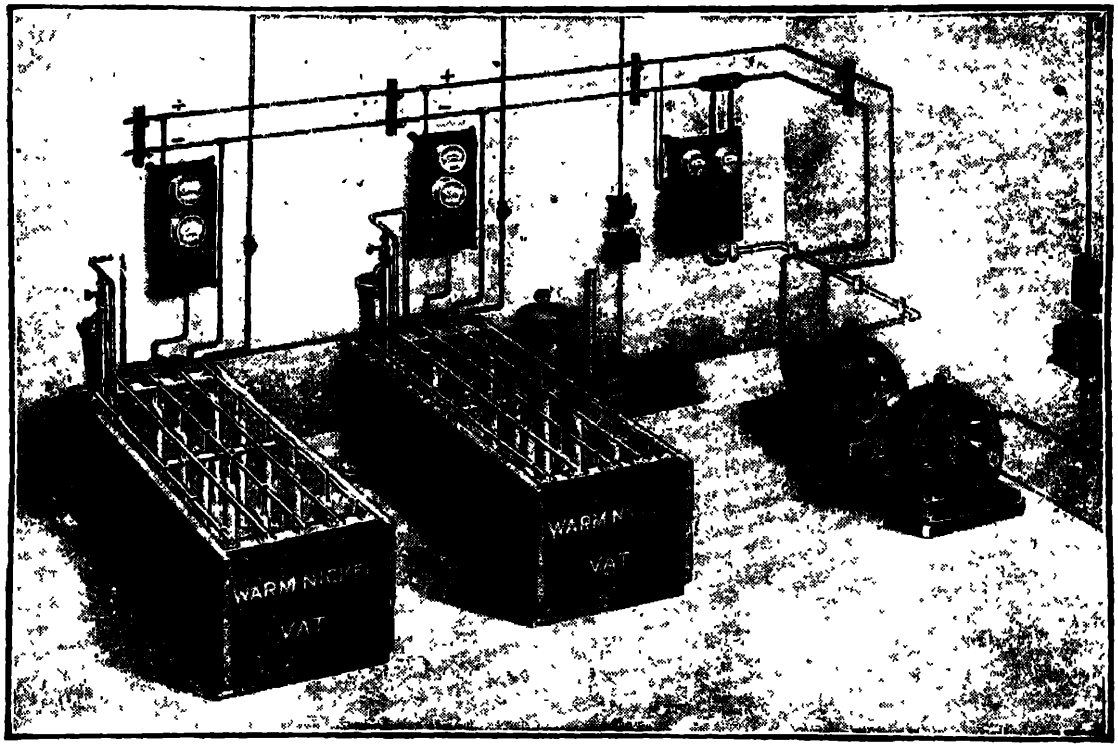
The constituents of the dissolved electrolyte are not always the final products of an electrolysis. There are two reasons for this. Firstly, water itself dissociates to some extent into hydrogen and hydroxyl ions.

$$H_2O \rightleftharpoons H^+ + OH^-.$$

The fraction of the water molecules dissociating in this way is very small, but there are so many of them that there is always a plentiful supply of hydrogen and hydroxyl ions in the neighbourhood of the electrodes in an electrolytic cell. Secondly, different atoms and groups differ greatly in the ease with which they lose or gain electrons. If a piece of iron is dipped into a solution of copper sulphate it becomes coated with copper, and by adding sufficient iron to such a solution the whole of the copper may be deposited from it and the solution is transformed into one of iron sulphate. The reason for this is that iron atoms give up electrons more readily than copper atoms, so when iron atoms are in contact with a solution containing copper ions there is a tendency for electrons to be transferred from the former to the latter, with the result that iron ions go into solution and copper atoms are precipitated.

$$Fe + Cu^{++} = Fe^{++} + Cu.$$

From the results of similar experiments, and of other types of experiment which cannot be dealt with here, it is possible to arrange the elements in an order known as the **electro-chemical series.** A short list of elements so arranged is given below.



Courtesy of W. J. Canning, Birmingham.

A NICKEL-PLATING PLANT.

Note pure nickel anodes suspended in the vats.

ELECTRO-POSITIVE.—Potassium, sodium, calcium, magnesium, aluminium, zinc, iron, lead, hydrogen, copper, mercury, silver, platinum, gold.

ELECTRO-NEGATIVE.—Carbon, nitrogen, phosphorus, sulphur, iodine, bromine, chlorine, oxygen, fluorine.

Atoms of those elements in the electro-positive list tend to lose electrons and become positive ions, and this tendency is stronger in any particular element than in those which follow it in the list. Those elements in the electro-negative list tend to add additional electrons to their atoms and produce negative ions, and this tendency is stronger in a given element than in those which precede it in the

If two different kinds of positive ions are near the cathode in an electrolytic cell, the one which is later in the electro-positive list will take electrons from the cathode and be deposited as an atom more readily than the other. Similarly, of two different kinds of negative ion near the anode, the one earlier in the electro-negative list rather than the later one will give up its additional electrons to the anode and become an electrically neutral particle. It may even happen that atoms of the material of the anode release electrons more easily than the negative ions in the solution, in which case, instead of the negative ions being discharged when they reach the anode, atoms from the anode liberate electrons to carry on the current and pass into the solution as positive ions. Thus the material of the anode may affect the final result of the electrolysis. A few examples in which these principles are applied will now be given. [illegible]

[illegible]

(1) Solution of Cuprous Chloride.—This will contain Cu^{++} and Cl^- ions from the dissolved salt and H^+ and OH^- ions from the water. If it is electrolysed between a copper cathode and carbon anode, as indicated on page 514, the Cu^{++} and Cl^- ions will be discharged, since copper is less electro-positive than hydrogen and chlorine less electro-negative than hydroxyl. Thus copper is deposited on the cathode and chlorine gas bubbles from the anode.

If, however, both electrodes are copper plates, the chlorine ions are not discharged, since copper atoms liberate electrons more readily and so positive copper ions pass into the solution from the anode. As equal masses of copper are deposited on the cathode and liberated from the anode, the composition of the solution is unchanged and the final result is that a quantity of copper is transferred from the anode to the cathode.

(2) Dilute Sulphuric Acid between Platinum Electrodes.—The acid ionises according to the equation

$$H_2SO_4 = 2\,H^+ + SO_4^{--}$$

and H^+ and OH^- ions are present from the water. Thus hydrogen ions are the only positive ions present and these are discharged at the cathode from which hydrogen gas bubbles off. At the anode the hydroxyl ions and not the SO_4^{--} ions are discharged, since the former liberate their additional electrons more readily than the latter. After discharge, the hydroxyl groups, which are not capable of existing as independent molecules, react with one another according to the equation

$$4\,OH = 2\,H_2O + O_2,$$

so water is returned to the solution and oxygen bubbles from the anode. Although water is reformed from the hydroxyl groups, only two water molecules are formed from four hydroxyl groups to provide which four molecules of water must dissociate, so the quantity of water present diminishes while that of sulphuric acid remains constant, since the SO_4^{--} ions are not discharged. In consequence of this the experiment is often referred to as "the electrolysis of water."

(3) Copper Sulphate Solution between Copper Electrodes.—The result is similar to that for cuprous chloride between copper electrodes. The copper sulphate ionises according to the equation

$$CuSO_4 = Cu^{++} + SO_4^{--}.$$

At the cathode the copper ions are discharged more readily than the more electro-positive H^+ ions and copper atoms are deposited. At the anode copper atoms give up electrons more readily than either the SO_4^{--} or OH^- ions, so positive Cu^{++} ions pass into solution and neither of the negative ions present is discharged. Thus once more the composition of the solution is unchanged and copper is transferred from the anode to the cathode.

(4) Silver Nitrate Solution between Silver Electrodes.—This is similar to case (3). The ionisation of the salt is represented by

$$AgNO_3 = Ag^+ + NO_3^-.$$

At the cathode silver is deposited rather than the more electro-positive hydrogen. At the anode Ag^+ ions pass into the solution owing to silver atoms giving up electrons more easily than NO_3^- or OH^- ions. Thus there is no change in the composition of the solution and silver is transferred from anode to cathode.

(5) Sodium Hydroxide Solution between Iron Electrodes.—The sodium hydroxide ionises according to

$$NaOH = Na^+ + OH^-.$$

Since sodium is more electro-positive than hydrogen, H^+ ions and not Na^+ ions are discharged at the cathode from which bubbles of hydrogen arise. At the anode the OH^- ions are discharged and the hydroxyl groups react with one another as in case (2). Thus there is no change in the quantity of sodium hydroxide in the solution, but the quantity of water diminishes so, as in case (2), this might be called the "electrolysis of water." This process is used on a large scale for manufacturing oxygen and hydrogen.

As a general summary of these results the following statements which will act as guides in considering other cases may be made:—

1. If a positive ion of a metal preceding hydrogen in the electrochemical series is present hydrogen will be liberated at the cathode, but if the metal follows hydrogen the metal itself will be deposited. It should be noted, however, that a high current density, *i.e.* a big current per unit area of electrode, may cause metals preceding hydrogen to be deposited on the cathode.

2. At the anode Cl^- ions are discharged more readily than OH^- ions, but SO_4^{--} ions less readily than OH^-.

3. All metals in the list given liberate electrons more readily than the Cl^- ion, so anodes made of them will dissolve if an electrolyte producing Cl^- ions is electrolysed. This is not the case with a carbon anode. Similarly, anodes of the metals preceding platinum in the list will be dissolved if SO_4^{--} or NO_3^{--} ions are present, but with platinum or gold anodes the OH^- ions are discharged.

Applications of Electrolysis

(1) Measuring currents and chemical equivalents has already been described.

(2) ELECTROPLATING.—The object of electroplating is to give articles made of one metal a coating of another metal, either to improve their appearance or to protect them against rusting and tarnishing.

The method is to electrolyse a solution of a salt of the plating metal, using the article to be plated as the cathode and a piece of the plating metal as anode. The acid radicle of the salt must be one forming ions which give up electrons less readily than the anode atoms, so that the latter will dissolve as positive ions and keep the bath charged with the salt. The conditions are then similar to cases 3 and 4 of the preceding section. In this way table forks, spoons, and ornamental objects made of inferior metal can be coated with silver to improve their appearance. Iron articles may be coated with nickel or chromium to protect them against rusting. They are usually first coated with copper, as nickel and chromium adhere better to copper than to iron. "Galvanised" iron sheets, used for roofing, etc., have a coating of zinc.

This may be given by dipping the sheets of iron in molten zinc, but a more even coating is obtained and less zinc used if it is applied electrolytically.

The picture on page 515 shows a nickel-plating plant. Rods of nickel are shown hanging from metal rods across the top of the baths. These rods are connected with the positive pole of the source, so that they form a compound anode. The other rods across the vat are connected to the negative pole, and the articles to be plated are hung from them to form the cathode. The electrolyte is a solution of a rather complex salt of nickel.

(3) Electrotyping.—This is a special application of electroplating. An impression of a page of type is made on a sheet of wax which is then coated with graphite to give it a conducting surface. It is used as cathode in a copper voltameter and becomes coated with a thin layer of copper. This is stripped off the wax and the back filled in with molten type metal. Thus an exact reproduction of the page of type is obtained. Note also the application of this method in the manufacture of gramophone records (page 401).

(4) Purifying Metals.—It was shown that in both the copper and silver voltameters metal was transferred to the cathode from the anode. If the anode consists of impure metal and a thin strip of pure metal is used for cathode, the impurities will not be transferred, and a block of pure metal will be built up on the cathode. This method is used when specially pure copper for electrical apparatus is required.

(5) Manufacture of Chemicals.—The electrolytic manufacture of hydrogen and oxygen has been mentioned. Details of many electrolytic methods of manufacture will be found in textbooks of chemistry.

QUESTIONS ON CHAPTER XLIII

1. Explain the meanings of the terms *electrolysis, anode, cathode.*
Describe one case of electrolysis you have seen. State what was observed and what products were obtained, and give a brief explanation of what took place.

2. State the two laws of electrolysis.
Briefly describe one experiment to illustrate each law.

3. What do you understand by the "chemical effect of electricity"? Describe one good experiment to illustrate this effect, and give such explanation as you think necessary for a proper understanding of the experiment. [L.U.

4. Explain the meanings of the terms *current strength* and *quantity of electricity*. Name and define units in which each is measured.

Describe how you would use a copper voltameter to measure the strength of a steady current.

5. Give a brief account of what occurs when a current of electricity is passed through a solution of copper sulphate, the electrodes being (*a*) of copper, (*b*) of platinum. [L.U.

6. State the laws of electrolysis.

A copper voltameter is connected in series with a source of electric supply and a current is passed for 1 hour. It is found that one electrode gains 1·1 grm. of copper in this time. Calculate the current and draw a circuit diagram which shows which electrode increases in weight. (E.C.E. of copper = 0·00033 grm./coulomb.) [L.U.

7. State the laws of electrolysis.

Describe, with a circuit diagram, the method you would use to find the electro-chemical equivalent of copper.

If the electro-chemical equivalent of copper is taken as 0·00033 grm. per coulomb, how long would it take to deposit 0·1 mm. thickness of copper on one side of a circular plate of metal of radius 2·5 cm., if the current passing is 1·25 amp.? (The density of copper is 8·9 grm. per c.cm.) [L.U.

8. Explain the electrical principles involved in the process of electroplating, referring particularly to that of copper plating. [L.U.

CHAPTER XLIV

CELLS

In electrolytic actions electrical energy is used to bring about chemical changes. In a cell this is reversed, and chemical actions are used to generate electrical energy.

The Simple Cell

Volta (1800) was the first to set up a cell which would give a continuous current. His cell consisted of a plate of zinc and a plate of silver placed in contact with a solution of salt. Similar results can be obtained from a plate of zinc and a plate of copper dipping into dilute sulphuric acid. Such simple cells are never used to-day, but their study throws light on the principles of cells.

You probably know that commercial zinc readily dissolves in dilute sulphuric acid, hydrogen being liberated. The action is usually represented by the equation

$$Zn + H_2SO_4 = ZnSO_4 + H_2.$$

If very pure zinc is used there is, however, no action. Also dilute sulphuric acid does not attack copper.

Into a vessel of dilute suphuric acid dip a plate of pure zinc. No action will be observed. Into the same vessel dip a plate of copper, keeping the two plates apart. Still there will be no action. Now allow the two plates to touch. The zinc at once begins to dissolve and hydrogen bubbles appear *on the copper plate.* You may not be able to see that the zinc is dissolving, but if the plate is weighed before being put into the solution and washed, dried, and reweighed after it has been in contact with the copper for some time, it will be found to have lost weight.

A similar result is obtained if the two plates are joined up by a copper wire outside the solution instead of being allowed to touch in the solution. If part of the wire is arranged to run north and south and a compass needle is placed just below it, the needle will be deflected when contact is made with the plates, showing that an electric current is running through the wire. It is convenient to have a contact key in the circuit so that everything may be arranged and then the circuit closed by means of the key.

It is clear from this that the two plates must acquire different potentials when placed in contact with the sulphuric acid, and. it can be shown that the copper plate is at the higher potential and the current flows in the direction indicated in Fig. 431.

The setting up of this potential difference may be explained as follows. A dilute solution of sulphuric acid contains positive hydrogen ions (H^+) and negative "sulphate" ions (SO_4^{--}). If a plate of zinc is dipped into it, owing to the electro-positive character of that element, there is a tendency for positive zinc ions (Zn^{++}) to escape from the surface of the plate into the solution, each of them leaving two electrons behind on the plate. The osmotic pressure due to the ions in the solution tends to drive positive hydrogen ions on to the plate, but the former tendency is the stronger so that some zinc ions do escape from the plate which becomes negatively charged by the electrons which are left behind. This results in the plate having a lower potential than the solution. The escape of ions from the plate also results in there being an excess of positive charge in a very thin layer of the solution in contact with the surface of the plate, and the difference of potential between this layer and the plate opposes the escape of positive ions into the solution, which is brought to a stop before any visible solution of the zinc has taken place.

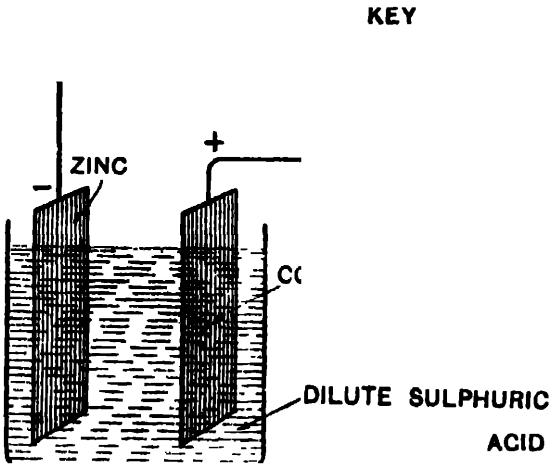


FIG. 431. SIMPLE CELL.

If a copper plate is dipped into the acid there is again a tendency for positive metal ions (Cu^{++}) to escape into the solution and for positive hydrogen ions to be driven on to the plate. But copper is less electro-positive than zinc and so its tendency to form ions is not so strong and is less than the tendency for hydrogen ions to be driven out of the solution. Hence hydrogen ions are driven on to the plate, and since hydrogen is less electro-positive than copper, electrons are transferred from copper atoms to the hydrogen ions, converting them into hydrogen atoms. The loss of electrons from the copper plate causes it to be positively charged and hence raised to a higher potential than the solution. The removal of hydrogen ions from the thin layer of solution in contact with the copper leaves it with an excess of SO_4^{--} ions so that it becomes negatively charged, and, as in the case of the zinc plate, the difference of potential between the plate and the adjacent solution layer rapidly stops the action and there is no visible liberation of hydrogen.

It follows from this that if a plate of zinc and a plate of copper are both dipped into the same vessel containing dilute sulphuric acid, the zinc will be lower in potential and the copper higher in potential than the solution, and so the copper plate will have a higher potential than the zinc plate. If then the plates are connected by a wire a current will flow from one plate to the other. It is usual to say that a positive current flows through the wire from the copper plate (high potential) to the zinc plate (low potential), though, as explained on page 502, what actually happens is that electrons flow from the low potential plate to that at high potential. The flow of electrons from the zinc plate reduces its negative charge and the potential difference between it and the surface layer of solution so more positive ions can escape from it. Also, as electrons flow through the wire into the copper plate they neutralise some of the positive charge on it and reduce the potential difference at its surface, which allows more hydrogen ions to be driven on to the plate. Thus the actions which have been described can go on continuously so long as the plates are connected by an external conductor, and the plates will be kept at different potentials and a continuous current will flow. Also zinc will be continuously dissolved and hydrogen continuously liberated at the copper plate so long as the current flows.

Local Action

The ready dissolving of impure zinc in dilute sulphuric acid may be explained by regarding the particles of impurities as playing the part of the copper plate in the simple cell. By the process already explained the particles of impurity on the surface of the zinc become charged to a higher potential than the zinc. As they are in contact with the zinc small local currents flow from the particles of impurity to the neighbouring zinc and the dissolving of the zinc is continuous, bubbles of hydrogen rising from the particles of impurity.

This local action is troublesome if impure zinc plates are used in cells. It results in the zinc plate continuing to dissolve even when the cell is not yielding current, thus wasting zinc. It also means that, even when the cell is working, much of the chemical energy that should be transformed into electrical energy to drive the current in the external circuit is used up in driving the little local currents in the cell and finally converted to heat energy causing a rise in temperature of the cell.

Local action can be prevented by *amalgamating* the zinc plate. It is dipped into sulphuric acid and then rubbed with mercury. The mercury dissolves some of the zinc forming a solution known as zinc amalgam which coats the plate. As the mercury does not dissolve the impurities they are kept from contact with the acid in the cell by the coating of amalgam over them.

Polarisation

Set up a simple cell and connect in series with it a sensitive ammeter and a switch key. Close the switch and watch the readings of the ammeter. It will be observed that the strength of the current falls off rapidly and in a short time it may become zero. When this happens inspection will show that the copper plate is covered with hydrogen bubbles. Remove these bubbles by brushing with a camel-hair brush. The current will recover somewhat, but will quickly fall off again as a fresh layer of bubbles forms. This shows clearly that the falling off of the current is due to the layer of hydrogen which prevents contact between the copper and the acid and so stops the action of the cell. This defect of a simple cell is called *polarisation,* and cells which are required to give current for any length of time must have some provision for *depolarising,* that is, preventing a layer of hydrogen from forming on the positive pole.

To some extent polarisation may be prevented by giving the positive pole a rough surface from which the hydrogen bubbles break away more readily than from a smooth surface. Most cells, however, utilise chemical actions for depolarising. To illustrate this allow the simple cell to become polarised, and then by means of a fountain pen filler squirt strong copper sulphate solution over the copper plate. The hydrogen bubbles will disappear and the current will regain strength.

It may be noted in passing that the deposition of hydrogen at the copper is not the *only* cause of "polarisation" in a cell.

Some Common Cells

(1) THE DANIELL CELL.—This is illustrated in Fig. 432. It is a modification of the simple cell with copper sulphate solution as

depolariser. Instead of a copper plate a copper vessel which contains the fluid of the cell is used, the vessel itself being the positive pole. The negative pole is an amalgamated zinc rod which stands in a porous pot inside the copper vessel. Dilute sulphuric acid is placed in the porous ıt and a saturated solution of copper sulphate fills the outer vessel around the top of which is fixed a perforated shelf on which crystals of copper sulphate are placed to maintain the solution in a saturated condition. The two fluids are in contact through the pores of the porous pot.

The primary actions, producing the potential difference between the poles, are those of the simple cell. The copper sulphate solution contains Cu^{++} and SO_4^{--} ions. Thus the positive ions which are driven on to the copper plate are copper ions instead of hydrogen ions and no layer of hydrogen bubbles is formed on the surface of the plate and polarisation is prevented.

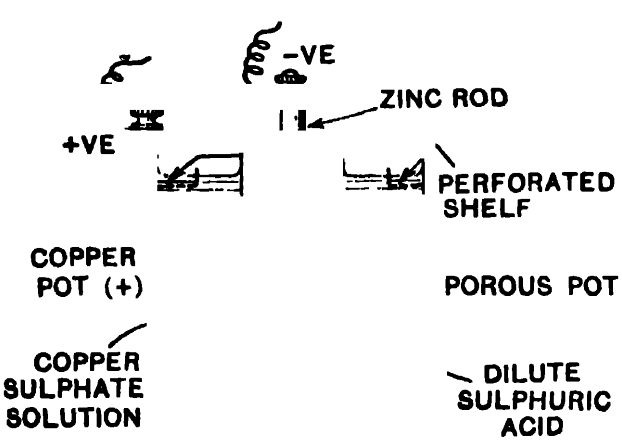


Fig. 432. Daniell Cell.

When the cell is not in use the porous pot should be removed and emptied or the two liquids will gradually diffuse into one another. Also the zinc rod should occasionally be re-amalgamated or there will be an excessive dissolving of zinc by the acid.

Daniell cells are not much used nowadays, but before the development of the accumulator they were frequently used when a source which would give a steady current for a considerable time was required.

(2) The Leclanché Cell.—This is shown in Fig. 433. The negative pole is a zinc rod which dips into a solution of sal-ammoniac. The positive pole is a carbon rod which is packed into a mixture of manganese dioxide and carbon in a porous pot. The zinc becomes negatively charged by the escape of positive ions as already explained. The ammonium chloride in the solution ionises according to the equation

$$NH_4Cl = NH_4^+ + Cl^-,$$

so the positive ions which are driven on to the carbon rod are ammonium ions. As these are discharged by taking electrons from the carbon atoms, the resulting ammonium groups react in pairs according to the equation

$$2\,NH_4 = 2\,NH_3 + H_2.$$

The ammonia dissolves in the water in the cell and the hydrogen is oxidised to water by the manganese dioxide ($2\,MnO_2 + H_2 = Mn_2O_3 + H_2O$). The Mn_2O_3 is oxidised back to MnO_2 by oxygen from the air when the cell is not working, and so it will continue to act as depolariser for a long time.

These cells require little attention beyond the addition of water occasionally to make up for loss by evaporation and the renewal of the sal-ammoniac as it gets used up. They are not, however, suitable for giving currents of long duration. This is readily shown by joining a Leclanché cell in series with an ammeter and a variable resistance. Adjust the current to give a suitable reading on the ammeter and take that reading at half-minute intervals. In a little time the current will be found to drop rapidly. If the circuit is broken for half-an-hour and then completed again the cell will be found to have recovered and to be giving about the same current as at first. The reason for this is that the oxidising action of the manganese dioxide is slow, so that the hydrogen is not removed as fast as it is produced when the cell is working. Hence, if the cell goes on working for some time it does become polarised. On standing the polarising hydrogen will gradually be oxidised and the cell recovers.

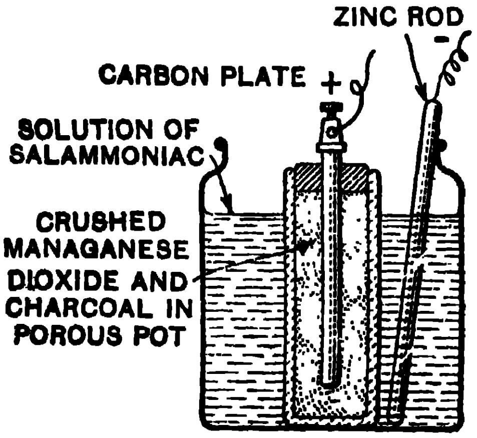


FIG. 433. LECLANCHÉ CELL.

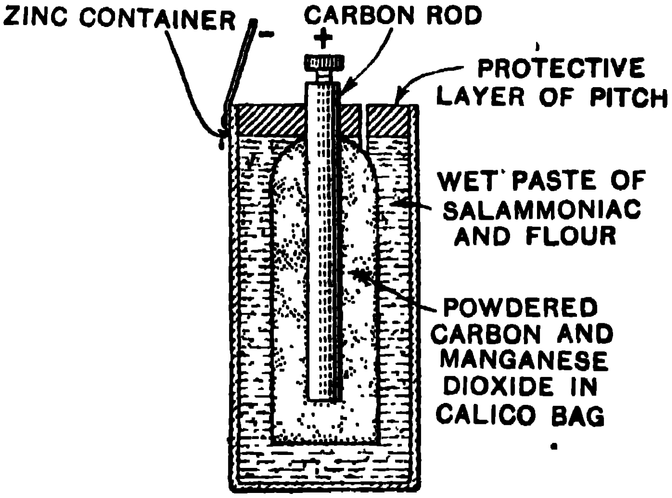

ZINC CONTAINER
−
CARBON ROD
+
PROTECTIVE LAYER OF PITCH
WET PASTE OF SALAMMONIAC AND FLOUR
POWDERED CARBON AND MANGANESE DIOXIDE IN CALICO BAG

FIG. 434. DRY CELL.

Because they need so little attention, Leclanché cells are still frequently used, but only for such purposes as ringing door-bells where intermittent currents of short duration are required.

(3) Dry Cells.—These, as illustrated in Fig. 434, are modifications of the Leclanché cell. The container is made of zinc and acts as the negative pole. Inside it is a paste made of flour or plaster of Paris moistened with sal-ammoniac solution. A calico bag replaces the porous pot. The actions are the same as in the Leclanché cell, but there is no free liquid to be spilled if the cell is upset. They are much used where that is an advantage, for example, in pocket torches, cycle lamps, wireless sets, and so on. For such purposes they are frequently made up into batteries, the zinc containers being separated by cardboard insulation and the positive pole of one cell being connected by wire to the negative pole of the next.

A modification of the Leclanché and dry cells employs ferric chloride as depolariser instead of manganese dioxide. This acts more quickly and enables the cell to be used continuously for a longer time.

(4) Accumulators.—The cells so far dealt with are called primary cells, since in them we merely bring into contact suitable substances which, by their chemical actions, will generate electrical energy. Accumulators are called secondary cells since, while it is chemical actions taking place in them which generate electrical energy when they are giving current, they were brought into condition to do this by first passing in a charging current. Accumulators are also said to be reversible cells because a charging current can reverse the chemical actions which take place in them when they are yielding current and bring the contents back to the condition they were in before discharge took place. The other cells are not reversible in this way. For example, when the ammonium chloride of a Leclanché cell has been used up, passing a current through the cell in the reverse way will not cause ammonium chloride to be formed in it, but a fresh supply must be placed in it.

The most common accumulator is the **lead accumulator**, the principle of which can be shown by electrolysing a solution of sulphuric acid between lead electrodes. Hydrogen will be liberated at the cathode and oxygen at the anode, as when platinum electrodes were used (page 516). In time, however, the anode will be found to have a brown colour due to the oxygen liberated at it oxidising the surface to lead peroxide, $Pb + O_2 = PbO_2$ [Fig. 435 (*a*)]. If now the original source of current is disconnected and the two plates are connected to an ammeter it will be found that current passes through it, the plate which was previously the anode acting as the positive pole of a cell; the cathode acting as the negative pole, the current flowing in the opposite direction to the original charging current [Fig. 435 (*b*)]. During this discharge both plates become white on the surface owing to the formation of lead sulphate on them. This is due to the ions of the sulphuric acid. The hydrogen liberated at the positive pole reduces the lead peroxide to litharge (PbO) which then reacts with the sulphuric acid to form lead sulphate ($PbSO_4$). At the negative pole there is a reaction between the SO_4^{--} ions and the lead also producing lead sulphate.

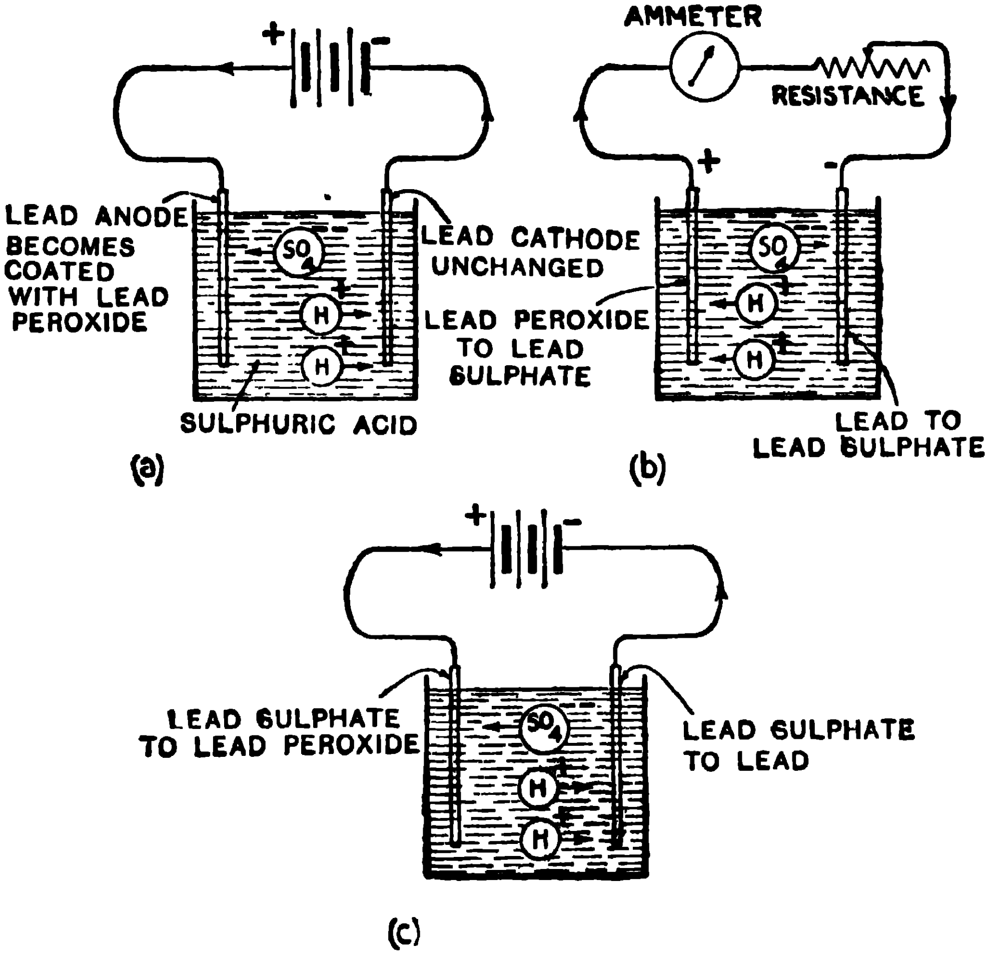


Fig. 435.

When the voltage of the cell has fallen it can be charged again by passing current into it once more in the original direction [Fig. 435 (*c*)]. The oxygen liberated at the positive pole oxidises the lead sulphate to lead peroxide and the hydrogen liberated at the negative pole reduces the lead sulphate there to lead and so the cell is brought back into the same condition as before discharge.

It will be found that after the second charging, which should be carried on until all the white lead sulphate has disappeared, the cell will give as big a current as after the first charging but for a longer time. This is due to the fact that, owing to the chemical changes which have taken place, the surface of the plates acquire a spongy texture so that the second time the ions can act on the plates to a greater depth and produce more active material. When accumulators have solid lead plates to start with they are charged and discharged a number of times before being used in order to increase their capacity. This process is called "forming" the plates. In modern portable accumulators the plates are generally lead grids such as that illustrated in Fig. 436, the grid being filled up with a paste made of sulphuric acid and oxides of lead. With such plates a considerable depth of lead peroxide and spongy lead is produced by the first charging. For stationary batteries, as used in telephone exchanges, power stations, and in various institutions for emergency lighting, the Planté type with positive plates formed from solid lead plates are still used as they are found to give more efficient service over long periods.

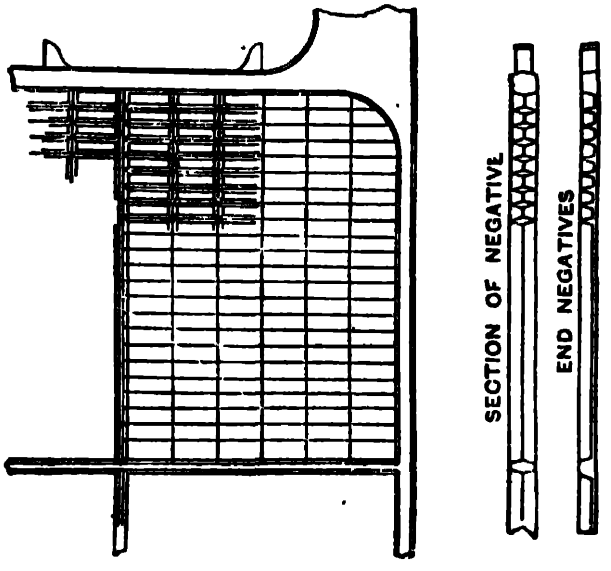


Fig. 436.

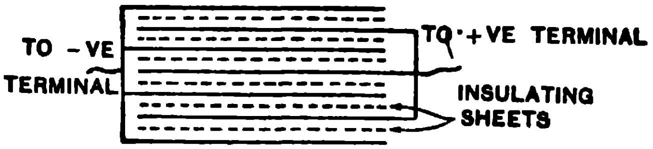


Fig. 437.

To obtain a large surface area, the plates are usually made in two interlocking sets, all of one set being connected to the positive terminal and all of the other set to the negative terminal. To prevent positives and negatives from making contact, sheets of insulating material are placed between them (see Fig. 437).

An accumulator is, of course, subject to the law of conservation of energy and cannot give out, during discharge, more electrical energy than was passed into it during charging. The capacity of an accumulator is generally stated in amp.-hours, that is the product of current given out by the number of hours for which it can be taken. Thus an accumulator with a capacity of 60 amp.-hours might when fully charged give a current of 3 amps. for 20 hours or of 5 amps. for 12 hours, and so on. Instructions are usually given with an accumulator that it should be charged with a current of a given strength. The number of amp.-hours required for charging is always greater than the capacity. With proper use the capacity should be about 90 per cent. of the charging amp.-hours and this is often said to be the amp.-hour efficiency of the accumulator. It is not a true mechanical efficiency for, as will be seen in a later chapter, amp.-hours are not units of energy. The energy efficiency of an accumulator should be about 75 per cent., that is the electrical energy given out in discharging should be about 75 per cent. of that used in charging.

The advantages of an accumulator are that it can give fairly large steady currents for a considerable time. Its disadvantages are that it is heavy and that it is easily damaged by careless use. If it is discharged too far or left standing for long unused, the acid attacks the plates and may form a very heavy deposit of lead sulphate on them which it is difficult to reconvert into lead peroxide and lead. If it is charged at too high a rate, particularly towards the end of the charging, considerable heat is generated in the plates and may buckle or crack them.

The formation of lead sulphate on the plates during discharge results in the removal of sulphuric acid from the solution, but this sulphuric acid is returned to the solution by the actions in which lead peroxide and lead are formed from the lead sulphate during charging. Thus the density of the electrolyte rises during charging and falls during discharge and the state of charge is shown by that density. Many accumulators are fitted with automatic devices which are operated by the change of density to show when recharging is necessary.

The **iron-nickel or Edison accumulator,** used to a considerable extent in America, is similar in principle. Its positive plates consist of nickel-plated iron tubes packed with nickel hydroxide. Its negative plate consists of similar tubes filled with finely divided iron. The electrolyte is a solution of potassium hydroxide which gives positive potassium and negative hydroxyl ions, $KOH = K^+ + OH^-$.

It has the advantage over the lead accumulator of being lighter and also of being more robust so that it can be charged at a higher rate without being damaged, but it is more bulky and does not give so steady a current. It is used for supplying current to motors in electric trucks, etc. In England a similar alkaline cell with nickel and cadmium as its elements is more used than the iron-nickel cell, but use of the lead accumulator is still much more widespread.

Electromotive Force

It is the potential difference between the poles of a cell which drives current round an *external* circuit connected to it. For this reason that potential difference is sometimes referred to as the electromotive force of a cell. This term should, however, only be used to denote the potential difference between the poles *when the cell is not giving current,* for the reasoning on page 536 will show that this potential difference tends to fall when current does flow, and only a portion of the total is available for driving current in the external circuit. When current flows the P.D. between the terminals is less than the E.M.F. of the cell; the P.D. between the terminals is only equal to the E.M.F. if no current is flowing.

Units of Potential Difference

As in the case of electrostatics we may say that *there is one unit of potential difference between two points when* 1 *unit of work has to be done in transferring* 1 *unit quantity of positive electricity from one point to the other.* If the coulomb is taken as the unit of quantity and the erg as unit of work in the definition a very small unit of potential difference is obtained, so for practical purposes a unit of work equal to 10,000,000 or 10^7 ergs is used. This is called a **joule.** **When 1 joule of work is required to transfer 1 coulomb of electricity from one point to another the potential difference between those points is said to be 1 volt.** This is smaller than the electrostatic unit of potential which equals 300 volts.

Standard Cells

For practical purposes it is useful to have a cell which has a constant known E.M.F. with which other potential differences can be compared in order to find their value. For many years the Daniell cell was used for this purpose, as if it is carefully prepared, it has an almost constant E.M.F. of 1·09 volts.

A more accurate standard is the **Weston cadmium cell** illustrated in Fig. 438. The cadmium in the amalgam on the left and the mercury on the right form the poles of the cell. The mercurous sulphate is the depolariser. Very exact instructions for making such a cell are issued by the National Physical Laboratory, and if they are followed the cell will have an E.M.F. of 1·0183 volts at 20° C. This voltage varies a little with temperature, but the coefficient is small and accurately known, so its E.M.F. at other temperatures can be calculated.

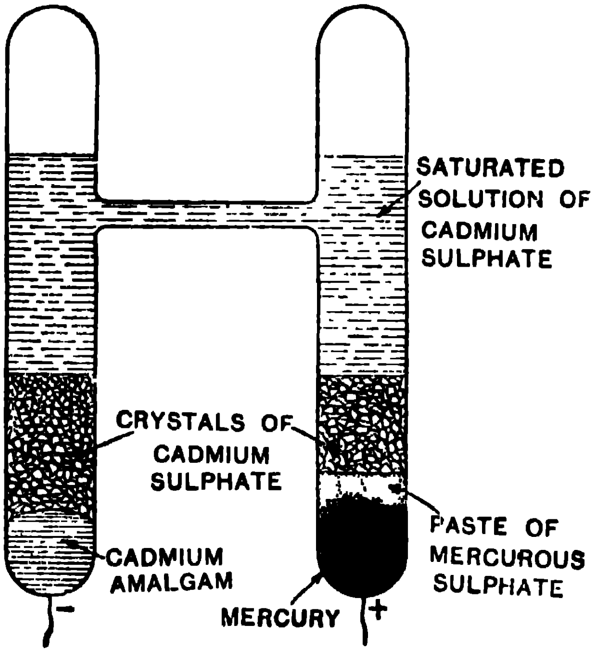

SATURATED SOLUTION OF CADMIUM SULPHATE
CRYSTALS OF CADMIUM SULPHATE
PASTE OF MERCUROUS SULPHATE
CADMIUM AMALGAM
MERCURY
−
+

FIG. 438. WESTON CADMIUM CELL.

The cadmium cell should never be used as a source of current as that would soon alter its E.M.F. It should only be used in methods in which it is momentarily put into a closed circuit for comparison with other cells. See "potentiometer," page 569.

Comparison of E.M.F.'s

The E.M.F.'s of two cells may be roughly compared by arranging them in the same circuit in such a way that they tend to drive current in opposite directions and finding which can drive current through the other.

Set up a wire running north and south and place a compass needle below it. Connect in series with it a variable resistance, a cell, and a key. When the key is depressed the compass needle is deflected. Adjust the resistance so that a deflection of about 45° is obtained. If now the cell connexions are reversed it will be found that the deflection of the needle is reversed. For instance, if in the first case the north pole was deflected to the east, in the second case it will be deflected to the west.

Place one of the cells to be compared in the circuit, depress the key, and note in which direction the needle is deflected [Fig. 439 (*a*)]. Now connect the second cell to the first, connecting their negative poles so that they tend to drive current in opposite directions round the circuit [Fig. 439 (*b*)]. Depress the key and note which way the needle is

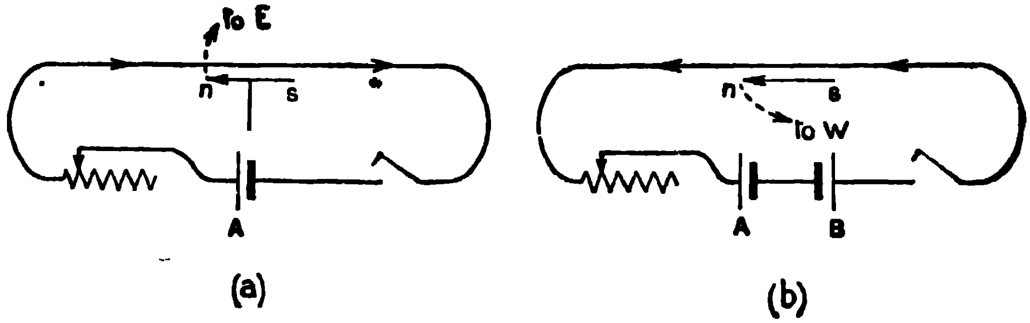


FIG. 439.

deflected this time. If it is in the same direction as before current is still travelling in the same direction, so the first cell is able to drive current against the second one and has the greater E.M.F. If the deflection of the needle is reversed the second cell has the greater E.M.F.

In this way compare cells of various types and see if the results agree with the following table.

CELL	E.M.F.
Daniell	1·09 volts
Leclanché	1·4-1·6 ,,
Lead accumulator	2·08 ,,

It is interesting to compare also cells of the same kind but of different sizes. It will be found that when two such cells are placed in opposition there is no deflection of the needle, showing that no current passes, and therefore the two E.M.F.'s are equal. This gives the important result that *the E.M.F. of a cell depends only on its materials and not on the size of the plates.* Large plates placed near together are an advantage in a cell as current has to be driven through the cell itself as well as through the external circuit, and there will be less resistance in the cell—see next chapter—if the plates are large and near together. Thus a cell with large plates near together will send a larger current through a given external circuit than a similar cell with smaller plates, though it has only the same E.M.F. In an accumulator a large plate surface also provides more active material and therefore a greater capacity.

Batteries

Study

Cells are often arranged in groups to form batteries. There are two main ways of connecting them.

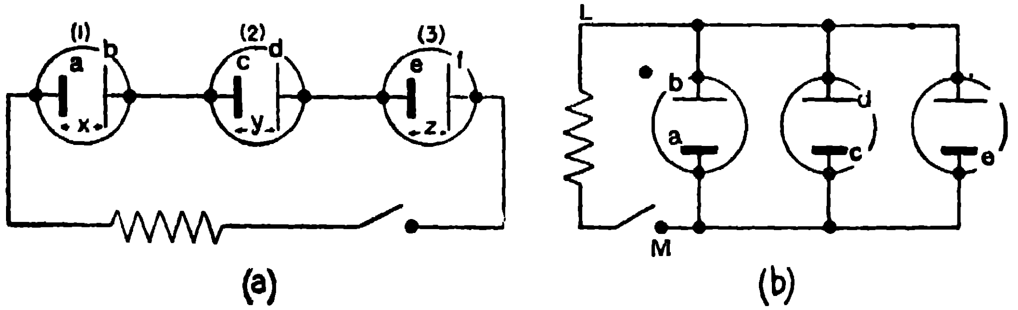


FIG. 440.

Study

(1) IN SERIES.—The negative pole of one is connected to the positive pole of the next throughout the battery [Fig 440 (*a*)]. In this case the total E.M.F. of the battery is the sum of the E.M.F.'s of the separate cells. Suppose 1, 2, and 3 have E.M.F.'s of x, y, and z volts respectively. Then b has a potential x volts higher than a. Because b is connected to c, they form one conductor and have the same potential. Therefore c is x volts higher than a in potential. But d has a potential y volts higher than c, and therefore $x + y$ volts higher than a. Similarly it can be shown that the potential of f is $x + y + z$ volts higher than that of a, and this process could be continued for any number of cells.

Study

(2) IN PARALLEL.—All the negative poles are joined to one point and all the positive poles to another point, these points becoming the poles of the battery [Fig. 440 (*b*).] Considering the case where all the cells have the same E.M.F., the E.M.F. of the battery is only equal to that of a single cell. For a, c, and e all being connected form one conductor and all have the same potential. Similarly b, d, and f all have the same potential. Hence the potential difference between L and M is the same as that between a and b. Although the parallel arrangement has only the same E.M.F. as a single cell it will send a bigger current round a given external circuit for it is equivalent to increasing the size of the plates.

Read

Car Batteries and High Tension Cells

Car batteries usually have E.M.F.'s of about 6 or 12 volts. They are composed of lead accumulators in series. Since each of these has an E.M.F. of about 2 volts, three are required for a 6-volt battery and six for a 12-volt battery.

In wireless sets, certain points require to have a high potential difference maintained between them. In battery sets a high-tension

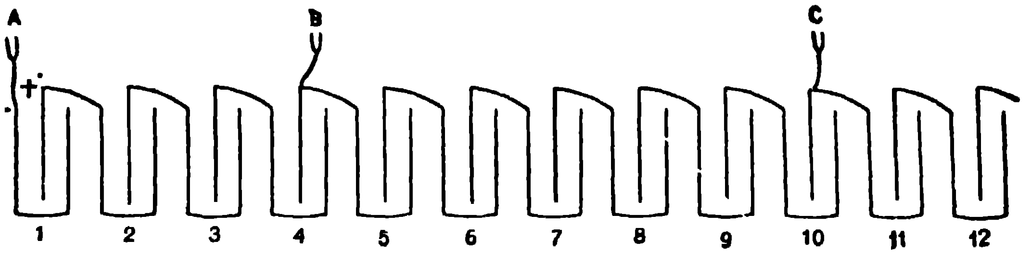


FIG. 441.

battery is used to secure this. It consists of a large number of dry cells connected in series. Each has an E.M.F. of 1·5 volts, so if n such cells are included in the battery the total E.M.F. is $n \times 1{\cdot}5$ volts. "Tappings" are usually provided so that connexion can be made at various points along the battery. Fig. 441 illustrates a small portion of such a battery. If conductors are plugged in at A and B they will have a potential difference of $4 \times 1{\cdot}5 = 6$ volts. When plugged in at A and C the potential difference will be $10 \times 1{\cdot}5 = 15$ volts.

Batteries for small pocket torches usually have two or three dry cells in series giving an E.M.F. of 3 volts or 4·5 volts. In the batteries for round torches they are usually placed end to end, the bottom of the zinc case of one making contact with a copper cap on the top of the carbon rod of the one below it.

QUESTIONS ON CHAPTER XLIV

1. State three properties which should be possessed by a good cell. Describe briefly how these properties are illustrated in each of the following: (*a*) Daniell cell, (*b*) Leclanché cell, (*c*) storage cell (accumulator).

Distinguish between *primary* and *secondary* cells. [L.U.

2. Describe and explain what is to be observed when (*a*) a copper and a zinc plate connected by a wire are dipped into dilute sulphuric acid, (*b*) a current is sent through a solution of copper sulphate, a copper and a platinum electrode being used, the latter as (i) the cathode, (ii) the anode.

Briefly describe *one* practical application of electrolysis. [J.M.B.

3. Describe the simple voltaic cell. State what is to be observed in the cell when its terminals are connected by a copper wire. Mention *three* disadvantages of this cell as a source of electric power.

Explain how the Daniell cell differs from this simple cell, and say why it is superior to it. Give *two* advantages of the accumulator over the Daniell cell. [J.M.B.

4. Explain the cause of (*a*) polarisation, (*b*) local action, in the case of a voltaic cell.

Describe how each of these defects is lessened in the case of a Daniell cell. [L.U.

5. Explain what is meant by (*a*) local action, (*b*) polarisation, in a voltaic cell. Describe and explain how these defects are minimised in one type of voltaic cell with which you are acquainted. [L.U.

6. Describe the construction and action of *one* type of primary cell in common use. Give reasons why you consider the cell you describe is suitable or unsuitable for use in a domestic bell or telephone circuit. [L.U.

7. What type of cell would you choose for each of the following purposes? (*a*) Lighting a room. (*b*) Working a bell circuit. (*c*) Lighting a cycle lamp. In each case give reasons for your choice.

8. What are the properties of a storage cell (accumulator) which make it so useful? Name two important uses of these cells in every day life.

It is most important that the terminals of a storage cell should not be short circuited. Explain the reason for this. [L.U.

9. Describe one useful form of primary (or voltaic) cell and a secondary cell (or accumulator).

Compare the advantages and disadvantages in the use of these two types of cell. [L.U.

10. Give an account of the construction, action, and use of any pattern of *either* (*a*) a Leclanché cell, *or* (*b*) a storage cell (accumulator).

What is the distinction between a primary cell and a secondary cell? [L.U.

11. What is meant by the electromotive force of a cell? State the name of the unit for measuring electromotive force and define that unit.

How would you find which was the greater (*a*) the E.M.F. of a Leclanché cell or that of a Daniell cell; (*b*) the E.M.F. of a small lead accumulator or a large one, each consisting of a single cell? What results would you expect to obtain? What is the advantage of using an accumulator with large plates?

CHAPTER XLV

MAGNETIC EFFECTS OF CURRENTS

In Chapter XLII. it was shown that magnetic effects could be detected in the neighbourhood of conductors carrying currents. Very many electrical devices, ranging from electric bells and telephones to powerful electro-magnetic cranes and motors, depend on these effects.

Magnetic Field Around a Straight Conductor

Fix a stout vertical wire passing through a small hole in a horizontal sheet of cardboard on which iron filings are sprinkled. Pass a current of 15 to 20 amps along the wire and gently tap the card. The filings arrange themselves in circles concentric with the wire, showing the form of the lines of magnetic force in the field which surrounds the wire when current is passing through it.

If the connexions are arranged so that the current passes *upwards* and small compass needles are placed around the wire, the directions in which their north poles are driven when the current is switched on will show that the lines have the direction shown by the arrows in Fig. 442. If the current is reversed the direction of the lines of force is also reversed.

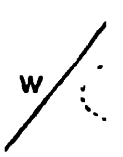


FIG. 442.
LINES OF FORCE ROUND A STRAIGHT WIRE CARRYING A CURRENT.

MAXWELL'S CORKSCREW RULE.—To help in memorising the above results Maxwell stated the rule, "*Imagine a corkscrew being screwed along the conductor in the direction of the current. The direction in which the ends of the handle move gives the direction of the lines of magnetic force.*"

Field Due to a Circular Coil

Thread about ten turns of stout insulated wire through two holes in a sheet of cardboard to form a close circular coil at right angles to the cardboard, with a diameter of about 4 inches. Pass a current of 3 amps. through the coil. By means of iron filings and compass needles it will be found that the magnetic field has the form shown in Fig. 443.

Note that the direction of the lines of force around each hole in the cardboard agrees with the corkscrew rule as applied to the part of the coil passing through that hole.

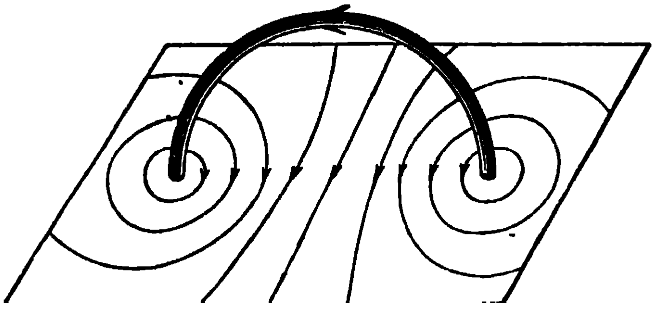


FIG. 443.

Field Due to a Solenoid

Along two parallel lines about 2 inches apart on a sheet of cardboard make holes about one-eighth of an inch apart and thread a stout wire to form a long drawn out coil, as shown in Fig. 444 Three or four turns should be passed through each pair of holes. Such a coil is called a solenoid. Pass a current of about 10 amps. through it and map the magnetic field produced. It will be found to have the form shown in the figure. Note that the lines of force inside the solenoid run parallel to its axis, and that the field outside it is similar to that of a bar magnet.

In all the above cases only one section of the field has been mapped. In the case of a straight wire all sections perpendicular to the wire, and in the case of a coil all passing through its axis, would be similar.

Comparison of Solenoid and Bar Magnet

The distribution of lines of force in Fig. 444 suggests that a solenoid carrying a current will behave like a magnet. This is investigated in the following experiments.

(1) Wind closely about four layers of thin insulated copper wire on a light cardboard tube 4 in. long and $\frac{1}{2}$ in. in diameter. Attach light flexible leads and suspend the coil by a silk thread attached half-way along it, so that its axis sets horizontally. Pass a current of about 2 amps. through it and bring the same pole of a bar magnet near each end in turn. One end of the coil will be repelled and the other will be attracted.

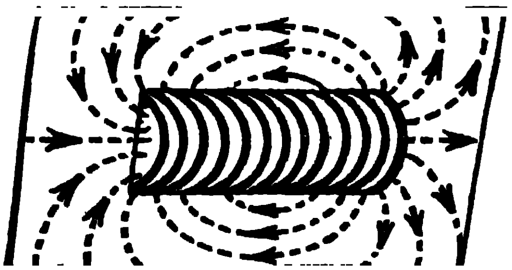


FIG 444.

MAGNETIC LINES OF FORCE ABOUT A COIL CARRYING A CURRENT.

(2) Fix the coil with its axis horizontal and suspend small pieces of clockspring near its ends. They will be attracted at both ends of the coil when current passes.

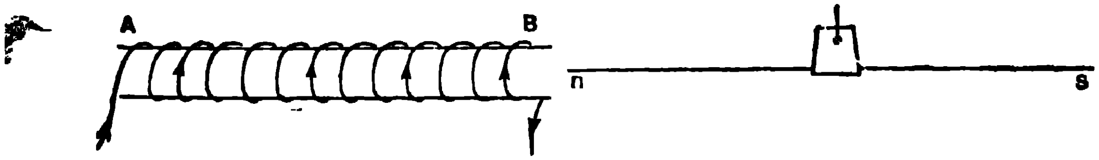


FIG. 445.

(3) Magnetise a knitting needle, suspend it in a sling, and allow it to set. Fix the coil with its axis in line with the needle as shown in Fig. 445. Switch on the current. With the coil opposite the north pole of the needle and current travelling in the direction indicated, the needle will be attracted into the coil. Move the coil, placing B opposite to the south pole, still passing current in at A and out at B. The needle will now be driven away from the coil. Repeat with A near the poles of the needle and *s* will now be attracted while *n* is repelled. Thus A behaves like a north magnetic pole and B like a south pole as shown in Fig. 446.

(4) Repeat the experiment with the coil and compass needle described on page 504. Again try the effect of both ends of the coil on the needle. The same conclusions as those under (3) should be reached.

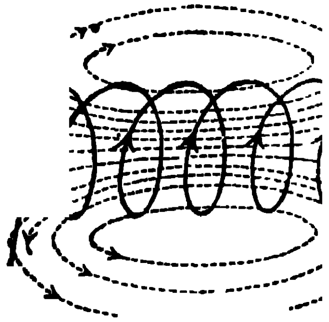


FIG. 446.

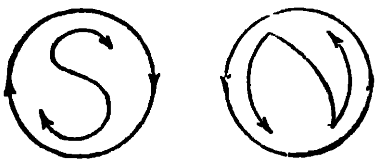


FIG. 447.

These experiments show conclusively that the solenoid when carrying a current behaves like a magnet with north polarity at one end and south polarity at the other. They also enable the polarity of each end of the coil to be determined. Verify in this way the following rule for remembering the relation between direction of current and the resulting polarity. "Look along the axis of the coil: **if the current goes round the near end in a clockwise direction that end will have south polarity; if it goes round in an anticlockwise direction the polarity will be north.** An easy device for remembering this rule is shown in Fig. 447.

Remembering that lines of force run outwards from north poles and inwards to south poles, examine Figs. 443, 444, and 446 and note that they are in agreement with this rule.

Coils with Iron Cores

Set up coil and compass needle as for Expt. 4 on page 539. Increase the distance of the coil from the needle until the latter only undergoes very slight deflection when current passes through the coil. Now gradually slide a rod of soft iron along the axis of the coil from the end away from the needle. The deflection will be considerably increased, showing that the intensity of the field along the axial line has been greatly increased by the presence of the iron core. This may be explained in two ways. It may be said that the magnetic lines pass more easily through the iron than through the air and so crowd into the bar making a more intense field along the axial line where the compass is situated. (Compare Fig. 358.) Alternatively it may be said that the field due to the coil " induces " magnetism in the iron bar, and the lines of force due to the magnetised iron are added to those due to the coil.

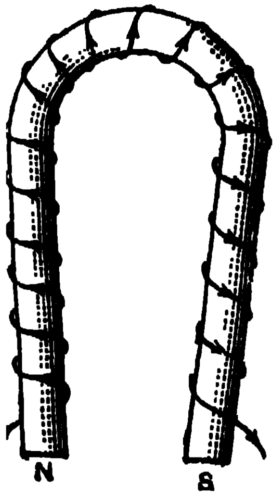


FIG. 448. AN ELECTROMAGNET.

If a rod of hard steel is used instead of one of soft iron, the effect on the field will not be so great but the rod will be found to be permanently magnetised when it is withdrawn, the poles being arranged as already indicated for the coil. This is the method by which bar magnets are usually made. The experiment also illustrates the fact that hard steel is more difficult to magnetise than soft iron but retains its magnetism better.

Electromagnets

The last section explains the principle of electromagnets. By wrapping a large number of turns of insulated wire around a soft iron core a very powerful magnetic effect is obtained when a small current is passed. Soft iron is used for the core both because it is so easily magnetised and because it tends to loose its magnetism again when the current is stopped. Fig. 448 shows how a horseshoe-shaped piece of iron may be wound so as to produce a horseshoe magnet by means of which the inductive effect of both poles can act on the same piece of iron thus increasing the lifting power.

Electromagnets used for lifting are often made in the form shown in Fig. 449. If the current is passed in such a way that the central projection becomes a north pole, the edges will have south polarity and all the lines of force will be concentrated on the piece of iron being lifted. Such electromagnets are useful in ironworks, since the load can be attached and released by switching the current on and off. The picture on page 542 shows several hundredweights of iron discs being lifted by such a magnet.

They can also be used for sorting out iron and steel articles from others made of non-magnetic material, and are sometimes used in this way to sort out tin cans, etc., from other rubbish at refuse dumps.

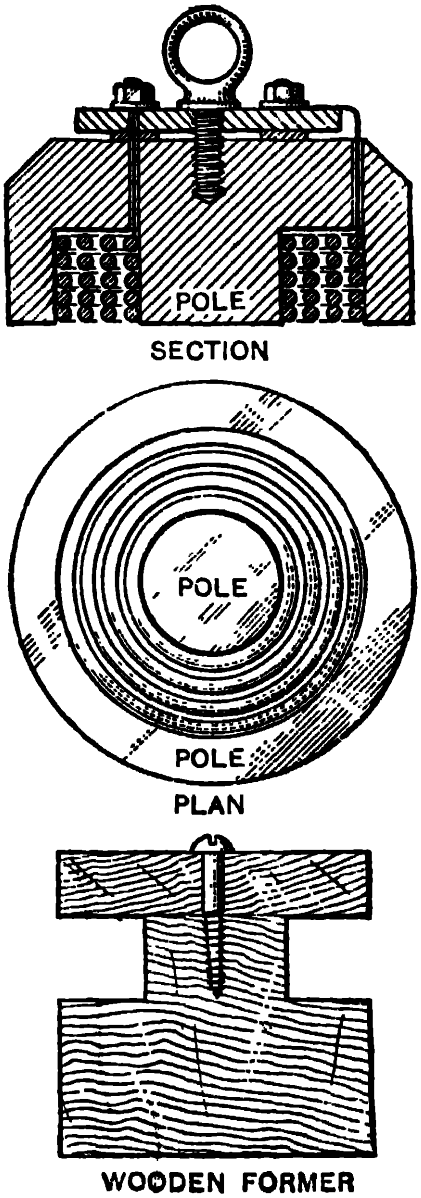


FIG. 449.

The Electric Bell

Fig. 450 illustrates the essentials of an electric bell. The leads are attached to the two terminals shown at the top. When current passes round the coils in the direction shown the iron core is magnetised, the upper pole piece becoming a north pole and the lower one a south pole. This attracts the piece of soft iron, A, causing the striker to hit the bell. The movement of A breaks the circuit at S so that the current ceases to flow and the core loses its magnetism. The spring by which A is attached to the screw above it now pulls A back, so that contact is made again at S and the process is repeated. This will continue so long as the circuit to the terminals is closed.

The Morse Sounder and Inker. Relays

THE MORSE SOUNDER.—In Fig. 451 the sending apparatus of a telegraph system is shown on the left, and the receiving apparatus on the right. When the key, K, is depressed and makes contact with M, current can pass from the battery, B, through the key, along the line, round the coils of the electromagnet, E, and back to the battery through the earth. The electromagnet attracts the piece of soft iron, X, making L strike P_1. On releasing K, the spring S_1 pulls it from contact with M, breaking the circuit so that the attraction of E for X ceases. S_2 then pulls L away from P_1 and makes it strike P_2. The

Courtesy of G.E.C. Ltd.

AN ELECTRIC LIFTING MAGNET CARRYING A LOAD OF 1½ TONS.

interval between the taps on P_1 and P_2 is fixed by the length of time K is kept down, and thus the "long" and "short" signals of the Morse code can be transmitted.

The Morse Inker.—Morse messages sent as above may be automatically recorded by the arrangement shown in Fig. 452. The circuit arrangements are similar to those on the sounder. When E is magnetised it pulls up the piece of soft iron below it, thus pressing the inky point of the inker down on the moving paper tape. By holding K down for long and short intervals, long and short marks can be made on the tape.

Relays.—The currents transmitted along telegraph wires are very small and may not be sufficiently powerful to work an inker mechanism. In that case, as shown in Fig. 453, E is made to close a light switch which completes a second circuit with its own battery sending a strong current round the coils of the inker magnet.

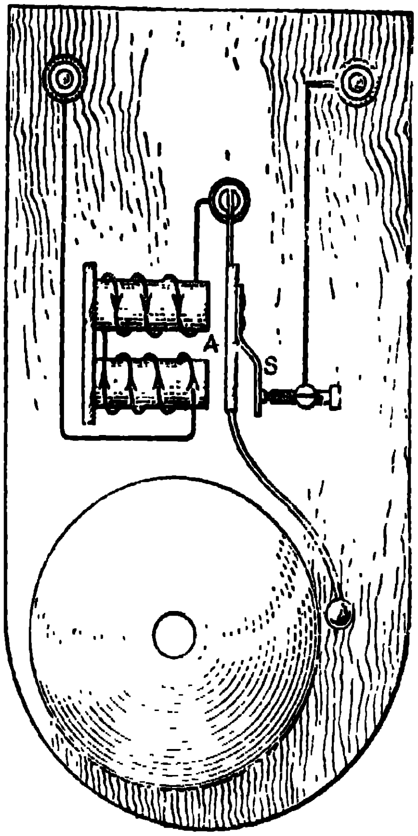


Fig. 450. The Electric Bell.

The Telephone Receiver

As shown in Fig. 454, the receiver contains a permanent magnet, M, to which pole pieces, P, of soft iron are attached. Around these are

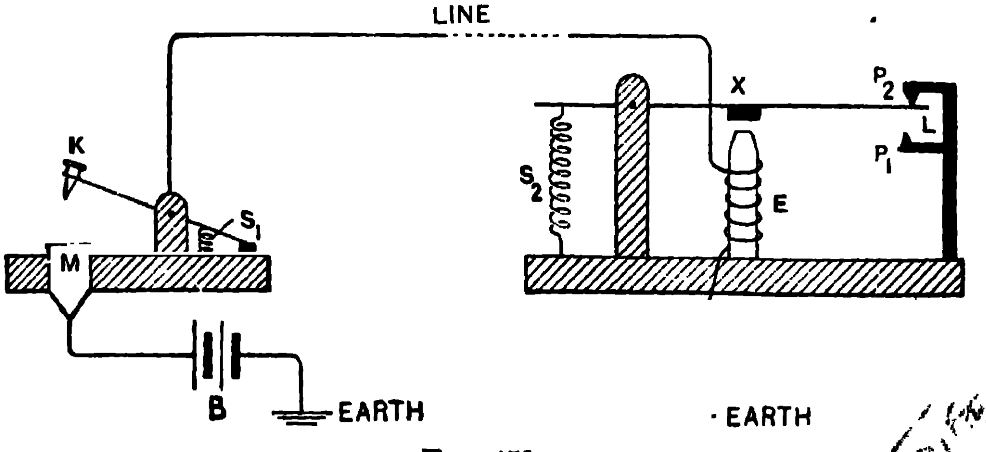
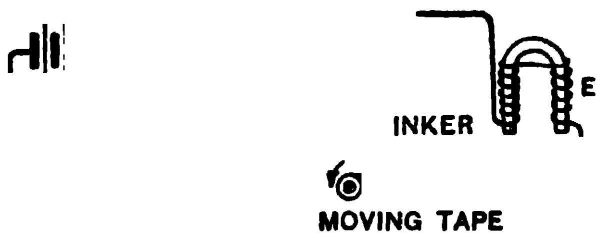


Fig. 451.



CIRCUIT COMPLETED THROUGH EARTH

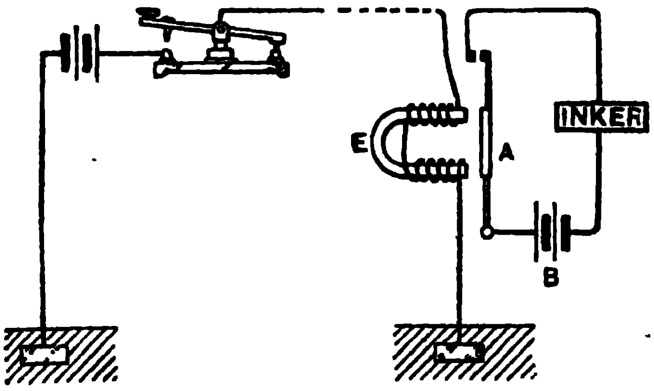
Fig. 452. Simple Telegraphic Circuit.

coils, C. A thin diaphragm of soft iron, D, is stretched across the earpiece near these pole pieces. When the transmitter is spoken into a similar diaphragm is set into vibration. Its movements cause changes in the strength of the current passing between the two instruments and round the coils, C. This causes variations in the magnetic strength of the pole pieces. D is thus subjected to varying attractions which cause it to spring backwards and forwards, reproducing the movements of the transmitter diaphragm and so sending reproductions of the original sound waves to the ear applied to the opening.

Loud Speakers. Motor Car Direction Indicators

(1) Moving Coil Loud Speaker.—This depends on the fact that a coil carrying a current will be attracted or repelled by a magnet. As shown in Fig. 455, a light coil attached to a stiff paper cone is inserted between the poles of a circular magnet which may be a permanent magnet or an electromagnet. Variations in the current passing round the coil cause variations in the attraction between it and the magnet, so setting the coil and the cone in vibration and reproducing the sound waves which gave rise to the current variations.

(2) Automatic Direction Indicators on Cars.—The arm is pivoted at P (Fig. 456). The iron weight, W, is just too light to pull the arm up. When current is passed through the coil C the magnetic field set up acts inductively on W, magnetising it in such a way that it is attracted into C and the arm is raised. When the current is stopped the magnetic effects disappear and the arm drops again.



CIRCUIT COMPLETED THROUGH EARTH

Fig. 453. The Use of a Relay.

Effect of Magnetic Field on a Conductor Carrying Current

It has been shown that a conductor carrying current exerts forces on magnetic poles near it. From the law of action and reaction it will follow that the magnetic poles exert forces on the conductor.

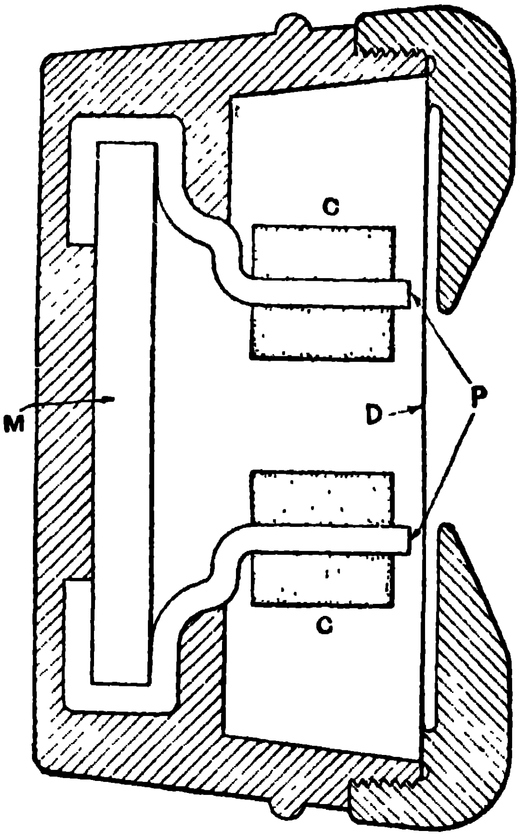


Fig. 454.

If the straight conductor, AB, in Fig. 457 carries current in a downward direction the lines of force around it would have the direction shown. Hence a magnetic north pole placed at P would tend to be driven along PR. Since reaction is opposite to action, if the pole at P is fixed and AB is free to move, the latter will be driven in the direction OX. Note that the motion of the conductor is in a direction at right angles to both the lines of force due to the pole at P—broken lines—and to the direction of the current. If the current is reversed the direction of motion will be reversed.

This effect may be shown experimentally by the apparatus shown in Fig. 458. AC is a piece of stout copper wire attached by a flexible wire to W. It dips into mercury in the trough M. NS is a magnet inserted through the base of the stand with its north pole upwards. Terminal Y is connected through B to W. Terminal X is connected to the mercury in M. When positive current is passed in at Y and out at X the wire AC will rotate as indicated in Fig. 457. When the current is reversed it will rotate in the opposite direction.

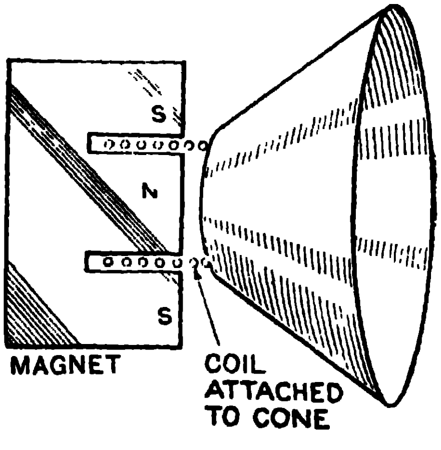


Fig. 455.

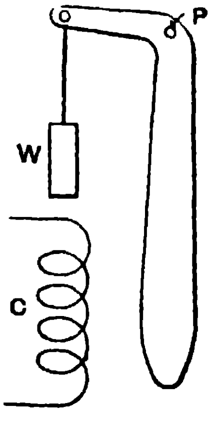


Fig. 456.

The direction of motion can be remembered by **Fleming's Left Hand Rule**, viz. *Hold the thumb, first and second fingers of the left hand so as to be mutually at right angles. Point the first finger in the direction of the field and the second in the direction of the current. The thumb will then point in the direction in which the conductor tends to move* (Fig. 459).

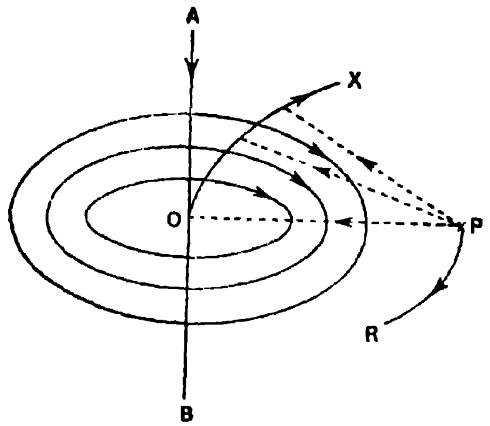


FIG. 457.

Galvanometers

Any arrangement, such as the magnetic needle which has been mentioned several times, which will indicate the presence of a current is called a **galvanoscope.** If readings taken from it allow current strengths to be compared it is a **galvanometer.** When direct readings of current strengths in amperes or milliamperes (thousandths of amps.) can be obtained from it it is an **ammeter** or a **milliammeter.**

The measurement of current by silver and copper voltameters, while very accurate if carefully conducted, is a long and tedious process and is only used in practice to test the readings of other instruments. Immediate readings may be made from instruments depending on magnetic effects.

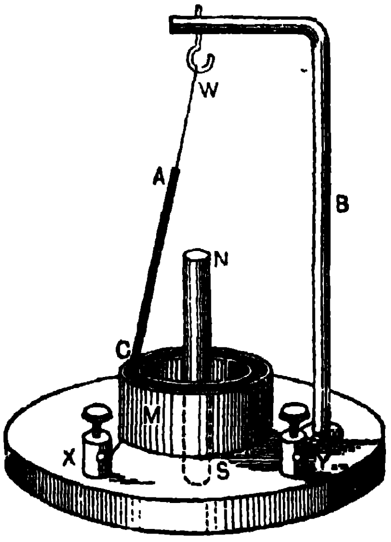


FIG. 458.

MOVING NEEDLE GALVANOMETERS.—The **tangent galvanometer** shown in Fig. 460 is typical of these. It has a coil in a vertical plane through which the current to be measured is passed. Suspended at the centre of the coil is a short magnet carrying a light aluminium pointer at right angles to it with a circular scale of degrees below it.

Before passing the current the coil is turned so that its plane coincides with that of the magnetic meridian, that is, so that the magnet sets in the plane of the coil. When current passes through the coil the resulting field at its centre will act along its axial line (see page 538). Thus the magnet will be under the influence of the horizontal component of the earth's field, H, and a field of intensity, F, due to the coil and at right angles to the earth's field (Fig. 461). This will cause a deflection, θ, of the needle. The conditions are exactly the same as in the case of the magnetometer —page 463— and by the same type of reasoning it can be shown that $\frac{F}{H} = \tan\theta$. Therefore, if two different currents, C_1 and C_2, produce fields F_1 and F_2 respectively resulting in deflections θ_1 and θ_2, we may write $\frac{F_1}{F_2} = \frac{\tan\theta_1}{\tan\theta_2}$. But the fields will be proportional to the currents producing them, so $\frac{C_1}{C_2} = \frac{\tan\theta_1}{\tan\theta_2}$. Hence the strengths of the two currents may be compared.

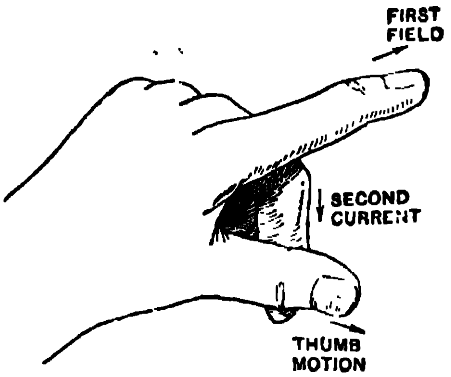


FIG. 459.

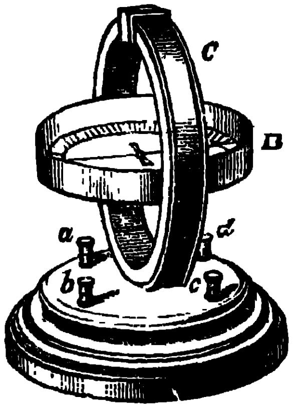


FIG. 460.

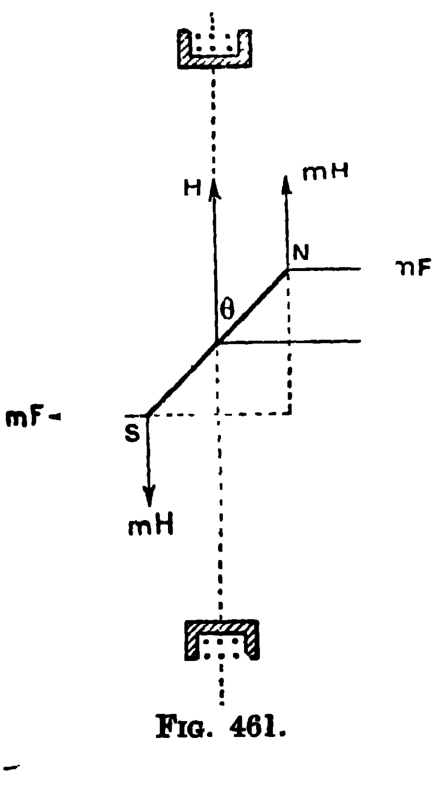


FIG. 461.

From the above it will be seen that *at a given place* the tangent of the angle of deflection is proportional to the current in the coil, though if the galvanometer is used in different places the deflection produced by a given current will vary owing to the variation of the horizontal component of the earth's field. At a fixed place the relation between current and deflection may be written

Current = K tan θ,

where K is a constant. This constant is known as the **reduction factor** for the instrument at the given place. It may be determined by arranging the galvanometer and a copper voltameter in the same

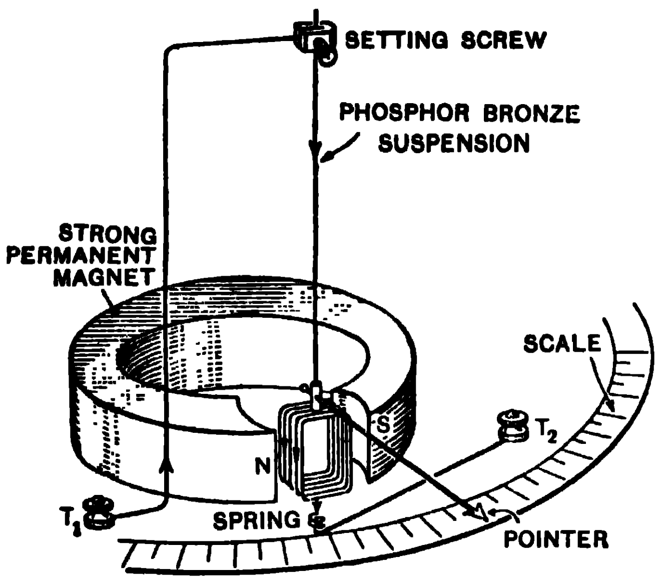


Fig. 462. A Moving Coil Galvanometer.

circuit. The actual value of the current passed can be calculated from the increase in weight of the cathode of the voltameter (see page 509) and the corresponding value of θ read from the galvanometer, and then K can be calculated. Once K is known the actual value of currents passed through the galvanometer can be calculated from the observed deflections.

The tangent galvanometer is not much used in laboratories now, but it is still used commercially for testing other instruments.

Moving Coil Galvanometers.—In these a light coil is suspended between the poles of a powerful permanent magnet bent into a circular form. It is suspended from a strip of phosphor bronze and connected below to a light spring. The current to be measured can be passed through it from terminal T_1 to T_2 by way of the suspension and spring. The reaction between the field of the magnet and that of the coil will then cause the coil to be rotated. Torsion in the suspension and spring oppose this rotation, and as the force due to torsion is proportional to the angle of twist, the amount of deflection will be proportional to the moment of the forces producing it. The coil may carry a light pointer to show the amount of deflection produced.

If a square coil is set with its plane parallel to the lines of force of a magnetic field, Fleming's left-hand rule will show that forces as indicated in Fig. 463 act on it when current passes through it. That on BC is directed forwards; that on AD towards the back, both being at right angles to the plane of the coil. These constitute a couple of moment $F \times DC$ tending to rotate the coil. As it rotates the forces will no longer be perpendicular to the plane

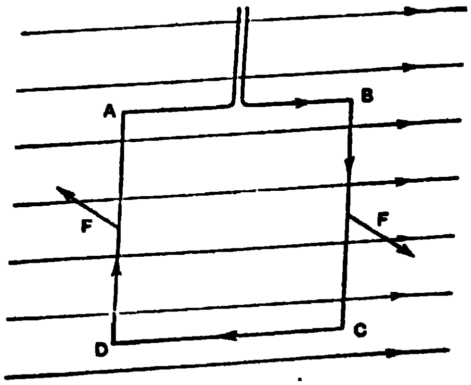


Fig. 463.

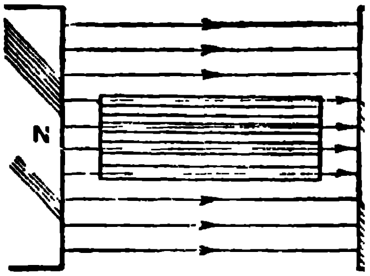


FIG. 464.

of the coil and the moment of the couple will decrease.

These conditions obtain in a galvanometer if the poles of the magnet are straight and parallel, as shown in Fig. 464. Hence, the more the coil rotates when a given current is passed the smaller will be the moment of the couple causing the rotation. The result of this is that the rotations are not proportional to the currents, but, for example, a current of 2 amps. will cause less than twice the deflection caused by a current of 1 amp.

Deflections which are proportional to the currents producing them are obtained if the poles are made concave and a soft iron cylinder is mounted at the centre of the coil (Fig. 465). The magnet acts inductively on this core and, as indicated, a radial field is obtained. Thus the plane of the coil will always coincide with that of the lines of force passing through it, and the deflecting couple will have a constant moment for any particular value of the current as the coil rotates.

The scale over which the pointer works may be standardised by placing the galvanometer and a copper voltameter in series while current is passed through them. By means of the voltameter the actual strength of the current used can be determined and thus the actual current indicated by the galvanometer deflection may be marked on the scale. When this has been done for a number of points on the scale the galvanometer becomes an ammeter. If the magnet poles are square the scale will not be evenly spaced, but for the reasons already given the graduations will be nearer together in the regions registering high currents than in those marking low ones. The scale of a radial field ammeter will be equally spaced.

Moving coil galvanometers have displaced moving needle instruments because the latter are controlled by the earth's field and so give different readings for equal currents at different places and have to be set up with their coils

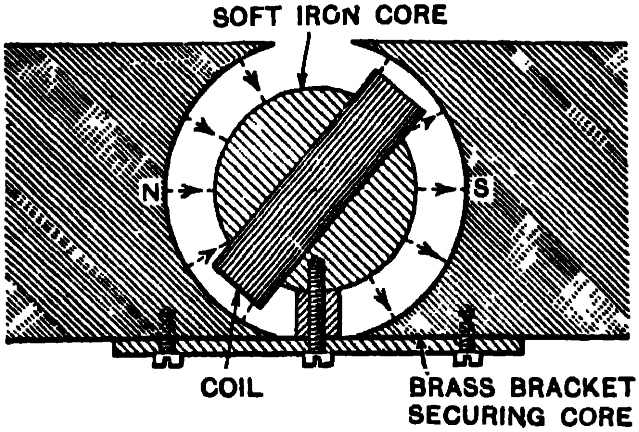


FIG. 465.
METHOD OF OBTAINING A RADIAL FIELD.

in a particular direction. Also their readings are much affected by the fields of any other magnets which may be near them. The field between the poles of a moving coil galvanometer is so powerful that outside fields have comparatively negligible effects on the movement of the coil.

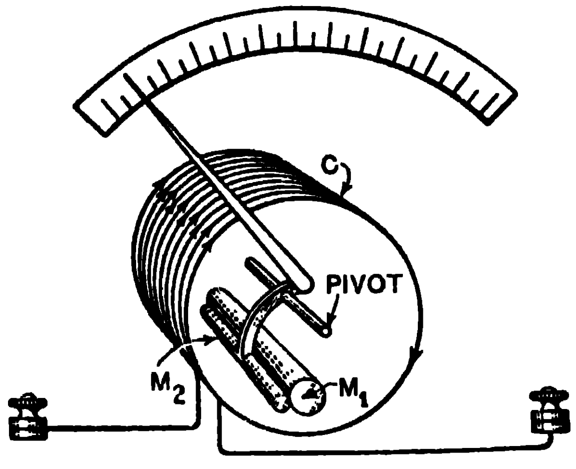


FIG. 466. MOVING IRON GALVANOMETER.

Moving Iron Ammeters

The construction of these is indicated in Fig. 466. Inside a coil and parallel to its axis is a fixed rod of soft iron, M_1. Another rod of soft iron, M_2, is mounted parallel to this on a pivoted arm, the other end of which carries a pointer passing over a scale. When current is passed through the coil both iron rods will be magnetised by induction by the field set up by the current and will have like poles in contact. Hence M_2 will be repelled from M_1 and the pointer will be moved. The stronger the current, the greater will this repulsion be and the greater the reading on the scale.

Moving iron instruments have the advantage of being able to measure alternating currents, that is currents which first flow one way and then the other. In a moving coil instrument this would tend to fling the coil first one way and then the other so that if the alternations were rapid it would just quiver without undergoing a definite deflection. In a moving iron instrument alternations of current do reverse the polarity of the iron rods, but since it is reversed in both rods, like poles will still be at neighbouring ends and the repulsion will persist.

QUESTIONS ON CHAPTER XLV

1. Describe, with the aid of clear diagrams, the chief features of the magnetic fields due to a current flowing in (*a*) a long straight wire, (*b*) a large circular wire, (*c*) a solenoid. [L.U.

2. Describe two experiments which illustrate the *magnetic effect* produced by an electric current, and point out two practical applications of this effect. [L.U.

3. Give a diagram and brief explanation of a circuit you would set up in order to demonstrate two effects produced by an electric current.

How can you use one of these effects to determine the direction of the current? [L.U.

4. Explain how you would use a magnetised needle to determine the direction of flow of a steady current of electricity in a straight wire, which must not be disturbed, when the wire is (*a*) horizontal and along the magnetic meridian, (*b*) vertical. State any rule that you would apply in the test, and sketch the arrangement of the lines of magnetic force round such a conductor. [L.U.

5. Describe an experiment to show that a current flowing in a wire produces a magnetic field. Show on a diagram the relation between the direction of the current and the direction of the field.

Describe briefly any instrument which depends for its action on the magnetic effect of a current. [L.U.

6. Describe two experiments you would perform in order to find the nature of the magnetic field associated with a current flowing in (*a*) a straight wire, (*b*) a circular coil, and also the relative directions of the fields and the currents.

Draw a diagram summarising the results you expect to obtain, and state a rule which expresses these results. [L.U.

7. A long straight wire conveys an electric current vertically downwards through a hole in a drawing board. Draw a map of the lines of magnetic force, marking their direction, which may be obtained on the board, and describe how you would experimentally produce this diagram.

Draw a second diagram showing the magnetic field on the board when there are two long wires about 5 cm. apart both conveying the same current downwards. In this case neglect the effect of the earth's field. [L.U.

8. A small rectangular coil of copper wire is placed in the magnetic meridian with its short sides vertical. A magnetised needle is pivoted at the centre of the coil, which is then connected in series with a key and a simple cell.

Sketch the arrangement and explain carefully what you would expect to observe in the cell and in the behaviour of the needle when the circuit is closed. [L.U.

9. Describe and explain the action of any instrument with which you are acquainted for measuring or detecting an electric current. Give an example of its use. [L.U.

10. Describe the construction and action of some form of ammeter with which you are acquainted.

If you were provided with a copper voltameter how would you proceed to test the accuracy of the ammeter reading at a particular point of the scale ? [L.U.

11. Give a careful diagram of an electric bell and explain clearly how it works.

Point out two common causes of failure in a bell circuit. How can these defects be remedied ? [L.U.

12. Describe the construction of an electromagnet. Show on a careful diagram the resulting polarity and the arrangement of the lines of magnetic force.

Give one practical application of an electromagnet. [L.U.

13. Describe and explain the principles underlying the action of a simple telephone receiver. [L.U.

14. Describe, with aid of a diagram, the construction and mode of action of an electric bell State the type of battery you would use with a bell circuit and point out the advantages and disadvantages of this battery. [L.U.

15. With the aid of a diagram describe an electric bell and explain its operation.

Draw a circuit diagram showing the bell circuit of a house in which cells are used to supply the current for the bells, there being a push in each of two rooms and at 1 door. State the type of cell which would be used and give *two* reasons why it would be chosen. [J.M.B.

16. Describe how telegraphic signals can be transmitted by a Morse sounder system.

Explain the modifications made in that system for automatic recording of the signals, a relay being used. What is the reason for using a relay ?

17. Briefly describe the main principles on which the action of moving needle, moving iron, and moving coil galvanometers depend, and discuss their relative advantages and disadvantages.

CHAPTER XLVI

OHM'S LAW. RESISTANCE

Ohm's Law

Different potential differences applied to the ends of the same conductor will drive different currents through it, and the same potential difference applied to different conductors will produce different currents. This was investigated by G. S. Ohm in 1827, and as a result of his work the following law was established. **The current passing through a "metallic conductor" at constant temperature is proportional to the potential difference applied to its ends.** In that statement the term "metallic conductor" includes conductors such as carbon which are not chemically affected by currents but excludes electrolytes. The law does apply to electrolytes under suitable conditions but this is not always obvious owing to polarisation effects.

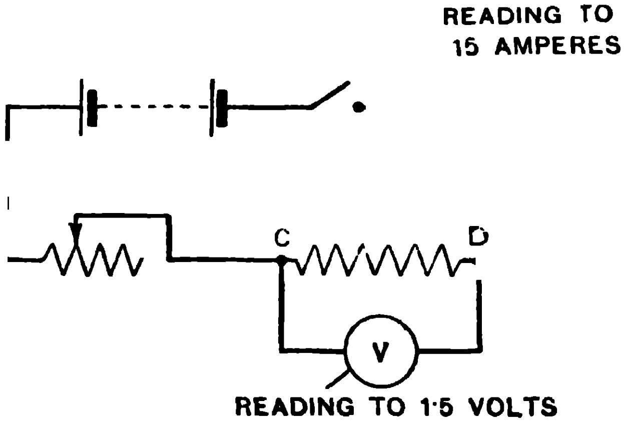


Fig. 467.

Illustration of Ohm's Law.—Set up a circuit as indicated in Fig. 467. CD is a coil of wire. A is an ammeter which will indicate the strength of current in the circuit and therefore the current through CD. V is a voltmeter which measures potential differences and so will give the potential difference between C and D. In series with CD is a variable resistance by means of which the current can be varied.

Commence with a large resistance, close the circuit and take readings of both A and V. Reduce the resistance a little and both readings will rise. Note their new values. Proceed in this way until a number of sets of readings have been obtained. Tabulate your results in the form:

Voltmeter Reading = P.D. between C and D	Ammeter Reading = Current through CD	$\frac{\text{P.D. between C and D}}{\text{Current through CD}}$

An approximately constant value should be obtained for the ratio in the last column. This result agrees with the law.

The method should not be stated to be a verification of the law, for as will be shown later the voltmeter is graduated on the assumption that Ohm's Law does apply. Verification can be obtained by using instruments designed to measure potential differences and based on electrostatic principles not involving Ohm's Law. These instruments are beyond the scope of this book.

Resistance

Ohm's Law as applied to the current in any particular conductor may be expressed by the equation

$$\frac{\textbf{Potential Difference}}{\textbf{Current}} = R,$$

R being a constant for the given conductor. This constant is termed the **resistance** of the conductor. From the form of the equation it is clear that when R is big the conductor requires a large potential difference to be applied to its ends to send a unit of current through it. Hence R may be said to indicate the resistance which the conductor offers to the passage of current through it.

If the potential difference is measured in volts and the current in amperes, the resistance is said to be given in **ohms** by the ratio in the above equation.

$$\frac{\text{Volts}}{\text{Amps.}} = \text{Ohms.}$$

Hence an ohm may be defined as **the resistance of a conductor which requires a potential difference of one volt to be applied to its ends to send a current of one ampere through it.** The Board of Trade define the ohm as *the resistance of a column of mercury, at the temperature of melting ice,* 14·4521 *grm. in mass, of constant cross-sectional area, and of length* 106·3 *cm.* This is a practical definition which merely describes a conductor which has been found to have a resistance of 1 ohm and which can be set up at any time for comparison with other resistances.

The general relation for a conductor

$$\frac{\text{Potential difference}}{\text{Current}} = \text{Resistance}$$

may also be written Pot. diff. = Current × Resistance or $\frac{\text{Pot. diff.}}{\text{Resistance}}$ = Current. Thus a potential difference of 10 volts applied to a conductor of 5 ohms resistance will send through it a current of $\frac{10}{5}$ amps. = 2 amps., and when a current of 3 amps. is flowing through a conductor of 4 ohms resistance, a potential difference of 4 × 3 volts = 12 volts must exist between its ends.

Factors on which Resistance Depends

The experiment described on page 553 may be regarded as an experiment to find the resistance of the coil since Pot. diff./Current = Resistance. In this way compare the resistances of (*a*) wires of the same substance and thickness but of different lengths, (*b*) wires of same substance and lengths but different thicknesses, (*c*) wires of the same length and thickness but of different substances.

Your results should indicate (1) That the resistance of a wire is proportional to its length. (2) It is inversely proportional to the cross-sectional area. (3) It depends on the substance of the wire. From this it follows that the fraction $\frac{\text{Resistance} \times \text{Cross-sectional area}}{\text{Length}}$ for any conductor, that is, **the resistance of 1 cm. of a conductor of 1 sq. cm. cross-sectional area** is a number which depends on the substance. It is called the **specific resistance** or **resistivity** of the substance.

EXAMPLE.—*A copper wire* 100 *cm. long and* 2 *sq. mm. in cross-section passed a current of* 0·2 *amp. when a potential difference of* 2 *millivolts was applied to its ends. What is the specific resistance of copper?*

$$\text{Resistance of wire} = \frac{\cdot 002}{\cdot 2} = \cdot 01 \text{ ohms;}$$

$$\therefore \text{ Specific resistance} = \frac{\cdot 01 \times \cdot 02}{100} = \cdot 000002 \text{ ohms per cm. cube.}$$

Since a conductor with small resistance will readily allow large currents to pass through it, the reciprocal of the resistance of a conductor is termed its **conductivity**, and the reciprocal of the specific resistance of a substance is its **specific conductivity.** Conductivity is said to be measured in reciprocal ohms or mhos. Thus the conductivity of the wire mentioned in the last example would be $\frac{1}{\cdot 01}$ mhos = 100 mhos, and the specific conductivity of copper would be $\frac{1}{\cdot 000002} =$ 500,000 mhos per cm. cube.

Resistance and Temperature.—Usually the resistance of a conductor rises as its temperature rises. The passage of a current itself heats a wire, so in experiments where resistances are being measured current should never be passed for a long time but a key should be used and the current allowed to pass only for a sufficient length of time for instruments to be read.

Equivalent Resistance

If a number of conductors are joined together the whole system will offer a certain resistance to current. We can imagine one conductor being substituted for the system and offering the same resistance. The resistance which that conductor would have is said to be the equivalent resistance of the system.

Conductors in Series.—When a number of conductors are joined end to end so that the current has to pass through each in turn, they are said to be joined in series. Thus the resistance coil P, the lamp L, and the ammeter A in Fig. 468 are joined in series.

Connecting conductors in series is equivalent to lengthening the conductor, so each conductor added to the series adds to its resistance. The current has to overcome the resistance of each conductor in turn and so **the equivalent resistance of the system is the sum of the separate resistances.** In the case shown the equivalent resistance would be 50 + 100 + 5 ohms = 155 ohms. The result may be put in the

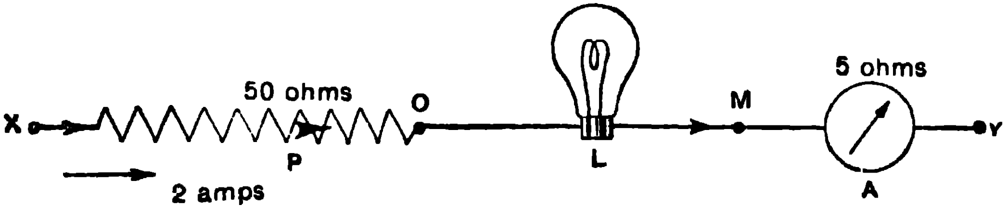


Fig. 468.

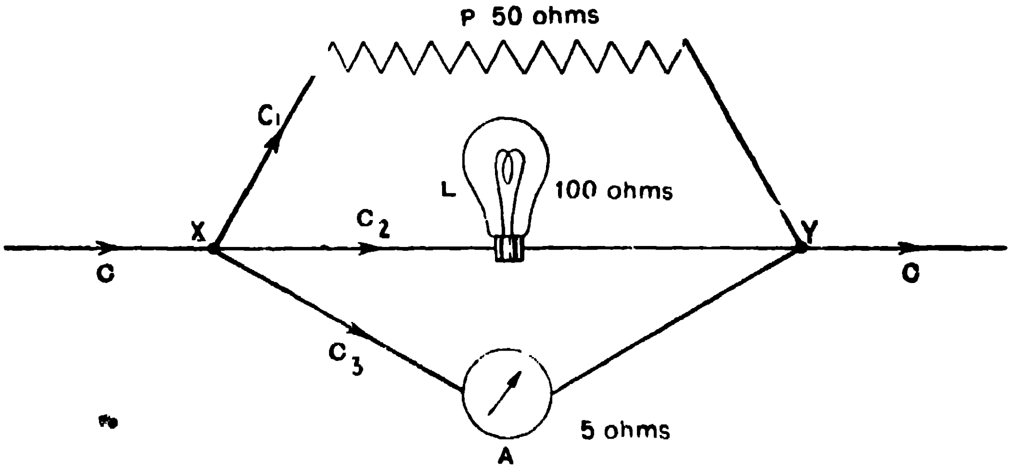


FIG. 469.

general form $R = r_1 + r_2 + r_3$, etc., where R is the equivalent resistance and r_1, r_2, r_3, etc., are the separate resistances of the conductors.

A more formal proof is as follows:- Suppose the equivalent resistance of the system in Fig. 468 is R ohms, and that a current of 2 amps. is flowing from X to Y. Then, since Pot. diff. = Current × Resistance,

P.D. between X and Y = 2R volts.
Also, P.D. between X and O = 2 × 50 volts
P.D. between O and M = 2 × 100 volts
P.D. between M and Y = 2 × 5 volts;

$$\therefore\ 2R = (2 \times 50) + (2 \times 100) + (2 \times 5) = 2\,(50 + 100 + 5);$$

$$\therefore\ R = 50 + 100 + 5.$$

If 50, 100, and 5 are replaced by r_1, r_2, r_3 we obtain

$$R = r_1 + r_2 + r_3.$$

CONDUCTORS IN PARALLEL.—When a number of conductors join the same two points, so that current passing between those points divides itself between then, as in Fig. 469, they are said to be in parallel.

Connecting conductors in parallel is equivalent to increasing the cross-section of the conductor, and so each conductor added *decreases* the equivalent resistance. The relation in this case is that the sum of the reciprocals of the separate resistances equals the reciprocal of the equivalent resistance which may be put in symbols

$$\frac{1}{R} = \frac{1}{r_1} + \frac{1}{r_2} + \frac{1}{r_3}, \text{ etc.}$$

Proof. Suppose in Fig. XY, there is a potential difference of 10 volts between X and Y. Then:—

Current through P = $\frac{10}{50}$ amp.
Current through L = $\frac{10}{100}$ amp.
Current through A = $\frac{10}{5}$ amp.;

$\therefore$ Total current from X to Y $= \frac{10}{50} + \frac{10}{100} + \frac{10}{5} = 10\left(\frac{1}{50} + \frac{1}{100} + \frac{1}{5}\right)$ amp.

If the equivalent resistance between X and Y is R ohms, current from X to Y = 10/R amps.;

$$\therefore\ 10/R = 10\left(\tfrac{1}{50} + \tfrac{1}{100} + \tfrac{1}{5}\right),\ \textit{i.e.}\ 1/R = \tfrac{1}{50} + \tfrac{1}{100} + \tfrac{1}{5}.$$

Putting r_1, r_2, r_3 for 50, 100, 5, we obtain

$$\frac{1}{R} = \frac{1}{r_1} + \frac{1}{r_2} + \frac{1}{r_3}.$$

Examples.—(1) *Find the current between* X *and* Y (*Fig.* 468) *when a potential difference of* 30 *volts is applied.*

$$\text{Equivalent resistance} = 50 + 100 + 5 \text{ ohms;}$$

$$\therefore\ \text{Current} = \tfrac{30}{155} \text{ amp.} = 1{\cdot}94 \text{ amps.}$$

(2) *Find the equivalent resistance between* X *and* Y (*Fig.* 469). *Also find the P.D between* X *and* Y *and the current through each conductor when the total current is* 3 *amps.*

Let equiv. resistance = R ohms. Then:—

$$\frac{1}{R} = \frac{1}{50} + \frac{1}{100} + \frac{1}{5} = \frac{2 + 1 + 20}{100} = \frac{23}{100};$$

$$\therefore\ R = \tfrac{100}{23} = 4{\cdot}35.$$

$$\text{Equiv. resistance} = 4{\cdot}35 \text{ ohms.}$$

If total current is 3 amps.,

$$\text{P.D. between X and Y} = 3 \times 4{\cdot}35 = 13{\cdot}05 \text{ volts;}$$

$$\therefore\ \text{Current through P} = \frac{13{\cdot}05}{50} = {\cdot}261 \text{ amp.}$$

$$\text{Current through L} = \frac{13{\cdot}05}{100} \text{ amp.} = {\cdot}1305 \text{ amp.}$$

$$\text{Current through A} = \frac{13{\cdot}05}{5} \text{ amps.} = 2{\cdot}61 \text{ amps.}$$

[Check. ·261 + ·1305 + 2·61 = 3·0015 = 3 (approx.). The sum of the separate currents should be equal to the total current. This is not quite the case in the above as the answers have been approximated and are not exact.]

Variable Resistances

A common type of variable resistance, shown in Fig. 470, consists of a coil of wire wound on an insulating former. The turns of the wire are either spaced out or insulated from one another. The current passes in at one end of the coil and out through a terminal which can slide along a bar and make contact at various points along the coil. Thus, only the resistance between the end where the current enters and the sliding contact has any effect on the current passing.

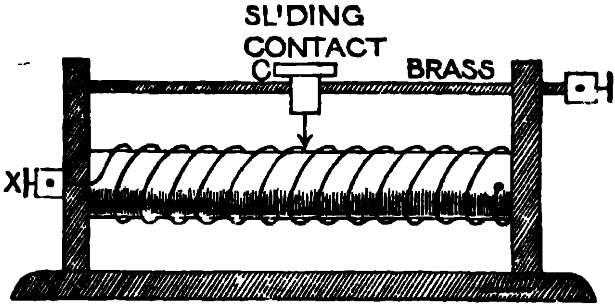


FIG. 470.

Another common type consists of a number of flat carbon blocks between two metal plates fitted with terminals. A screw passing through the framework holding the blocks can be made to press the blocks tightly together or allow them to lie loosely in the frame. When they are loose they make contact with one another at a few points only and so have a high resistance. Tightening the screw improves the contact and so reduces the resistance.

Lamp resistances are frequently used in establishments where accumulator charging, etc., is carried on. These consist of a number of lamp sockets arranged in parallel (Fig. 471). Suppose a number of lamps, each of 100 ohms resistance, are available. If one is inserted in a socket current can only pass through that one branch and the resistance between A and B is 100 ohms. If now another lamp is inserted there are two conductors in parallel between A and B, each of 100 ohms, and calculation as on page 558 will show the equivalent resistance to be $\frac{100}{2}$ ohms. Similarly with three lamps the resistance will be $\frac{100}{3}$ ohms, and so on.

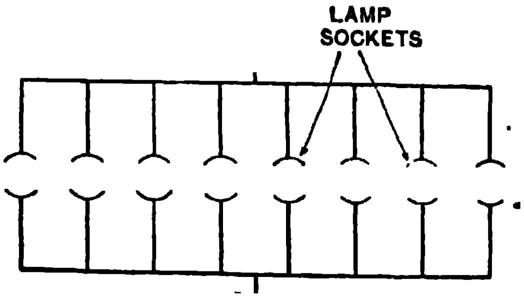


FIG. 471.

Current regulators of the above types are often called **rheostats.**

The **resistance box** is a variable resistance which enables standard known resistances to be added to a circuit. It contains a number of coils of known resistance connected in series by thick brass bars fixed

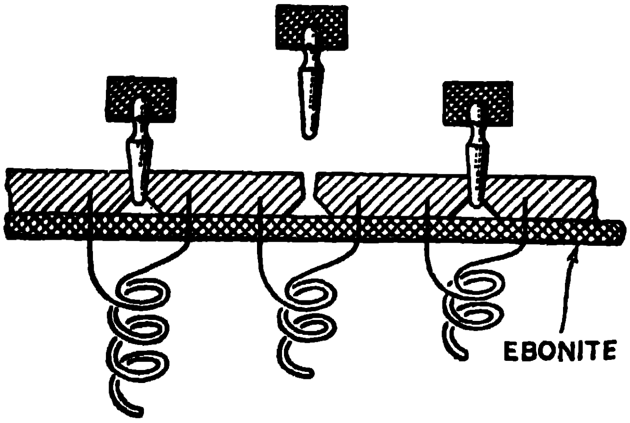


FIG. 472.
NON-INDUCTIVE COILS IN A RESISTANCE BOX.

on the insulating ebonite lid of the box. Metal plugs can be fitted in the gaps between these bars. The bars are thick so that their resistance is negligible. When a plug is removed current passing along the bars has to pass through the corresponding coil. Thus the resistance of that coil is added to the circuit. If two plugs are removed the current will have to pass through the two corresponding coils in series, and so the sum of their resistances is added to the circuit. The coils in order usually have resistances of 1, 2, 2, 5, 10, 20, 20, 50, etc., ohms. Thus, by withdrawing suitable plugs, any number of ohms resistance, up to the total for the box, can be made up.

The method of winding the coils shown in Fig. 472 should be noted. It ensures that at any point along the coil there are two neighbouring portions of current travelling in opposite directions. These will neutralise one another's magnetic effects so that no appreciable magnetic field is set up by the coil and effects due to self-induction—see Chapter XLVIII.—are avoided. Such winding is said to be *non-inductive.*

Shunts

It is sometimes necessary to arrange that only a portion of the current in a circuit shall pass through some instrument. The placing of a suitable resistance in parallel with the instrument secures this. A resistance so used is termed a shunt. Any desired fraction of the current may be made to pass through the instrument by choosing the shunt so that its resistance bears the proper ratio to

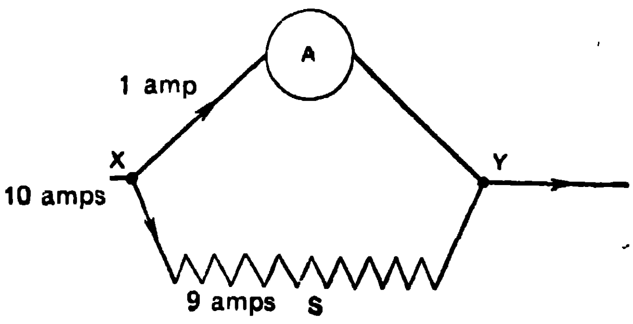


FIG. 473.

that of the instrument. For instance, suppose that Fig. 473 represents a portion of a circuit in which a current of 10 amps. is passing, and it is desired that only 1 amp. shall pass through A which has a resistance of 2 ohms. Then 9 amps. must pass through the shunt, S.

When 1 amp. passes through A the potential difference between X and Y must be 2 × 1 volts. Therefore, if 9 amps. are passing through S, the resistance of S must be $\frac{2}{9}$ ohms.

The range over which an ammeter will give readings is adjusted by fixing a suitable shunt in parallel with it. Suppose A, in Fig. 473, is an ammeter which gives a full scale deflection, that is the needle travels to the end of the scale, when a current of 15 milliamps (·015 amp.) passes through it and it is required to use it for measuring currents up to 15 amps. Also suppose its resistance to be 5 ohms. Then the shunt S must have such a resistance that, when 15 amps. is travelling around the main circuit, ·015 amp. will pass through A and 14·985 amps. through S.

With ·015 amp. passing through A, we have that the P.D. between X and Y = 5 × ·015 volts = ·075 volt.

Hence for 14·985 amps. to pass through S it follows that the resistance of S must be:—

$$\frac{\cdot 075}{14\cdot 985} = \cdot 005005 \text{ ohms.}$$

It will be seen from this example that the shunt will have a low resistance and the shunt and ammeter together will have a still lower one. An ammeter should have a low resistance or a considerable amount of the electrical energy supplied to the circuit will be used up in driving current through it instead of performing work in other parts of the circuit.

The proper shunt to give the range marked on the scale is usually incorporated in an ammeter. Sometimes additional shunts known as *multipliers* are provided separately so that the same instrument can be used for various ranges of current as desired.

Voltmeters

By connecting a suitable resistance in series with a galvanometer it may be adapted to measure potential differences and so become a voltmeter.

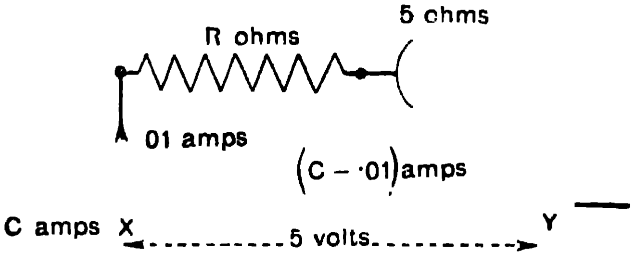


FIG. 474.

Suppose the galvanometer, G, in Fig. 474, gives a full scale deflection when 10 milliamps pass through it, and it is desired to use it as a voltmeter to measure potential differences up to 5 volts. A resistance R must be connected in series with it and must be of such a magnitude that, when the terminals of the complete instrument are connected to points X and Y between which there is a potential difference of 5 volts, a current of ·01 amp. will pass through it.

With P.D. = 5 volts across its terminals,

$$\text{Current through RG} = \frac{5}{R+5} \text{ amps.};$$

$$\therefore \frac{5}{R+5} = \cdot 01;$$

$$\therefore \cdot 01R + \cdot 05 = 5; \quad \therefore \cdot 01R = 4\cdot 95; \quad \therefore R = 495.$$

So the required resistance is 495 ohms.

It will be noticed that the resistance in a voltmeter is high. This is an advantage since it means that only a small fraction of the total current will pass through it and therefore the current in the part of the circuit across which it is connected will be approximately the same as that in the rest of the circuit.

Ohm's Law for a Complete Circuit

When a conductor is connected across the terminals of a cell, the E.M.F. of the cell is used in driving current around the circuit. But the cell itself has an internal resistance and part of the E.M.F. is required to drive the current against that resistance so that only a portion is available as a potential difference to drive the current round the external circuit.

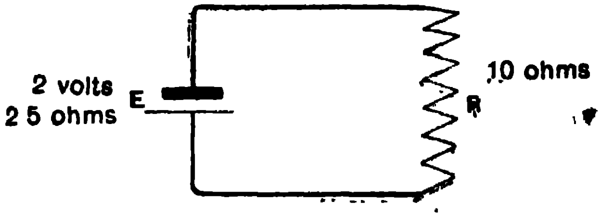


FIG. 475.

In Fig. 475, E represents a battery with E.M.F. of 2 volts and an internal resistance of 2·5 ohms to which a conductor of resistance 10 ohms has been connected.

$$\text{Total resistance of circuit} = 10 + 2{\cdot}5 \text{ ohms};$$

$$\therefore \text{ Current flowing} = \frac{2}{12{\cdot}5} \text{ amps.};$$

$$\therefore \text{ P.D. driving current through external circuit} = \frac{2 \times 10}{12{\cdot}5} = 1{\cdot}6 \text{ volts.}$$

$$\text{P.D. driving current through cell} = \frac{2 \times 2{\cdot}5}{12{\cdot}5} = {\cdot}4 \text{ volt.}$$

Since a voltmeter only measures the potential difference which is driving current through itself, it will not give an accurate measurement of the E.M.F. of a cell. For example, if a voltmeter of resistance 500 ohms is connected to the poles of the above cell,

$$\text{Total resistance of circuit} = 502{\cdot}5 \text{ ohms};$$

$$\therefore \text{ Current} = \frac{2}{502{\cdot}5} \text{ amp.};$$

$$\therefore \text{ P.D. through voltmeter} = \frac{2 \times 500}{502{\cdot}5} = 1{\cdot}992 \text{ volts.}$$

This would be the reading given by the voltmeter.

From the calculation it may be seen that the voltmeter reading will be very little less than the E.M.F. of the cell if the voltmeter has a resistance which is very high compared with the internal resistance of the cell.

Arrangement of Cells

The values of both internal and external resistance must be taken into account in deciding whether a series or parallel arrangement of cells will give the larger current. Consider the following examples.

EXAMPLES.—(1) *Three cells each of E.M.F.* 2 *volts and internal resistance* 2·5 *ohms are taken. Find the current they will send through a conductor of* 100 *ohms resistance* (*a*) *when connected in series*, (*b*) *when connected in parallel.*

(*a*) *In series.* Total resistance $= 100 + 2{\cdot}5 + 2{\cdot}5 + 2{\cdot}5$
$= 107{\cdot}5$ ohms.

$$\text{E.M.F. of battery} = 2 + 2 + 2 = 6 \text{ volts};$$

$$\therefore \text{ Current} = \frac{6}{107{\cdot}5} = {\cdot}0558 \text{ amp.}$$

(*b*) *In parallel.* Let R ohms be equivalent resistance of battery.

$$\frac{1}{R} = \frac{1}{2{\cdot}5} + \frac{1}{2{\cdot}5} + \frac{1}{2{\cdot}5} \quad \frac{3}{2{\cdot}5}; \qquad R = \frac{2{\cdot}5}{3};$$

$$\therefore \text{ Total resistance of circuit} = 100 + \frac{2{\cdot}5}{3} = \frac{302{\cdot}5}{3} \text{ ohms.}$$

$$\text{E.M.F. of battery} = 2 \text{ volts};$$

$$\therefore \text{ Current} = \frac{2}{\frac{302{\cdot}5}{3}} = \frac{6}{302{\cdot}5} = {\cdot}0132 \text{ amp.}$$

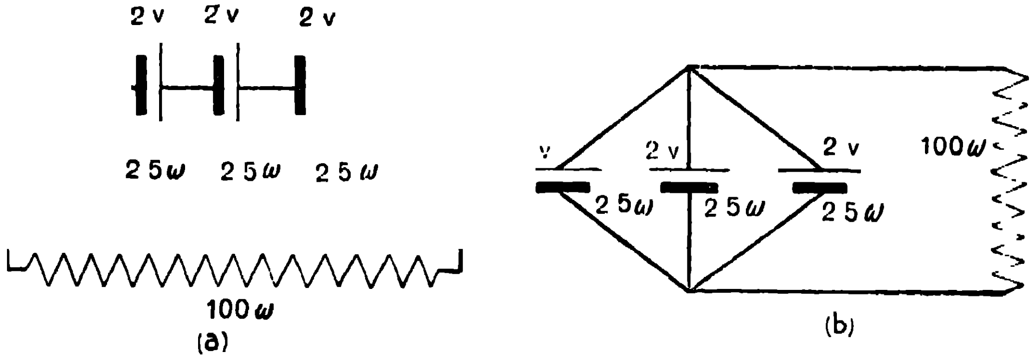


FIG. 476.

(2) *What would be the currents in the above examples if the external resistance were* ·01 *ohm ?*

(*a*) *In series.* Total resistance $= {\cdot}01 + 2{\cdot}5 + 2{\cdot}5 + 2{\cdot}5$
$= 7{\cdot}51$ ohms.

$$\text{E.M.F. of battery} = 6 \text{ volts};$$

$$\therefore \text{ Current} = \frac{6}{7{\cdot}51} = {\cdot}799 \text{ amp.}$$

(*b*) *In parallel.* Total resistance $= \frac{2{\cdot}5}{3} + {\cdot}01 = \frac{2{\cdot}53}{3}$ ohms.

$$\text{E.M.F. of battery} = 2 \text{ volts};$$

$$\therefore \text{ Current} = \frac{2}{\frac{2{\cdot}53}{3}} = \frac{6}{2{\cdot}53} = 2{\cdot}37 \text{ amp.}$$

Comparison of the two examples will show that the larger current is given by the series arrangement when external resistance is large compared with internal resistance, and the parallel arrangement gives the larger current when the external resistance is small compared with internal resistance.

Measurement of Resistance

(*a*) By Ammeter and Voltmeter.—The method of page 553 will give fairly good results for the resistance of a conductor provided that the resistance of the voltmeter is high compared with that of CD. If this is not the case a considerable fraction of the current measured by the ammeter will pass through the voltmeter and the ammeter reading will not be an accurate measurement of the current through CD.

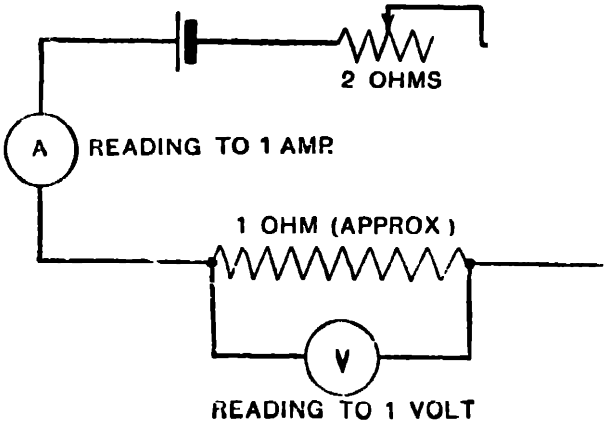


Fig. 477.

(*b*) By Substitution.—Set up the circuit shown in Fig. 478. X is the conductor whose resistance is to be determined, R a variable resistance, and G a galvanometer. The two-way switch enables the circuit to be completed either through X or through the resistance box.

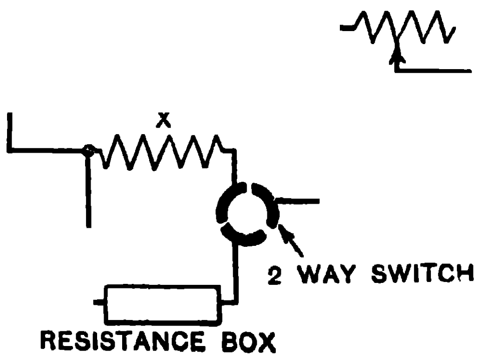


Fig. 478.

Set the switch so that the current passes through X and not through the resistance box. Adjust R so that a readable deflection of the galvanometer is obtained.

Change over the switch so that the current passes through the resistance box instead of through X. Without altering R again, take plugs out of the box until the galvanometer gives the same reading as before. This means that the same current will now flow round the

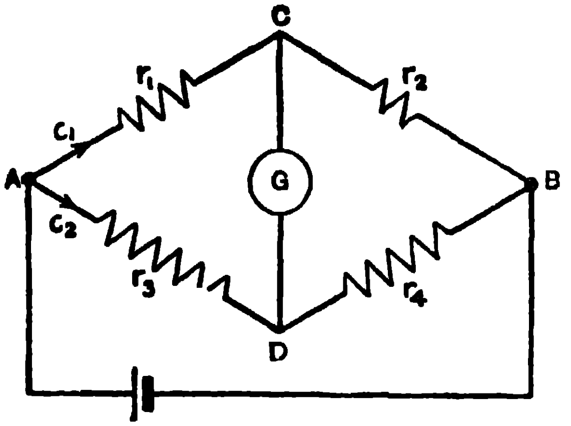


FIG. 479.

circuit whether X or the resistance box is included. Hence X must have a resistance equal to the sum of the resistances taken from the box.

This gives only a very rough measurement since X can seldom be exactly matched by the resistances available in the box.

(c) BY THE WHEATSTONE BRIDGE.—The arrangement of conductors shown in Fig. 479 is known as the Wheatstone bridge circuit. If the four resistances r_1, r_2, r_3, r_4 have relative values such that $\frac{r_1}{r_2} = \frac{r_3}{r_4}$ no current will pass through CD. Conversely, if we find that no current is passing through CD, as shown by there being no deflection in the galvanometer G, we know that the above relation holds between the four resistances. Hence if the values of three of them are known the fourth can be calculated. Actually it is not necessary, for the values of r_3 and r_4 to be known so long as their ratio is known. For example, suppose r_2 is 5 ohms and the ratio of r_3 to r_4 when the circuit is balanced is 4 : 3. Then since $\frac{r_1}{r_2} = \frac{r_3}{r_4}$; $\frac{r_1}{5} = \frac{4}{3}$; $\therefore r_1 = \frac{4}{3} \times 5 = \frac{20}{3} = 6{\cdot}67$. Therefore the resistance of r_1 is 6·67 ohms.

Proof of Formula. If no current passes through G, C and D must be at the same potential and the current through r_2 must equal that through r_1, whilst that through r_4 must equal that through r_3.

At A suppose the current to split into parts c_1 flowing in r_1 and c_2 flowing in r_3. Then the difference of potential between A and C = c_1r_1, and that between A and $D = c_2r_3$.

But C and D have the same potential;

$$c_1r_1 = c_2r_3 \quad \ldots\ldots\ldots\ldots(1)$$

Also difference of potential between C and B = c_1r_2, and that between D and B = c_2r_4;

$$\therefore c_1r_2 = c_2r_4 \quad \ldots\ldots\ldots\ldots(2)$$

Dividing (1) by (2)

$$\frac{c_1r_1}{c_1r_1} = \frac{c_2r_3}{}:$$

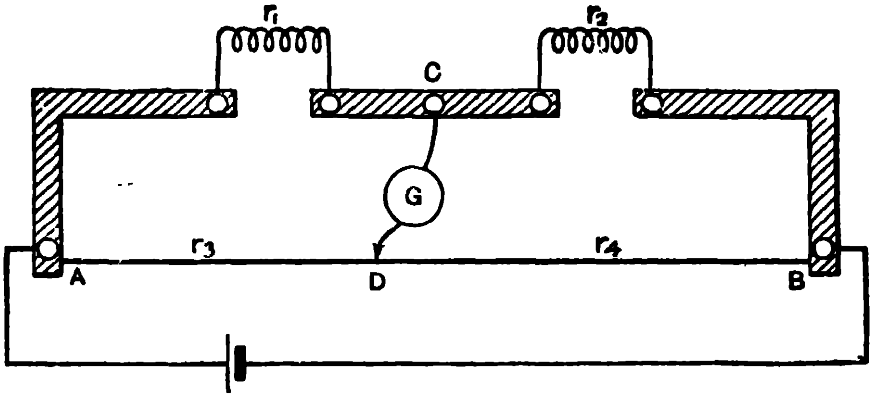


FIG. 480. A WHEATSTONE BRIDGE.

The metre bridge shown in Fig. 480 is an arrangement for measuring resistances using the Wheatstone Bridge relation. AB is a uniform wire 1 metre long with a metre scale alongside it. The conductor whose resistance is to be measured is r_1 while r_2 is a standard conductor of known resistance. The shaded portions are thick brass or copper bars of negligible resistance. G is a sensitive galvanometer to which is connected a movable contact, D, which can slide along AB. An accumulator is connected to the ends of AB. The whole then forms a Wheatstone bridge circuit, the points in Fig. 480 corresponding with those marked with the same letters in Fig. 479. D is made to slide along AB until a point is found such that the needle of G gives no kick when contact is made, and the lengths AD and DB are read from the scale. Since the galvanometer shows that no current is passing along CD, $\frac{r_1}{r_2} = \frac{r_3}{r_4}$.

Also, since the wire AB is uniform, $\frac{r_3}{r_4} = \frac{\text{Length AD}}{\text{Length DB}}$;

$$\frac{AD}{DB}; \qquad r_1 = r_2 \times \frac{AD}{DB}.$$

All three quantities on the right are known, so r_1 may be calculated.

As the wire may not be quite uniform, it is best to take another reading with r_1 and r_2 interchanged and take the average of the two values obtained.

Measurement of E.M.F.'s of Cells

(1) BY VOLTMETER.—It has already been explained that an approximate value is given by direct reading of a voltmeter if it has a very

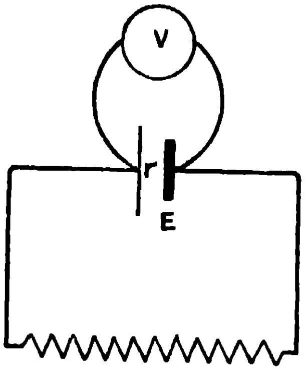


FIG. 481.

high resistance compared with that of the cell. The following method will give both a measurement of the E.M.F. and the internal resistance of the cell.

To the poles of the cell connect a voltmeter with a very high resistance. Also connect the poles to a standard resistance of, say, 10 ohms. Note the reading of the voltmeter. Since the current passing through the voltmeter will be negligible, its reading will give the potential difference driving current through the 10 ohm resistance.

Replace the 10 ohm resistance by one of 15 ohms and read the voltmeter again. Suppose the voltmeter reading in the first case was 1·6 volts and in the second case 1·8 volts.

Then current in 1st case $= \frac{1{\cdot}6}{10}$ amps. = ·16 amp., and current in 2nd case $= \frac{1{\cdot}8}{15}$ amps. = ·12 amp.

Now suppose the E.M.F. of the cell is E volts and its internal resistance is r ohms.

In the first case total resistance $= 10 + r$ ohms;

$$\text{Current} = \frac{E}{10 + r} \text{ amps.}$$

$$\frac{E}{10 + r} = {\cdot}16, \text{ i.e. } E = 1{\cdot}6 + {\cdot}16r. \qquad .(1)$$

Similarly in second case

$$\frac{E}{15 + r} = {\cdot}12, \text{ i.e. } E = 1{\cdot}8 + {\cdot}12r \quad \ldots\ldots\ldots\ldots \qquad .(2)$$

$$\therefore\ 1{\cdot}6 + {\cdot}16r = 1{\cdot}8 + {\cdot}12r \text{ (each} = E\text{)};$$

$$\therefore\ {\cdot}04r = {\cdot}2; \quad \therefore\ r = 5.$$

By substitution in (1)

$$E = 1{\cdot}6 + ({\cdot}16 \times 5) = 1{\cdot}6 + {\cdot}8 = 2{\cdot}4;$$

∴ The E.M.F. of the cell is 2·4 volts and its internal resistance is ·5 ohm.

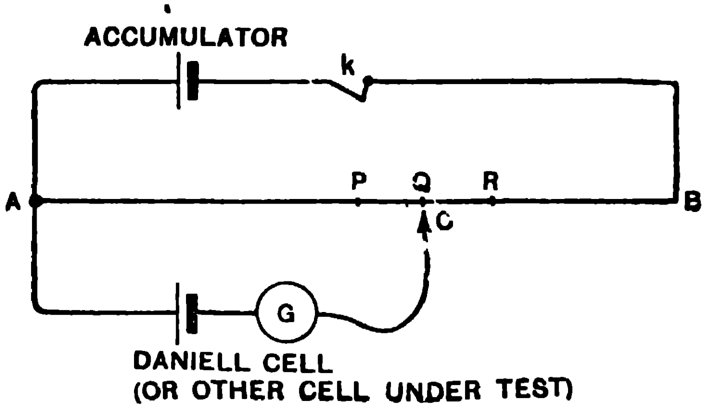


FIG. 482.

(2) BY POTENTIOMETER.—This is a more accurate method of measuring E.M.F.'s. The potentiometer consists of a long uniform wire mounted on a board with a scale by which lengths of the wire can be measured. Its ends are connected by thick leads of negligible resistance to the poles of an accumulator, a key being included in the circuit. When the key is depressed current will flow through the wire and the potential difference between its ends will be equal to that between the poles of the accumulator, say 2 volts. As the wire is uniform, the potential fall along it will be uniform, so that the potential fall from A to P, the mid-point of the wire, will be $\frac{1}{2}$ of 2 volts, and that from A to R will be $\frac{AR}{AB} \times 2$ volts. Even if the actual potential difference between A and B is not known we can say

$$\frac{\text{P.D. between A and Q}}{\text{P.D. between A and R}} = \frac{\text{Length AQ}}{\text{Length AR}}.$$

If now we connect a galvanometer G to A and Q the potential difference between A and Q will tend to drive current from A to Q through G. If in this second circuit we insert a Daniell cell with *its positive pole connected to* A—note that the positive pole of the accumulator was also connected to A—it will tend to drive current in the direction QGA. Thus the cell and the P.D. between A and Q are tending to drive currents in opposite directions through G, and if the P.D. between A and Q is equal to the E.M.F. of the cell, no current will pass through G which will therefore give no deflection.

The method of using the potentiometer then is to set up the circuit as shown, C being a sliding contact. K is closed and C brought into contact with AB. If the needle of G gives a kick C is moved and the process repeated until the point Q is found such that the needle gives no kick when contact is made. Hence no current passes through G, and as shown above,

$$\text{E.M.F. of Daniell cell} = \text{P.D. between A and Q}.$$

Suppose now a Leclanché cell is substituted for the Daniell cell, and C has to be moved to R for no current to pass through G. Then:—

$$\text{E.M.F. of Leclanché cell} = \text{P.D. between A and R;}$$

$$\therefore \frac{\text{E.M.F. of Leclanché cell}}{\text{E.M.F. of Daniell cell}} = \frac{\text{P.D. between A and R}}{\text{P.D. between A and Q}} = \frac{\text{Length AR}}{\text{Length AQ}}.$$

Hence the ratio of the E.M.F.'s of the two cells is found, and if that of one of them is known, the other can be calculated.

EXAMPLE.—*A standard cadmium cell of E.M.F.* 1·018 *volts gave no current when* C *was* 30·5 *cm. from* A. *For a Leclanché cell* C *had to be* 45·7 *cm. from A. What is the E.M.F. of the Leclanché cell ?*

$$\frac{\text{E.M.F. of Leclanché cell}}{1{\cdot}018 \text{ volts}} = \frac{45{\cdot}7}{30{\cdot}5};$$

$$\therefore \text{ E.M.F. of Leclanché cell} = \frac{45{\cdot}7}{30{\cdot}5} \times 1{\cdot}018 = 1{\cdot}52 \text{ volts.}$$

This method gives a true measurement of E.M.F. because the cell is not giving any current when the measurement is taken, and hence the P.D. between its poles is the same as its E.M.F.

The accumulator should not be allowed to run current continuously through AB as that would heat the wire and alter its resistance and so affect the potential drop along it. The key should be opened every time C is moved.

QUESTIONS ON CHAPTER XLVI

1. State *Ohm's Law.*

It is required to test the value of a resistance coil marked 5 ohms (2 amps.). Describe briefly how you would conduct the necessary test in the laboratory. Can you suggest a reason why the value 2 amps. is specified ? [L.U.

2. A uniform wire of resistance 2 ohms is bent into the form of a square, the free ends of the wire being joined. Calculate the resistance the wire would have if measured between opposite corners of the square.

Draw a diagram showing how you would use a metre bridge to measure this resistance practically, and indicate how the value of the resistance of the arrangement would be deduced from your observations. [L.U.

3. A metal wire of length 1 metre and uniform diameter has a resistance of 1·05 ohms; calculate the resistance of a coil made from wire of the same material 50 metres long but having twice the diameter.

Describe an experimental method of measuring the resistance of such a coil. [L.U.

4. You are provided with two fixed resistances of values 2 ohms and 4 ohms. By means of diagrams show the number of ways they can be introduced into a circuit, either singly or together, to vary the current. State the value of the resistance introduced into the circuit in each instance.

Draw a circuit diagram showing a battery of two cells in parallel being used to pass a current through a heating coil and include an ammeter to measure the current and a voltmeter to measure the potential drop in the coil.

What is the chief difference between the construction of an ammeter and a voltmeter? [L.U.

5. What factors determine (*a*) the electromotive force, (*b*) the resistance, of a voltaic cell?

Describe a method for comparing the electromotive force of two cells. [L.U.

6. Explain the use of a galvanometer shunt.

A galvanometer of 250 ohms resistance, a cell of electromotive force 1·4 volts and internal resistance 2 ohms, and a coil of resistance 70 ohms are connected in series. Calculate the current through the galvanometer. If the galvanometer is now shunted through a coil of resistance 10 ohms, what current passes through the galvanometer? [L.U.

7. Describe the construction and action of the Leclanché cell, pointing out its advantages and disadvantages.

Explain why in certain circuits it is better to use two such cells in parallel than in series to obtain an increase of current.

What kind of circuit would this apply to? [L.U.

8. Explain how a sensitive galvanometer can be modified for use as (*a*) an ammeter, (*b*) a voltmeter.

A galvanometer has a resistance of 5 ohms and a full scale deflection is produced by 15 milliamps. Calculate what resistance must be used with it to enable it to read (*a*) 1·5 amps., (*b*) 1·5 volts. State how the resistance must be connected in each case. [L.U.

9. It is required to send a current of 0·75 amp. through a resistance coil of 3 ohms. Each of the cells used in the battery has an E.M.F. of 1·5 volts and a resistance of 1 ohm. Show that 3 cells are required and that they must be fitted in series. [L.U.

10. Two resistances R_1 and R_2 are arranged in parallel; find an expression for their equivalent resistance.

A resistance of 500 ohms forms part of a circuit in which the current is 0·00001 ampere. If the current in the circuit is increased to 1 milliampere (= 0·001 ampere) what resistance must be placed in parallel with the 500 ohms, so that the current through this remains at its previous value. [L.U.

11. Describe, with the aid of a diagram, the construction *either* of a Leclanché cell *or* of a Daniell cell. Name *one* advantage which *each* of the cells has over the other.

How would you compare their electromotive forces, using a potentiometer ? Give a clear diagram of the connexions. [J.M.B.

12. Describe the construction of some form of moving coil galvanometer, and illustrate your answer with a diagram. Explain why the coil turns when a current is passed through it and why it returns to its original position when the current is switched off.

What attachments must be fitted to a sensitive galvanometer to convert it into an instrument suitable for use as (*a*) an ammeter, (*b*) a voltmeter ? Draw diagrams to show how these attachments should be connected. [J.M.B.

CHAPTER XLVII

THE HEATING EFFECT. ELECTRICAL ENERGY

The widespread domestic use of electricity for heating purposes and its industrial use for driving machines and doing mechanical work makes the relation between electrical and other forms of energy a matter of importance.

Units

Reference has been made to the **joule**, which equals 10^7 ergs, as a unit of work or energy frequently used in connexion with electrical measurements, and the volt has been defined as the potential difference between two points when one joule of work is required to transfer a coulomb of positive electricity from the point of lower to the point of higher potential. It follows that when a coulomb of positive electricity flows through a potential drop of one volt, one joule of energy is liberated and that x coulombs falling through y volts will liberate xy joules of energy. Thus for any flow of electricity we have the relation

Energy liberated	=	**Quantity**	×	**Potential fall.**
(*joules*)		(*coulombs*)		(*volts*)

This may be put in symbols,

$$E = QV \qquad \text{.........(1)}$$

where E is energy, Q is quantity of electricity transferred, and V is potential fall. This equation will apply to a whole circuit or to any particular conductor in that circuit.

Since coulombs = amperes × seconds, the relation may also be written

Energy = Current × Potential fall × Seconds,

$$\text{i.e. } E = CVt \qquad \text{.........(2)}$$

Further, for any particular conductor, potential fall = current × resistance, which may be written V = CR. Substituting this in equation (2) we obtain

$$E = C^2Rt \qquad \text{.........(3)}$$

Substituting this in

Power was defined in Chapter VII. as rate at which work is done. When one joule of work is performed in one second a power of one **watt** is said to be developed. From equations (2) and (3), the number of joules liberated per second in any conductor carrying current is equal to CV or C^2R. Hence we may write:—

$$\textbf{Power}\ (\text{watts}) = CV = C^2R.$$

The first form of this statement may be memorised as

$$\textbf{Watts} = \textbf{Amperes} \times \textbf{Volts.}$$

It can be shown that approximately 746 watts are equal to one horse-power, so that the watt is a somewhat small unit of power, and powers are often measured in **kilowatts** (1000 watts). A unit for measuring large quantities of energy is the **kilowatt-hour**, that is the energy supplied in one hour by a power of one kilowatt. Since working at one kilowatt means supplying 1000 joules per second, a kilowatt-hour is equal to 1000 × 60 × 60 joules.

The kilowatt-hour is the unit recognised by the Board of Trade in connexion with electricity supply. It is these units which are measured by the electric meters in our houses and for which the electricity companies are paid. They are sometimes referred to as Board of Trade Units (B.T.U.).

If electric lamps are examined they will be found to be marked with a rating in watts and a rating in volts. For example, the marking 100 W., 220 V. would mean that the lamp is intended to be used in a circuit which will apply a potential difference of 220 volts to it, and it will then consume electrical energy at the rate of 100 watts.

EXAMPLES.—(1) *Find (a) the current which passes through the above lamp when the proper voltage is applied to it and (b) its resistance. Find also the current which will flow through it and the power at which it will work when a potential difference of* 100 *volts is applied to it.*

$$\text{Watts} = \text{Volts} \times \text{Amps.}; \quad \therefore \text{Amps.} = \frac{\text{Watts}}{\text{Volts}};$$

$$\therefore \textbf{Current} = \tfrac{100}{220} = \cdot 455 \textbf{ amp.};$$

$$\therefore \textbf{Resistance} = \frac{220}{\cdot 455} = 483{\cdot}5 \textbf{ ohms.}$$

If a P.D. of 100 volts is applied,

$$\text{Current} = \frac{100}{483 \cdot 5} = \cdot 207 \text{ amp.};$$

$$\therefore \text{ Power} = \cdot 207 \times 100 = 20 \cdot 7 \text{ watts.}$$

(2) *Find the cost of lighting the above lamp for* 10 *hours at its proper voltage, supply being charged for at* 6*d. per unit.*

$$\text{Power} = 100 \text{ watts} = \tfrac{100}{1000} \text{ kilowatts};$$

$$\therefore \text{ Energy supplied} = \tfrac{100}{1000} \times 10 = 1 \text{ kilowatt-hour};$$

$$\therefore \text{ Charge} = 6\text{d.}$$

Most domestic electrical appliances, vacuum cleaners, heaters, etc., are similarly marked with watt and volt ratings and similar calculations may be made concerning them.

Electrical Energy and Heat

Unless it is made to do chemical work or mechanical work by being passed through a voltameter or a motor, most of the energy supplied to an electrical circuit is converted into heat which raises the temperature of the conductors and their surroundings.

In Chapter XXII. it was shown that the mechanical equivalent of heat is $4 \cdot 18 \times 10^7$ ergs per calorie. This is the same as 4·18 joules per calorie. Hence, when all the electrical energy is converted into heat, we may write from the equation on page 555:—

$$\text{Heat developed (calories)} = \frac{QV}{4 \cdot 18} = \frac{CVt}{4 \cdot 18} = \frac{C^2Rt}{4 \cdot 18} \text{ calories.}$$

From the last form of the expression it is clear that the heat developed is proportional to the square of the current and to the resistance of the conductor.

The relation between heat and current may be investigated as follows. Connect a small immersion heater, an ammeter, and a variable resistance in series. Allow the heater to dip into a measured quantity of water in a shielded calorimeter. Pass a small current for a given time, say 3 minutes, keeping the water stirred. Note the value of the current from the ammeter and the rise in temperature of the water from a thermometer dipping into it. Repeat a number of times varying the current by means of the variable resistance always starting with the same quantity of water at room temperature and heating for the same length of time. The quantity of heat developed each time can, of course, be calculated from the mass of water and its rise in temperature. Tabulate results as follows:—

HEAT DEVELOPED	CURRENT	$(\text{CURRENT})^2$	$\dfrac{(\text{CURRENT})^2}{\text{HEAT DEVELOPED}}$

A constant should be obtained in the last column.

It follows from the above that any considerable increase in the current passing through a conductor very greatly increases the rate at which heat is developed in it. Thus, if the current is increased to 10 times its former value heat is generated 100 times as fast as before. This may be dangerous. If through breaking down of insulation some part of a circuit becomes short-circuited so that the current does not have to pass through a considerable part of it, the current will suddenly increase because of reduced resistance and the heat developed in the remaining part of the circuit may be so great that the wires melt and the molten metal may start a dangerous fire.

For this reason fuses are usually inserted in circuits. These are thin wires made of metal with a low melting point. Their thickness is such that when current in excess of the safe current for the circuit passes through them sufficient heat will be produced in them to bring them to their melting point. They then fuse and the circuit is broken. The fuses are enclosed in porcelain cases which are themselves enclosed in an iron box so that the molten metal can do no damage.

Lamps and Heaters

An electric lamp contains a conducting filament which is raised to a high temperature by the current passing through it, and so is made to glow and give out light. The higher the temperature to which the filament is raised the whiter is the light it gives out. If, however, the filament temperature is raised too high it tends to give off vapour which condenses on the inner surface of the bulb and darkens it.

Since the filament would be liable to oxidise at its high temperature the bulb is either evacuated or filled with some such inactive gas as argon.

As the energy liberated in a conductor by a given current is proportional to its resistance, the filament should have a high resistance and the leads to it should be of low resistance. The leads therefore are made of stout, good conducting copper. This ensures that most of the energy of the circuit is liberated in the lamp.

The original filament lamps had carbon filaments in vacuum. They could be raised to about 1800° C. and gave a somewhat reddish light. Modern lamps have tungsten filaments in argon which can be heated to about 2500° C. and give a much whiter light.

Much of the energy liberated in a lamp is dissipated as heat, and only a fraction of it is converted into light. For ordinary purposes the *efficiency of a lamp* is said to be measured by its candle-power per watt. (Manufacturers now quote efficiencies as the " light in lumens per watt ": a lamp giving 1 candle-power in every direction is giving out 4π lumens of light energy per sec.) Modern lamps may give about 1·7 candle-power per watt. The old carbon lamps gave about $\frac{1}{4}$ candle-power per watt.

The efficiency of a lamp will vary with the voltage at which it is working. This may be shown by connecting a lamp in series with a variable resistance, an ammeter, and source of current and connecting a voltmeter of suitable range across its terminals (Fig. 483). Start with a high resistance in the circuit, note the voltmeter and ammeter readings, and determine the candle-power of the lamp by one of the methods described in Chapter XXX. The power at which the lamp is working is given by current × voltage across its terminals, and candle-power divided by this power gives efficiency. Reduce the resistance and take a fresh set of readings and continue until the proper voltage for the lamp is reached.

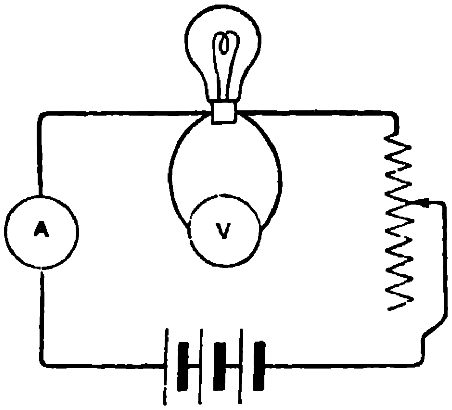


FIG. 483.

In electrical heating apparatus again the heating elements must have high resistance compared with the leads so that most of the energy will be released in them. They must also, in the case of electric fire elements, withstand oxidation at high temperatures as they must be exposed to the air. They are usually made of nickel-chrome alloys which possess those properties.

Household Circuits

Fig. 484 shows the type of circuit commonly found in houses. The connexions to the mains in most cases nowadays have a potential difference of 230 volts. One of them passes through the meter which measures the energy consumed in the house. Beyond this is a main switch by which current can be cut off from all circuits in the house, and just beyond this are main fuses which protect the mains against short circuiting in the main leads. From these main leads sets of branch leads are taken off, usually one to each floor in the case of a small house. Each of these branch circuits passes through fuse wires immediately after leaving the main leads.

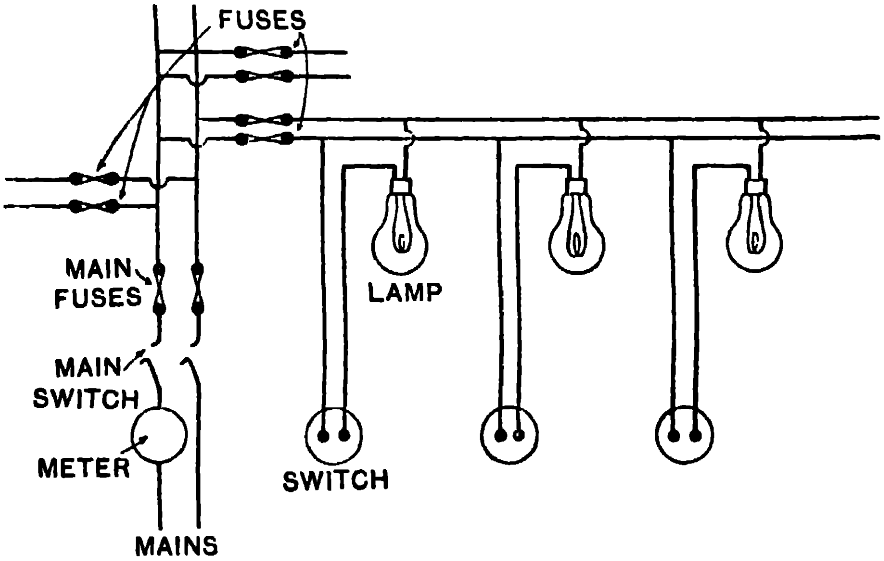


Fig. 484.

Lamps, etc., are connected in parallel across these branch leads, each having its own switch. Evidently the leads and fuses in a branch should be capable of carrying the current which will pass if all the lamps, etc., connected with it are turned on at once. Thus, a 100-watt lamp on a 230-volt circuit will take a little less than ½ amp. current. If there are 4 such lamps in one branch circuit there may be nearly 2 amps. flowing in that branch when all are turned on. Usually branch circuits are made capable of carrying 5 amps. The branch fuse should be one that will fuse as soon as the current exceeds 5 amps.

A " one unit " fire, that is one which works at 1 kilowatt, will take nearly five amps. from such a circuit. Usually separate branches capable of taking 15 amps. are wired for fires, stoves, etc.

Obviously the main leads and fuses should be capable of taking the sum of the maximum currents for all the branch circuits.

Electric Arc

Two rods of carbon mounted on insulating stands have stout copper wires wrapped round them and connected through a resistance to a source of over 40 volts potential difference. The ends of the rods are brought into contact so that current passes and are then drawn a little apart. The current will continue to pass across the gap and a very intense light is emitted. When the rods make contact the large resistance due to the few points of contact results in an intense local production of heat which causes some vapourisation of the carbon. When the rods are separated the current is carried across the gap by the vapour atoms. (The gaseous atoms in the gap really have electrons ejected from them by the intense heat, so that in the gap positive ions and electrons are produced: the ions move to the negative carbon and the electrons to the positive carbon.) The temperature of the arc itself may rise to about 3500° C. or 4000° C.: the tip of the positive carbon reaches about 3500° C. and of the negative about 2500° C. In the case of this direct current arc most of the *light* comes from the positive carbon tip.

Carbon arc lamps were much used for street lighting in the days when filament lamps sufficiently robust for such purposes had not been invented. They are still used for producing an intense light in optical lanterns.

The main modern use of the arc is arc welding. Arcs can be struck between rods of metal as well as between carbon rods. For welding, the pieces of metal to be joined are connected to one pole of the source. A rod of the metal is connected to the other pole. An arc is struck between the pieces of metal and the rod. The edges of the pieces of metal and the end of the rod both melt and thus the joint is filled up with molten metal.

Discharge Lamps

At ordinary pressures gases act as insulators, but when rarefied and subjected to high potential differences electric discharges can take place through them, and under suitable conditions a glow is produced in the gas. This is due to the fact that a few free electrons and positive ions formed by escape of electrons from molecules are present in the gas. When a large potential difference is applied to two electrodes in a tube containing a rarefied gas, the free electrons will be driven towards the anode and the positive ions towards the cathode and will acquire very high velocities, and the current is carried through the gas in this way. Collisions between the moving charged particles and other molecules of the gas tend to eject electrons from the latter, so producing more charged particles, and, as a result, once the current has been started it can be maintained by a lower potential difference. As the number of charged particles becomes greater there will be collisions between electrons and positive ions which will combine to form ordinary molecules again. In the ejection of electrons from molecules work is done and energy is absorbed. When recombination takes place an equivalent quantity of energy is released, some of it in the form of light radiation, and so the gas glows.

The type of radiation emitted, and hence the colour of the glow, depends on the nature of the gas. The long tubes largely used for advertising illumination usually contain neon at low pressures which emits a red-orange light. Other shades of colour can be obtained by using various coloured glasses for the tubes. Mercury vapour lamps, now often used for street lighting, give a blue light and emit much ultra-violet radiation. If neon is mixed with the mercury vapour a bluish-white light is produced, and if a suitable glass is used for the tube much of the ultra-violet radiation is converted to light in the visible part of the spectrum and a greenish light results. Sodium vapour lamps give out a bright yellow light.

In recent types of discharge lamps the electrodes are rods or filaments of metal which are themselves raised to white heat by the current. In this condition they eject electrons which by colliding with molecules assist in ionising the gas. As a result such hot electrode lamps will work at lower voltages than cold electrode lamps, and by using a suitable mixture of gases a type of light somewhat deficient in red but suitable for general illumination is obtained. The candle-power per watt efficiency of such lamps is two or three times that of a gas-filled filament lamp.

Hot-Wire Ammeter

Some ammeters depend on the heating effect of a current. Such an ammeter is illustrated in Fig. 485. XY is a fine wire through which the current to be measured passes. To its mid-point is attached a silk fibre which passes round an axle O and is then attached to the spring S which keeps it taut. When current passes through XY the heat developed causes the wire to expand. This allows the fibre to be pulled a little round O which is turned and carries round a pointer attached to it.

This ammeter can be used for alternating current as the heating effect does not depend on the direction of the current.

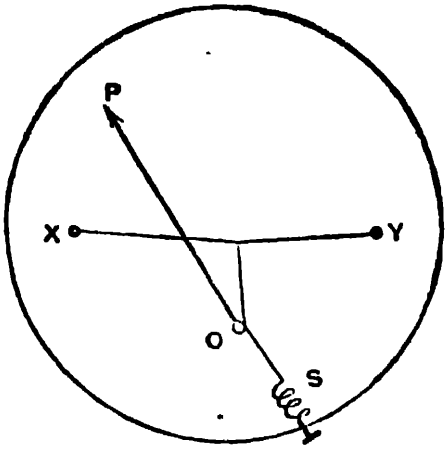


FIG. 485.

QUESTIONS ON CHAPTER XLVII

1. When a current of electricity passes through a wire its temperature is raised. What factors determine (1) the heat developed in the wire, (2) the temperature it attains?

Discuss the application of this heating effect to (1) a safety fuse (2) an electric radiator. [L.U.

2. Being supplied with an accumulator (storage cell), together with some resistance wire and a thermometer, describe how you could demonstrate the heating effect of a current.

How could you modify the experiment if it is required to measure the quantity of heat which has been produced? Upon what factors does this quantity of heat depend? [L.U.

3. Explain the terms *joule, watt,* and *kilowatt-hour.*

An electric kettle, which is marked "500 watts, 230 volts," is found to take 15 minutes to raise the temperature of 1 kilogramme of water from 15° C. to 100° C. Calculate the percentage of electrical energy which is employed in heating the water. [4·2 joules are equivalent to 1 calorie.] [L.U.

4. Define a unit in which power supplied to an electric circuit may be measured.

Two resistances of 80 and 120 ohms are connected (1) in series, (2) in parallel to a direct current supply of 100 volts. Calculate the power expended in each resistance in the two cases. [L.U.

5. Explain the action and use of a fuse used in wiring of the electric supply of a house.

A room has three points supplied from the same pair of leads from the 200-volts mains. These are used to supply electricity to: (*a*) a 2 kilowatt electric fire, (*b*) a 100 watt lamp, (*c*) a motor which costs 2d. per hour to run.

What current is taken in each of the three cases, and how much would it cost to run the whole for 3 hours at 3d. per unit? What would be a suitable fuse to use in the leads supplying the current? [L.U.

6. Describe the construction and action of a current measuring instrument which depends upon *either* the magnetic effect *or* the heating effect of an electric current. [L.U.

7. State the laws which govern the consumption of energy by an electric circuit. In what units is this energy measured?

An electrical machine, marked 80 V., 300 W., is to be used on a 120-volt circuit. Describe some form of the additional apparatus needed and give a diagram showing how it is connected in the circuit. Calculate (*a*) the resistance of the whole circuit, (*b*) the power wasted. [J.M.B.

8. A given electric lamp is marked 6 v., 12 w. Explain what this marking means and describe how you would use three accumulators (each of approximately 2·2 volts E.M.F.), an ammeter, a voltmeter, and any other necessary apparatus to check the accuracy of the marking of the lamp.

Two electric kettles A and B, having the same thermal capacity, are marked 200 V., 500 W., and 100 V., 500 W. respectively. Calculate the resistance of each heating element and the current flowing through each kettle when it is connected to the electric mains for which it is designed. What comparison is there between the rates at which these kettles can heat the same volume of water? Give reasons for your answer. [J.M.B.

9. Describe, with the aid of a diagram, some form of fuse which is used in the electric lighting circuit of a house. Explain why it is inserted and why a fuse wire must not be replaced by an ordinary wire.

A room is lighted by five 200-volt, 60-watt lamps, and heated by an electric fire, all connected in parallel. Fires are made which consume power in multiples of 500 watts (*e.g.* 500 watts, 1000 watts, and so on). Calculate the greatest permissible power of the fire so that a 15 amp. fuse, placed in the circuit leading to the room, is not burnt out when the fire and all the lamps are in use. [J.M.B.

10. Explain what is meant by the *power* of an electrical circuit. State and define the unit used in measuring such a power.

How does the power of a heating coil of fixed resistance depend on (*a*) the potential difference applied to its ends; (*b*) the current through it?

Describe an experiment to verify *one* of these relations.

11. Draw a diagram of the wiring of a household circuit in which two branches, each to take three 100-watt lamps in parallel, and a third to take a 2-unit heater are connected to the mains.

Fuse wires to take 2, 5, 10, or 15 amps. are available. Find by calculation which is the least of these suitable for inclusion (*a*) in a lighting branch; (*b*) in the heating branch, and (*c*) in the mains if the mains voltage is 250 volts.

12. An electrical water heater with a capacity of 12 gall. is marked 230 volts, 1000 watts.

Calculate (*a*) the resistance of the heating coil; (*b*) the time to the nearest minute taken for the contained water to be heated from 11° C. to 52° C., assuming that 90 per cent. of the heat generated goes into the water; (*c*) the cost of this heating at 3d. per unit.

[1 gal. of water weighs 10 lb.; 1 lb. = 454 grm.; J = 4·2 joules per cal.]

13. Describe an experiment for finding how the efficiency of an electric lamp expressed in candle-power per watt varies with the voltage applied to it. What would you expect the results of the experiment to show?

Why should the filament of a lamp have a high resistance but the leads to it a low resistance?

14. Give brief explanations of the way in which current passes (*a*) between carbon poles in an arc lamp; (*b*) through a gas discharge lamp. Why can the current through a discharge lamp be maintained with a lower voltage than that required to start it?

Briefly describe some practical applications of electric arcs and discharge lamps.

CHAPTER XLVIII

ELECTROMAGNETIC INDUCTION

In 1831 Michael Faraday performed a series of experiments which were of a very simple nature but which had most important results, for by means of them he discovered the principles on which the whole of the industry for generating and distributing electricity on a large scale was later built up. By performing similar experiments an understanding of these principles may be gained.

Action of Magnets on Closed Circuits

Wind 150 to 200 turns of insulated copper wire on a cardboard tube a few inches long and about one inch in diameter. Stand the coil upright and connect the ends of the wire to the terminals of a sensitive galvanometer. Hold a bar magnet above the coil with its north pole pointing downwards. Plunge this pole sharply into the coil and note that the galvanometer needle gives a kick as this is done. Sharply withdraw the magnet from the coil and note that a kick is registered in the opposite direction. Also note that there is no deflection of the galvanometer needle *except when the magnet is moving*. From this we arrive at the conclusion that *a current is induced in a closed circuit when a magnet moves with respect to it*. Note that this means that **the current is induced when changes are taking place in the magnetic field in which the coil is situated.** The production of currents by such means is termed **electromagnetic induction.**

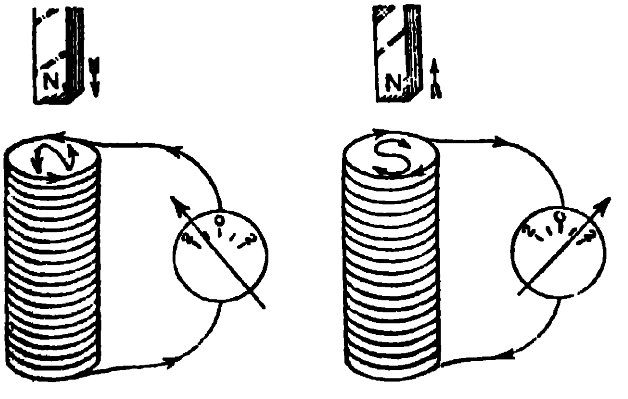


FIG. 486.

By placing the galvanometer in a circuit in which current is travelling in a known direction determine the directions in which current passes through it when it kicks to left and to right. From this determine the directions in which current passes round the coil when the north pole approaches and when it is withdrawn. They will be found to be as indicated in Fig. 486, and such that the near end of the coil becomes a north pole as the north pole of a magnet approaches, and a south pole as the north pole is withdrawn. If the south pole of the magnet is used, again a like pole will be produced by approach and an unlike pole by withdrawal. These results are summarised in **Lenz's Law,** which says that **an induced current always flows in such a direction that it opposes the motion producing it.** Note that this means that mechanical work has to be done against the magnetic forces in order to maintain the motion and this mechanical work is the source of the energy which is transformed into electrical energy to produce the current.

Investigate the results of (*a*) using a coil with more turns, (*b*) using a more powerful magnet, (*c*) moving the magnet more quickly. Each of these changes will be found to increase the strength of the induced current. If we think of the field of the magnet we shall realise that some of its lines of force pass through the coil. Owing to the arrangement of the lines in the field, as the magnet approaches more of them pass through the coil, and as it recedes fewer of them pass through. Increasing the number of turns is equivalent to increasing the number of lines passing through the coil since we should expect any effect of them to be produced on each turn. Thus it appears that the inductive effect is connected with changes in the number of lines of force passing through the coil, and (*c*) shows that it is connected with the *rate* at which that number changes. It has been established that **the induced E.M.F. is proportional to the rate of change in the number of magnetic lines passing through the circuit.** Thus, if the number of magnetic lines passing through a circuit increases or decreases by n in t seconds, the E.M.F. induced in the circuit will be proportional to $\frac{n}{t}$.

Action Between Two Adjacent Circuits

The passage of a current around a circuit has been shown to produce a magnetic field. Hence any change in the current in one circuit will tend to change the number of lines of force passing through a neighbouring circuit and so set up an induced E.M.F. in the latter.

This may be illustrated by arranging two coils end to end as indicated in Fig. 487. When the key, K, is closed current starts to flow through

A. This sets up a magnetic field increasing the number of lines of force passing through B and the galvanometer G gives a kick indicating the presence of induced current in B. On opening K again, the field due to the current in A disappears and G kicks in the opposite direction. Note that in each case the kick is only momentary. There is no induced current in B when the current in A is steady. *Induction only takes place while changes in the magnetic field are occurring.*

The arrows in Fig. 487 indicate the directions in which the currents will flow when K is closed. It passes round A in such a way as to produce a south pole at X. In accordance with Lenz's Law the induced current will pass round B in such a way as to produce a south pole at Y, thus tending to set up opposing lines of force. On breaking circuit at K the current in B will flow in a reverse way producing a north pole at Y and so tending to replace the lines of force which are disappearing owing to the disappearance of the south pole at X.

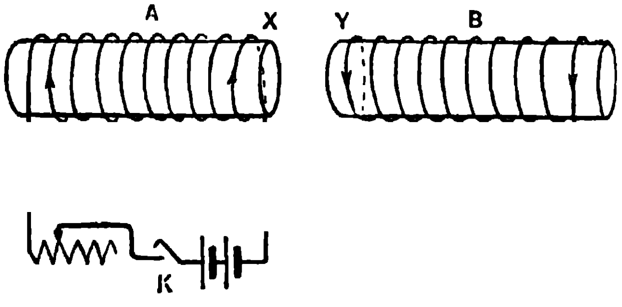


Fig. 487.

The building up of these induced currents in B will tend to induce E.M.F.'s in A which in the first case will tend to oppose the original current in A, and in the second case will tend to continue it. This interaction between neighbouring circuits is called **mutual induction.**

If A is connected to a source of alternating current an induced current will flow through B every time that in A changes, and thus a continuous alternating current can be obtained from B. A is then said to be the **primary** coil of the arrangement, and B the **secondary** coil.

Induction effects also take place in an isolated circuit. When a current starts to flow round such a circuit a magnetic field due to this current appears and so changes the number of lines of force passing through the circuit. This sets up an induced E.M.F. opposing the passage of the current so that on closing a circuit the current does not attain its maximum strength immediately. For this reason in Wheatstone bridge and potentiometer experiments the accumulator circuit should always be closed before the galvanometer circuit so that the current through the wire may have its maximum value before the galvanometer reading is taken. Owing to this effect, for a moment after closing a circuit, it will act as though its resistance is greater than its true value. The special winding of resistance box coils mentioned on page 560 ensures that the magnetic effect of one part of the coil is neutralised by that of another, so that there is no change of magnetic field on starting current through it, and this inductive effect is eliminated.

On breaking a circuit a spark is often observed to pass at the switch. The disappearance of the lines of force due to the current induces an E.M.F. tending to continue the current, and this, added to the E.M.F. already existing in the circuit may set up so great a potential difference across the switch gap that a discharge occurs across it.

These effects in a single circuit are referred to as **self-induction.**

Transformers

The insertion of a soft iron core in a coil considerably increases its inductive effect since it tends to concentrate the magnetic lines inside the coil and so increases the change in the number of lines passing through a given space as current is made or broken. This can be shown by inserting a bar of soft iron through the two coils shown in Fig. 487. The kick of G on making and breaking the circuit of A will be much increased.

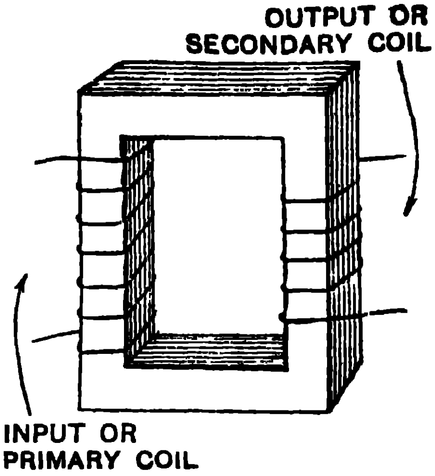


FIG. 488. THE TRANSFORMER.

Transformers usually consist of two coils of insulated wire, containing different numbers of turns, wound separately on a continuous soft iron core, as shown in Fig. 488. The magnetic lines emanating from one coil will almost all remain in this core and so pass through the other coil.

One coil, the input or primary coil, is connected to a source of alternating current. At every alternation the magnetic lines through the secondary or output coil will be reversed and so an alternating induced E.M.F. will be set up in it. It can be shown that for any such transformer

$$\frac{\text{Voltage of secondary circuit}}{\text{Voltage of primary circuit}} = \frac{\text{No. of turns in secondary coil}}{\text{No. of turns in primary coil}}.$$

Thus if the secondary coil contains fewer turns than the primary it applies to its circuit a lower voltage than that of the primary coil and is called a **step-down** transformer. One with more turns in the secondary coil than in the primary will give an increased secondary voltage, and is a **step-up** transformer. For example, in a mains wireless set it may be required to apply a potential difference of 4 volts to the valves. The mains supply has a voltage of 240. Hence

$$\frac{\text{Voltage required}}{\text{Mains voltage}} = \frac{4}{240} = \frac{1}{60},$$

so to obtain the required voltage a transformer should be used in which the primary coil has 60 times as many turns as the secondary.

Transmission of Electrical Energy.—It has been found to be more economical to transmit electrical energy at high potential than at low potential. At the big power stations it is generated at a voltage of about 10,000. For long distance transmission on the "grid" it is stepped up to 132,000 volts. At substations this is stepped down to 33,000 volts for distribution over an area, and at various points within the area it is stepped down again to 230 volts for distribution to the consumer.

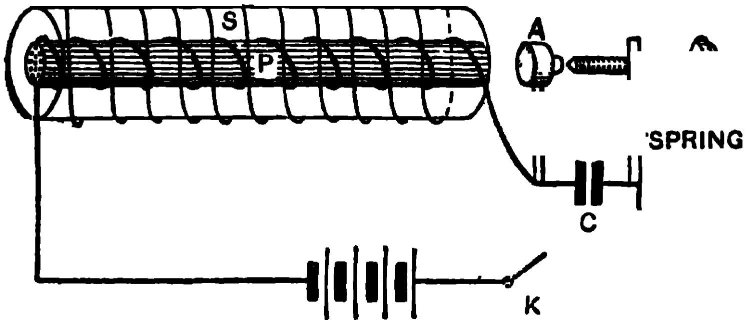


FIG. 489. THE INDUCTION COIL.

The Induction (Rumkorff) Coil

A type of transformer much used in laboratories is illustrated in Fig. 489. The primary coil, P, consists of a few turns of stout insulated wire wound on a soft iron core. The secondary coil, S, consists of a large number of turns of thin insulated wire wound on the outside of the primary coil. The primary is connected to a source of direct current and a key. The circuit is completed through a spring bearing a piece of soft iron A which just makes contact with a screw. A condenser, C, is usually included between the spring carrying A and the metal support of the screw. Two conductors with insulating handles, shown in the upper part of the diagram, are connected to the ends of the secondary coil.

When K is closed so that current passes through the primary coil a magnetic field is set up and the presence of the iron core intensifies this field. The magnetisation of the iron core results in A being attracted towards it away from the screw so that the primary circuit is broken, the current ceases, and the core loses its magnetism. The spring then brings A back into contact with the screw, so that the primary current flows once more and the above process is repeated.

Each time the primary current is made and broken there is a very rapid change in the number of magnetic lines passing through the secondary coil. As a result high induced E.M.F.'s develop in the secondary coil. The direction of the E.M.F. caused by breaking the primary circuit is opposite to that developed on making the circuit. Thus induced alternating current of high voltage may be taken from the secondary coil. It is more frequently used to produce sparks by placing the conductors joined to its ends at such a distance apart that discharge takes place across the gap between them.

The object of the condenser is to ensure a sudden disappearance of the primary current and the magnetic lines due to it when the break occurs. As explained on page 587, there will be self-inductive effects in the primary coil tending to maintain the current, and this might cause a sufficient potential difference between the screw and A to cause sparking between them, thus lengthening the time taken for the current to disappear and reducing the induced E.M.F. in the secondary. Owing to the capacity of the condenser a sufficient potential difference to cause sparking between the screw and A does not develop. On the other hand, self-inductive effects do take place on making the primary circuit, so that the current does not reach its maximum value instantaneously. Thus a higher E.M.F. is produced in the secondary at the break than at the make, and the secondary spark gap may be made of such a width that a spark will pass at the break but not at the make, a series of discharges which are all in the same direction being obtained.

Induction coils are used in laboratories for causing discharges through tubes of gases at low pressure. The ignition coil of a motor car is a type of induction coil.

Movement of Conductors in Magnetic Fields

Consider the rectangular coil ABCD placed vertically between unlike poles of horizontal magnets as shown in Fig. 490 (*a*). Let it rotate about the axis XY in the direction shown by the arrow at B. During the first quarter of a revolution the number of magnetic lines passing through it will be decreasing. According to Lenz's Law induced current will flow through it in such a way that the change tends to be neutralised, that is so that more lines in the direction of the field due to the magnets are produced. Thus the right-hand face of the coil tends to become a north pole and therefore current runs as shown in the direction ADCB.

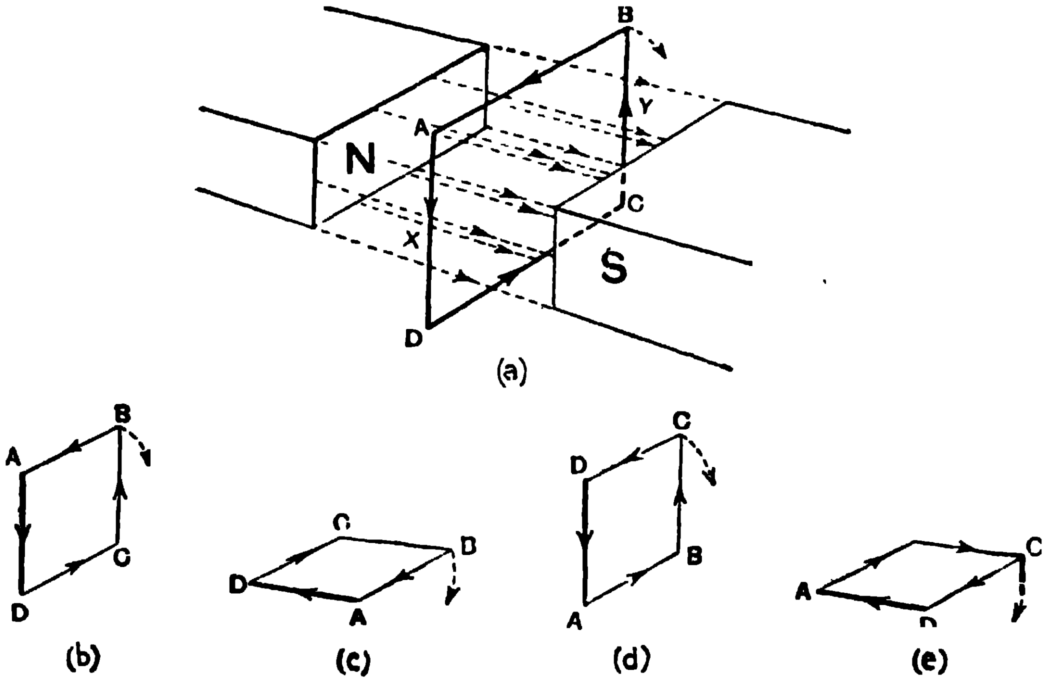


FIG. 490.

When the coil is horizontal the number of lines passing through it is a minimum and further rotation increases that number. The induced current will now tend to produce lines running the other way, and the right-hand face will become a south pole. But the face which was originally to the left is now turning to the right, so the current still runs in the direction ADCB [Fig. 490 (*c*)].

As the coil passes the vertical once more the right-hand face becomes a north pole again, so current flows now in the reverse direction ABCD around it [Fig. 490 (*d*)]. On passing the next horizontal position the right-hand face once more becomes a south pole, and as the original face is now turning to the right, current continues to flow in the direction ABCD.

Thus, if the coil is continuously rotated, alternating current will flow round it, the changes in direction of current occurring when the coil is in the vertical position, that is twice per revolution.

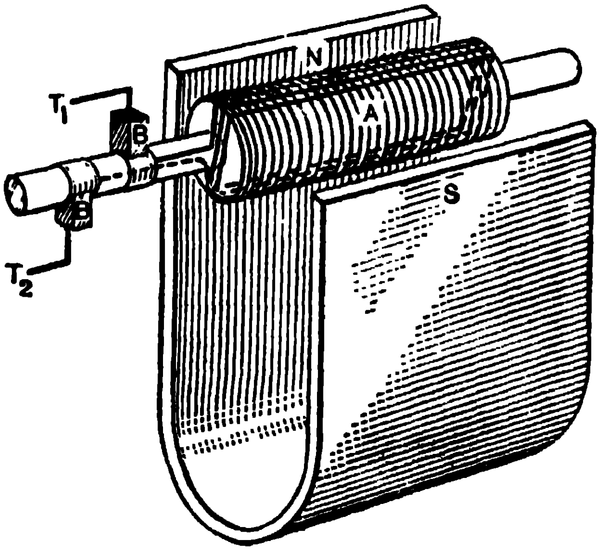


Fig. 491. A Simple Alternating Current Dynamo.

The Dynamo

(1) Alternating Current Machine or Alternator.—The last paragraph gives the principle of the dynamo. The coil is mounted on an axle. Its free ends make contact with two insulated metal rings, known as **slip rings** mounted on the axle. "Brushes" of carbon or copper are pressed against the slip rings by springs and connected to an external circuit. If the coil is mounted between the poles of a powerful magnet and rotated alternating current will flow through the external circuit.

The E.M.F. of the induced current will depend on the strength of the field magnet, the number of turns in the coil, and the speed with which it is rotated. Increasing each of these will increase the rate at which lines of force are cut and so increase the E.M.F. VERY:

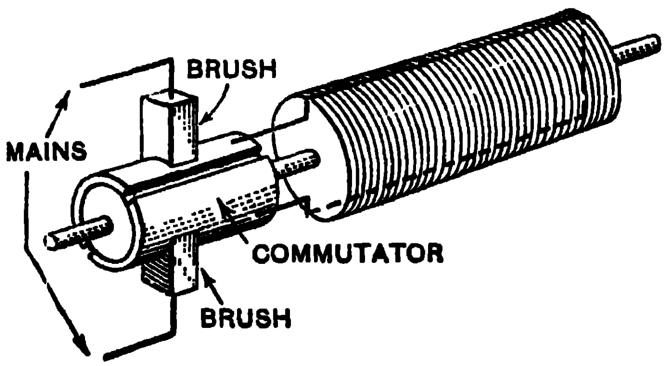


Fig. 492.

(2) Direct Current Machine.—If direct current is required from a dynamo an arrangement known as a split ring **commutator** is used instead of the slip rings. This, as shown in Fig. 492, consists of two segments of a metal ring mounted on the coil axle. Thus, as the axle rotates, the connexions between the brushes and the ends of the coil are interchanged twice

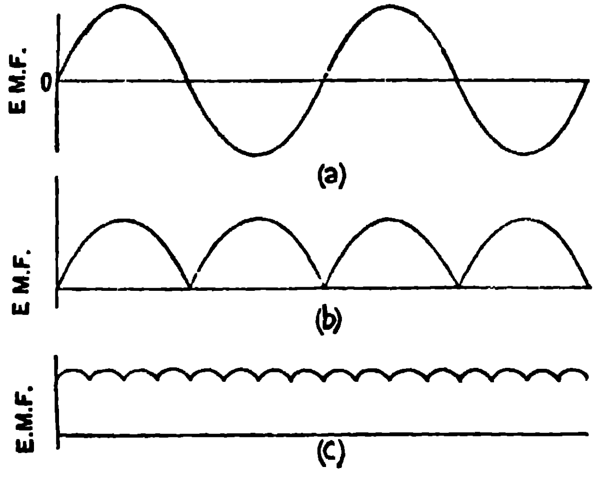


FIG. 493.

per revolution. The gaps are so placed that these interchanges occur just as the current is being reversed in the coil, and so current is always flowing in the same direction in the external circuit.

Consideration of Fig. 490 will show **that the change** in the number of magnetic lines passing through the coil is more rapid when the coil is nearly horizontal than when it is nearly vertical. Hence the E.M.F. varies in magnitude as well as alternating. The changes in E.M.F. during two revolutions may be illustrated by a graph such as Fig. 493 (*a*). With a split ring commutator the alternations are prevented but the changes in magnitude persist as shown in Fig. 493 (*b*), and a pulsating current is obtained.

For most purposes a steady E.M.F. and current are required. This is obtained by mounting a number of coils at different angles on the axle and using a split ring with a corresponding number of segments, each coil being connected to two segments. Thus, as the axle rotates the brushes are in contact with each coil only for a very short time. The commutator is so arranged that this time coincides with the period at which the coil has its maximum E.M.F. This gives a nearly constant E.M.F. as indicated in Fig. 493 (*c*). The picture on page 593 illustrates such a dynamo.

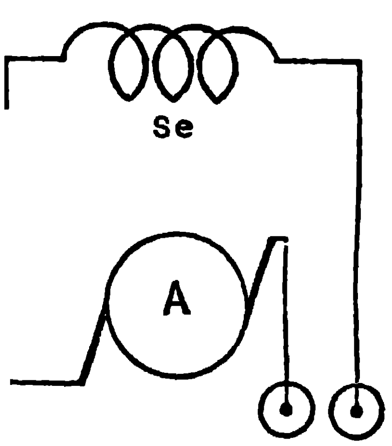


FIG. 494. A = ARMATURE. SE = FIELD COILS.

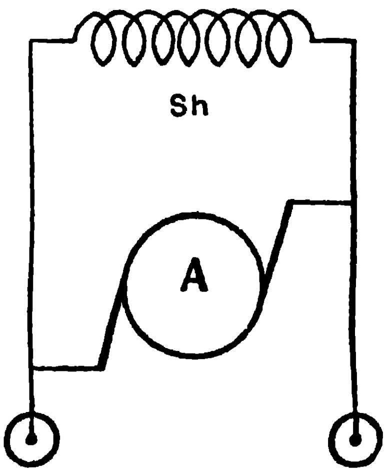
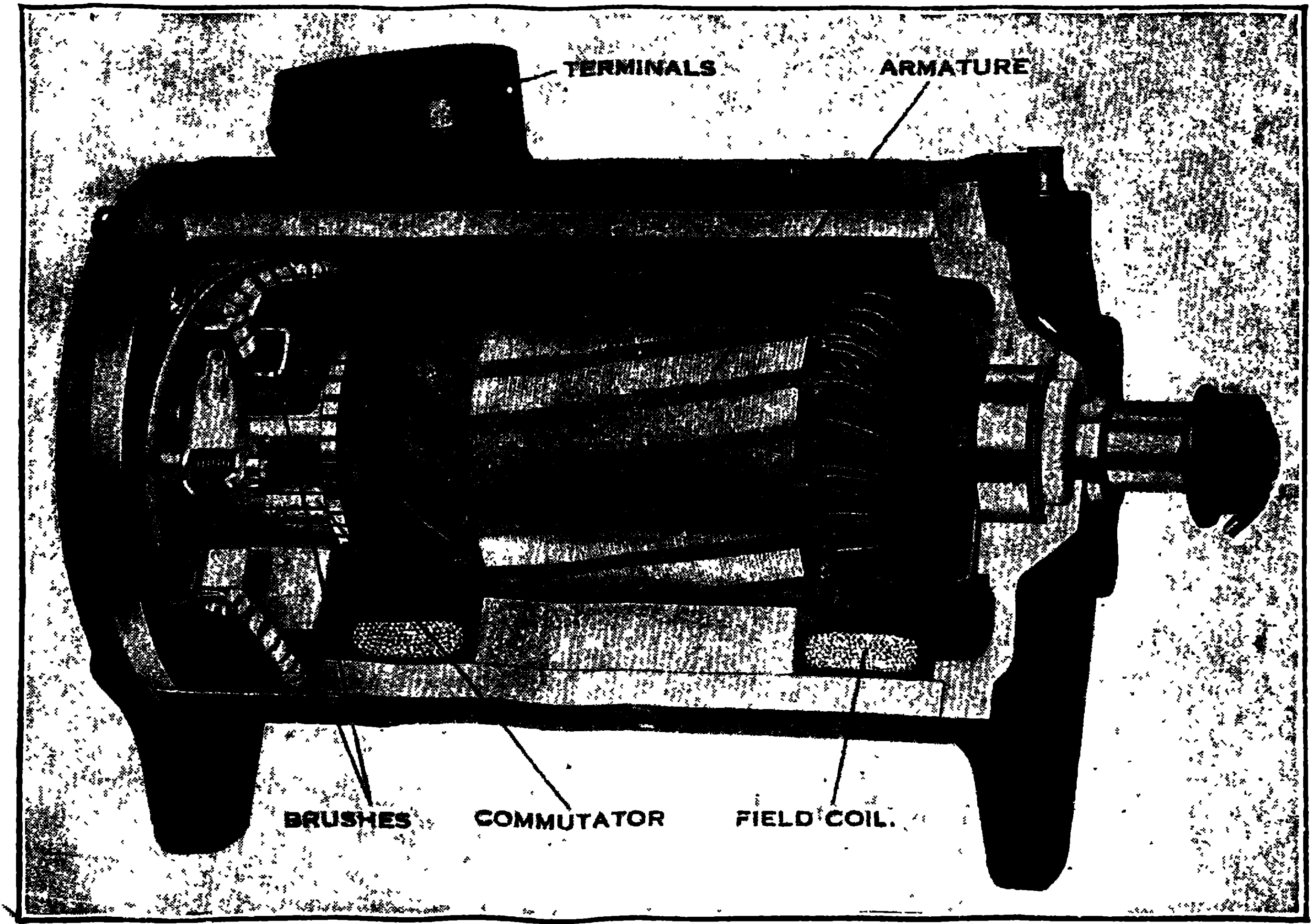


FIG. 495. SH = FIELD COILS.

In commercial dynamos the field magnet is usually an electromagnet. In a direct current dynamo the magnet coils are usually arranged



Courtesy of Messrs. Joseph Lucas & Co. Ltd., Birmingham.

A TYPICAL LUCAS CAR TYPE DYNAMO.

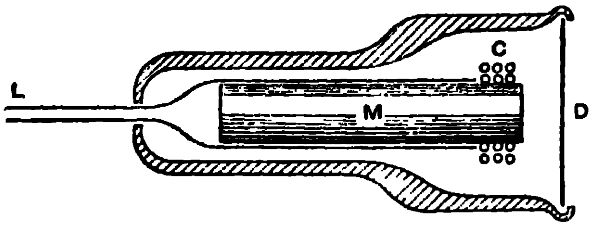


FIG. 496.

in series with, or as a shunt to, the mains from the commutator (Figs. 494 and 495). If in series they should be of low resistance so as to have little effect on the current in the mains. If shunt wound the coils should have high resistance, so that only a small fraction of the total current will flow through them.

Magnetos, used on some motor cars and cycles to produce the ignition sparks, are small dynamos with permanent magnets of hard steel.

Telephone Transmitters

Early telephone *transmitters* depended for their action on induced currents. A coil C was wound round the end of a permanent magnet M and connected to leads L joining it to the receiver. An iron diaphragm D was made to vibrate by the sound waves when words were spoken into the instrument. This motion caused changes in the field of the magnet which induced currents in the coil. Thus intermittent currents corresponding to the sound waves were transmitted to the receiver.

The currents produced by the above instrument were very weak and could not be transmitted over long distances. Edison's introduction of the carbon **microphone** made long distance telephony possible. In this instrument the space between the diaphragm, D, and the carbon block, B, is filled with carbon granules, G. The leads are connected to B and D respectively and between the transmitter and the receiver there is a battery. When the movement of the diaphragm presses the granules together the resistance of the circuit decreases and a larger current flows. Outward movement of D causes the granules to have only loose contact with one another, and so increases the resistance and decreases the current. Thus fluctuations of current corresponding to the movements of the diaphragm are transmitted to the receiver.

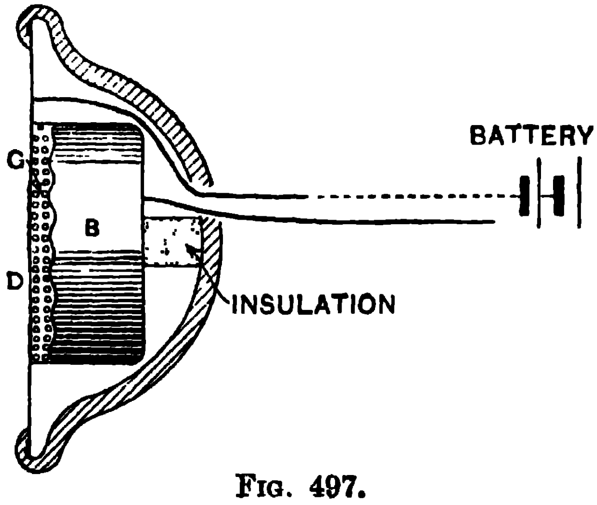


FIG. 497.

Motors

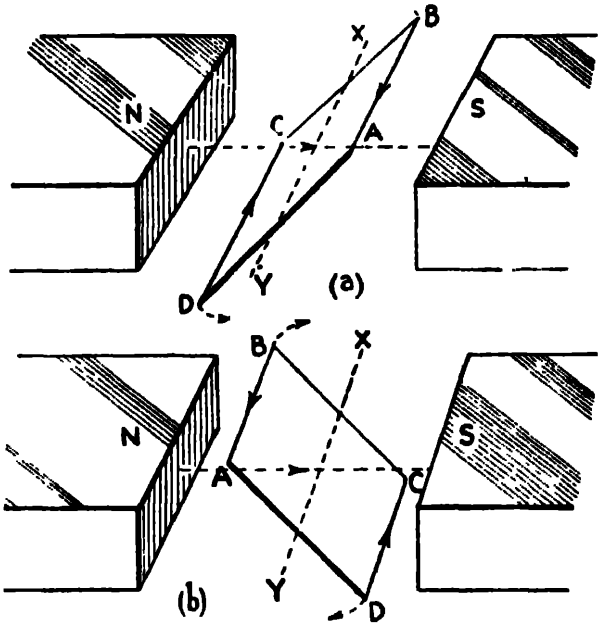


FIG. 498.

If current is sent through the armature coils of a direct current dynamo, the armature will revolve and may be made to drive other machinery. Thus it becomes a **motor.**

If in the coil ABCD in Fig. 498 (*a*) current is running in the direction BADC, Fleming's left hand rule indicates that the coil would tend to rotate about a horizontal axis XY in a counterclockwise direction.

Fig. 498 (*b*) indicates that if the current were still flowing in the same direction after the plane ABCD had passed the vertical the direction of motion would be reversed. Hence the armature coils must take current from a split ring commutator which will reverse the current in them every time they pass the vertical position.

Motor Starting Switch

The armature of a motor, rotating between the poles of a magnet, will act as a dynamo. Comparison of Figs. 498 and 490 will show that the induced E.M.F. tends to send current in the opposite direction to that driving the armature. Hence a greater P.D. has to be applied to the armature than would be the case if this dynamo effect were absent.

At the moment of switching current on to the armature there is no induced P.D. so if the full driving P.D. were immediately applied much too large a current for the coils might pass and the coils be burned out. To avoid this it is generally arranged that when the circuit is first closed there is a high resistance in it. As the starting switch is brought over it gradually cuts out this resistance, and so the current is gradually brought up to its full value.

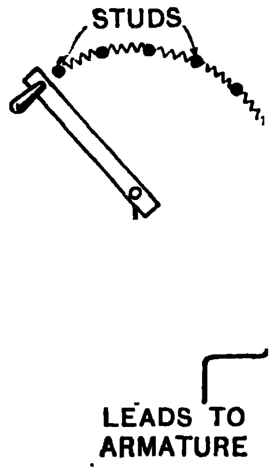


FIG. 499.

QUESTIONS ON CHAPTER XLVIII

1. Describe one method of producing an *induced current.* Upon what factors does the value of such a current depend? Name two practical applications of induced currents. [L.U.

2. Describe an experiment to illustrate the production of an induced current and give a diagram showing the direction of the current.

Describe a simple form of dynamo and explain how it works. [L.U.

3. State the chief facts relating to electromagnetic induction. Describe experiments to illustrate them and show on explanatory diagrams the directions of the induced currents obtained in the experiments you describe. [L.U.

4. A circular coil of wire is placed in a uniform field of magnetic force and rotated about an axis which lies in the plane of the coil, passes through its centre and is at right angles to the direction of the magnetic force. State the electrical effects produced in the coil as it is rotated through 360°, representing the results graphically if possible.

Describe the arrangements by which it would be possible to obtain from the coil (*a*) an alternating current, (*b*) a direct current. [L.U.

5. Describe a simple transformer that can be used to reduce an alternating current from a high voltage to a lower one. Explain the principles underlying its action. [L.U.

6. Make a careful drawing to illustrate an induction coil and give an explanation of the production of sparks from it.

7. Describe how sounds are transmitted by telephone, explaining the actions of both transmitter and receiver.

8. Describe, with the aid of a diagram, the construction of a moving coil ammeter.

In what respects does the construction of this instrument r[illegible] and differ from that of an electric motor?

9. What is meant by electromagnetic induction? State t[illegible] of electromagnetic induction and describe a simple form of [illegible]

10. If you were provided with two flat circular coils des[illegible] arrangements you would make to show that the alteration in current strength in one coil may produce a current in the other. State and explain a rule for determining the direction of the induced current and show how, if the centre of the coils are maintained at a fixed distance apart, they should be arranged to give (*a*) the greatest possible induced current, (*b*) the least possible. [L.U.

CHAPTER XLIX

ALTERNATING CURRENTS

It was shown on page 591 that a dynamo with plain slip rings would generate an E.M.F. which changed direction twice per revolution of the coil and which varied in magnitude as indicated in Fig. 493 (*a*). The current in the external circuit would undergo a corresponding series of alternations and changes in magnitude. Most public electric systems supply such alternating currents and this chapter will deal with some of their chief properties.

NOTE.—For practical work on A.C. it is advisable to have a step-down transformer so that E.M F.s of lower voltage than that of the lighting system can be used. Also a bicycle dynamo or some other alternator that can be worked by hand is useful for generating A.C. at varying frequencies. Exact details for experiments will not be given, as coils, etc., have to be adapted to voltages and frequencies available.

Magnetic Effects

If the experiments for demonstrating magnetic fields due to currents described on page 538 are repeated, using A.C., the iron filings patterns produced will be the same as those previously obtained with D.C. Also a solenoid carrying A.C. will attract a piece of iron suspended near one of its ends, showing that it behaves as a magnet. If, however, experiment (1) (page 538) is repeated with A.C., neither end of the suspended solenoid will be steadily attracted by a given pole of a magnet but, when a magnetic pole is brought near one of its ends, the solenoid will quiver slightly if the alternations are rapid and may show oscillations through a larger angle with slower altenations. This indicates that the end of the solenoid is alternately attracted and repelled by the magnetic pole, that is its polarity alternates with the alternations of current. These results indicate that alternating currents give rise to magnetic fields of the same form as those due to direct currents but, as might be expected, the directions of the lines of magnetic force and hence the polarity at the ends of a solenoid carrying alternating current reverse each time the current changes direction. The solenoid still acts as a magnet towards a piece of iron since it will act inductively on the iron, and each time the polarity of the solenoid reverses the induced poles in the iron will be reversed. Thus the pole at the end of the solenoid and that facing it on the piece of iron will always be unlike poles and will attract one another.

The **moving iron ammeter,** described on page 550, is based on this magnetic effect and, since the strength of the magnetic field depends on the strength of the current and not on its direction, the instrument will give a measurement of an alternating current. It should be noted that the strength as well as the direction of the current varies but the changes are so rapid, many public supplies going through the whole series of changes fifty times per second, that the instrument will show a steady deflection indicating an *average* strength of the current.

Heating Effect

Here again we have an effect that depends on the size of the current and not on its direction. This heating effect may be demonstrated in the same way as it was for direct currents (see page 575) and the same relations between heat developed and current will be obtained. The **hot wire ammeter** described on page 581 is based on the heating effect and, as in the case of the moving iron ammeter, owing to the rapidity of the alternations it will give a steady reading indicating an average value of the current with an alternating current.

Since the heating effect is proportional to the square of the current (see page 576) the direct current which has the same heating effect as a given alternating current will be equal to the square root of the average value of the square of the A.C. which differs somewhat from the average value of the current. It is this so-called "root-mean-square" value of an A.C. which is measured by a hot wire ammeter and it can be shown that it is equal to $\frac{1}{\sqrt{2}}$ times the maximum value of the current.

Chemical Effect

Connect the water voltameter (page 506) in series with a moving iron or hot wire ammeter, a rheostat and a source of alternating current. Adjust the rheostat so that a current of about 0·5 amp. passes through the voltameter. It will be found that the volumes of gases collected in the two tubes are equal. Compare this with the result given on page 507. When a considerable quantity of gas has collected cork one of the tubes while its mouth is still under water, invert it, *wrap it in* *a thick duster for protection in case it shatters*, remove the cork and apply a light to the gas. A sharp explosion will indicate that the gas is a mixture of hydrogen and oxygen.

The explanation of the action is the same as that given in Chapter XLIII. except that at each alternation of current the electrodes will interchange the parts they play as anode and cathode. Thus, in the bulk of the electrolyte, while the positive and negative ions will always be moving in opposite directions, each set will keep reversing its direction of motion and there will be no continuous separation of them. But some ions near the electrodes during their oscillations will make contact with the electrodes and be discharged and, owing to the alternations, oxygen and hydrogen will alternately be liberated at each electrode.

Measurements of the volumes of gases liberated in different times by a steady current, as on page 507, will show that Faraday's Laws apply to electrolysis by alternating currents as well as by direct currents.

If alternating current is passed through a copper voltameter there will be no change of weight in either electrode, since the electrode actions described on page 517 will take place alternately at each electrode and each will have equal masses of copper deposited on it and dissolved from it.

Alternating Current in Solenoids, Chokes

For the following experiments a solenoid made by winding closely 10 to 12 layers of thin cotton-covered copper wire on a tube about six inches long and half an inch in diameter, is required. An iron core made up of a bundle of wires or strips of soft iron which can easily be slipped in and out of the coil should be prepared.

1. Connect the solenoid without the core in series with a lamp and a rheostat and a source of direct current. Adjust the rheostat until the lamp glows brightly. If the core is rapidly pushed into the solenoid the lamp will be dimmed for a moment but will resume its steady glow when the core is stationary. If the core is now rapidly withdrawn the lamp will give a very bright flash.

2. Repeat the experiment, using alternating current. The lamp will be dimmed when the core is inserted and remain dim so long as the core is inside the solenoid. Also, if the core is inserted or removed in stages it will be found that the greater the length of core inside the solenoid the greater is the dimming effect.

3. Similar effects may also be shown by setting up a circuit as shown in Fig. 500. S is the solenoid with its iron core and R is a wire whose resistance is large compared with that of S. Apply a direct E.M.F. to XY and the lamp A will light up but not B since most of the current will pass through the branch SA which has the lower resistance. With an alternating E.M.F. B and not A will light up, showing that most of the alternating current passes through the branch RB.

These experiments illustrate the fact that a solenoid, particularly one with an iron core, acts towards alternating current as though its resistance is greater than that determined with direct current. This effect is due to the self-induction of the coil (see page 587). In experiment (1), as the core approaches the solenoid it becomes magnetised by induction. This increases the number of lines of magnetic force passing through the coil and this change in the magnetic field will set up an induced E.M.F. in the solenoid which, in accordance with Lenz's Law, will oppose the effect producing it, that is, it will act in the opposite direction to the E.M.F. driving current through the solenoid. Thus the current is momentarily reduced and the lamp is dimmed. But this inductive effect only lasts so long as the magnetic field through the solenoid is changing, so when the core becomes stationary the induced E.M.F. disappears and the current returns to its steady value. On withdrawing the core from the solenoid the number of lines of force through it is reduced and an induced E.M.F. momentarily increases the current to oppose this change and causes the bright flash of the lamp.

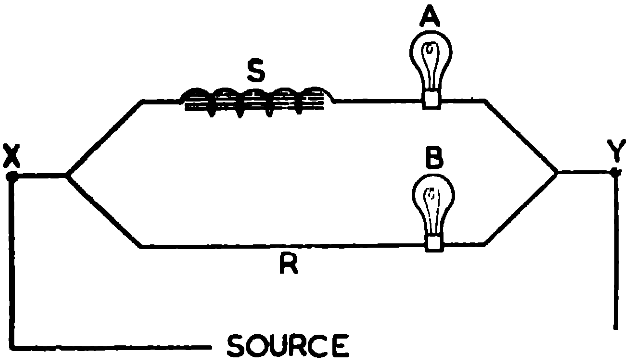


Fig. 500.

In experiment (2) each change in direction of the current will reverse the magnetisation of the core and so reverse the direction of the lines of force through the solenoid. Thus there is a continuously changing magnetic field inside the solenoid and the continuous production of induced E.M.F. opposing the passage of the current which is therefore reduced so long as the core is present. Also, the degree to which the core is magnetised and hence the magnitude of the change in the magnetic field at each alternation will become greater as more of the core is inserted into the solenoid so that if it is pushed in steadily there will be a continuous reduction of the current.

It should be noted that these effects will be present to some extent even when the solenoid has no iron core though the presence of the latter much increases the effect owing to the increase in the changes of magnetic field it produces. When a solenoid, with or without an iron core, is introduced into an A.C. circuit in order to reduce the current it is called a **choke.** The apparent resistance to A.C. of a circuit containing a choke is termed the **impedance** of the circuit, and for A.C. the Ohm's Law equation is replaced by $\frac{\text{Potential Difference}}{\text{Impedance}} = \text{Current}$. The impedance depends partly on the resistance of the circuit determined for direct current according to Ohm's Law and partly on a quantity called the **reactance** of the choke which depends on its self-inductive properties and it can be shown that

$$(\text{Impedance})^2 = (\text{Resistance})^2 + (\text{Reactance})^2.$$

Thus, if the resistance of the circuit is very small the impedance is approximately equal to the reactance. Since the induced E.M.F. is proportional to the *rate* at which the magnetic field changes the reactance of a choke is increased by increasing the number of turns in it or by the provision of an iron core. It is also increased by increasing the frequency of the A.C. that is the number of times per second that the current goes through its complete series or cycle of changes. Thus in the case of very high frequency A.C., such as is found in certain parts of radio circuits where there may be frequencies of about one million cycles per second, a solenoid with only a few turns and without an iron core may have a very large reactance and produce a very large impedance to the current so that very high frequency currents may be almost entirely suppressed by such a choke.

Alternating Current and Condensers

If a condenser is connected in series with a lamp in a circuit to which a direct E.M.F. is applied the lamp will not give a continuous light as the current cannot pass through the dielectric between the plates of the condenser. If an alternating E.M.F. is substituted for the direct one the lamp will light up. Because of this and other similar results it is often said that A.C. will flow through a condenser. This is not strictly true and the reason for the lamp lighting with A.C. can be illustrated as follows.

A circuit as shown in Fig. 501 is set up, E being a high-tension battery, C a condenser of large capacity, and L a lamp. The switch *x* can make contact with either *b* or *c*. When *x* is brought in contact with *b* there will be a momentary flash from the lamp. The lamp will flash again when *x* is switched over from *b* to *c*. When the connexion with *b* is made there will be a flow of electricity to the condenser until the potential difference between its plates is equal to that between the poles of the source. The actual flow will be one of electrons from the negative pole of the source to the condenser plate to which it is connected which thus becomes negatively charged and a flow of electrons from the other condenser plate to the positive pole of the cell leaving the second plate positively charged. When the connexion is switched from *b* to *c* the condenser will discharge, electrons flowing from its negatively-charged plate to neutralise the positive charge on the other plate. Thus there is a momentary flow of electricity through the lamp causing it to flash each time the connexion is switched over.

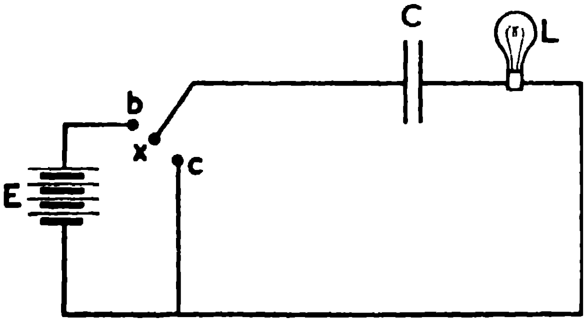


FIG. 501.

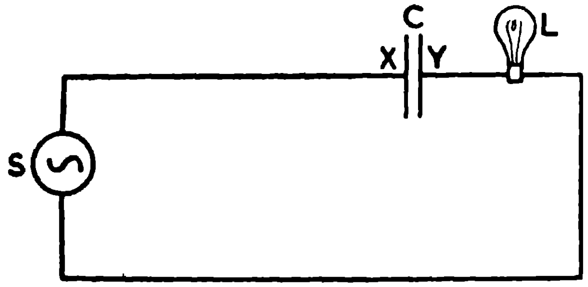


FIG. 502.

When a source of A.C. is connected in series with a condenser and lamp (Fig. 502) similar results are obtained. At one moment electrons will be flowing from the source to plate X, charging it negatively, and to the source from plate Y which becomes positively charged. But a moment later the direction of the E.M.F. of the source will be reversed and so electrons will flow from X to the source and from the source to Y, reversing the charges on the plates. Thus, there is a continuous flow of electricity through the lamp, though it changes its direction each time there is an alternation of the source. From this it will be seen that the current does not really flow through the condenser but there is a continuous series of surges of current from one condenser plate to the other, the direction changing with every alternation of the source, giving the same effect as if A.C. were flowing right round the circuit.

For an A.C. circuit containing a condenser as for one containing a choke we may write—

$$\frac{\text{Potential Difference}}{\text{Impedance}} = \text{Current},$$

and again

$$(\text{Impedance})^2 = (\text{Resistance})^2 + (\text{Reactance})^2.$$

If experiments are made with condensers of different capacities it will be found that the lamp glows more brightly as the capacity of the condenser is increased and it can be shown that the reactance is inversely proportional to the capacity of the condenser. Also, if currents of varying frequency are used it will be found that the reactance of a given condenser is less for high-frequency currents than for low-frequency currents. Hence a condenser of moderate capacity in a circuit will offer little impedance to very high frequency currents but may suppress low-frequency currents.

Electrical Transmission. Advantages of A.C.

The various transformations of voltages during electrical transmission were mentioned on page 588. There are two reasons why high voltage is more economical than low voltage for transmission over long distances, (*a*) the energy lost through the generation of heat in the cables is less, and (*b*) thinner cables can be used and so there is a saving in the cost of metal for them. This may be illustrated as follows. Suppose it is required to transmit electrical energy at the rate of 500,000 watts from one place to another. Since watts = volts × amps., this might be done by sending a current of 5 amps. at 100,000 volts or a current of 500 amps. at 1000 volts. But the rate at which heat is generated in the cables is proportional to the square of the current through them. Hence the rate of loss of electrical energy through its conversion to heat in the cables would be 10,000 times as much with the second current as with the first, also, to carry the much bigger current safely a much thicker cable would be required.

Alternating current is much more suitable than direct current for this transmission with various stages of voltage change since the changes can be effected by means of transformers (see page 587) which do not require much attention and which have a high efficiency. The energy transmitted by the output circuit of a transformer is usually more than 85 per cent. of that dissipated in the input circuit. Direct current cannot have its voltage changed by transformers, but each voltage change would require the use of a motor and dynamo which would require much more attention than a transformer and in which there would be much greater energy losses.

QUESTIONS ON CHAPTER XLIX

1. Illustrate by graphs the difference between an alternating current and a direct current of electricity. Use the graph to explain what is meant by the terms *cycle* and *frequency* as applied to *alternating* current.

2. Compare direct current and alternating current with regard to (*a*) magnetic effects, (*b*) heating effects.

Describe and explain the action of an instrument which will measure alternating current by means of one of these effects.

3. Describe and explain the different effects produced by passing (*a*) direct current, (*b*) alternating current, through a water voltameter.

Could you measure an alternating current with a copper voltameter? Give reasons for your answer.

4. Write notes on the meaning of the terms *resistance*, *impedance*, and *reactance* and give the relation between these three quantities in a circuit carrying alternating current.

5. What is a *choke*? Describe experiments to show the effects of a choke on (*a*) direct, and (*b*) alternating currents. Give an explanation of the effects.

6. If you wished to arrange for the gradual dimming of the lights on a stage, what would you include in the circuit and how would you operate it if the electrical supply were (*a*) alternating current, (*b*) direct current?

7. Is it correct to say that alternating current can pass through a condenser? Describe an experiment to illustrate why this is frequently said and give an explanation of what really happens.

8. A lamp is connected in series with two condensers, one on each side of it, a hot wire ammeter, a switch, and a source of current. Draw a diagram of the circuit.

What will be observed when the switch is closed if the source has (*a*) a direct E.M.F., (*b*) an alternating E.M.F.? Explain these results.

9. How would you set up circuits (*a*) to pass very high-frequency currents but stop low-frequency currents, (*b*) to pass low-frequency currents but to stop very high-frequency currents?

CHAPTER L

DISCHARGE TUBES, X-RAY TUBES, AND VALVES

Discharge through Gases. Cathode Rays

In Chapter XLVII. some account of electrical discharges through gases was given in connexion with discharge lamps. Further details of the process will now be given.

At atmospheric pressure a voltage gradient of about 30,000 volts per cm. is required to produce sparking across an air gap. The molecules of gas are so near together at such a pressure that a very high potential difference is needed to give such ions as may be present sufficient velocity in between collisions to ionise molecules with which they collide (see page 580). At low pressures the "free paths," *i.e.* the distances between molecules, are much longer so the ions travel further in between collisions and while travelling these longer paths a lower potential difference will accelerate them to the required velocity and the gas becomes more conducting.

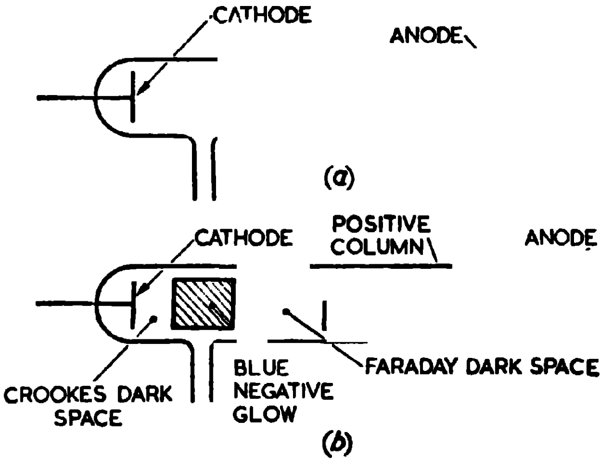


FIG. 503.

Several interesting phenomena may be observed by passing a discharge through a tube such as that illustrated in Fig. 503 (*a*). The anode and cathode are connected to the terminals of the secondary winding of an induction coil to apply a sufficient voltage to them and the side tube is connected to an air-pump so that the pressure may be reduced in the tube by pumping air from it. When the pressure is reduced to 2 or 3 cm. of mercury the spark discharge is seen to take the form of violet streamers between the two electrodes. As the pressure is further reduced the spark path broadens and at about 1 cm. pressure there is a steadier discharge and at a pressure of about 4 mm. an orange glow, known as the positive column, extends from the anode almost all the way to the cathode which is surrounded by a blue glow, the negative glow. A dark space, known as the Faraday dark space, separates the two glows. At between 1 and 2 mm. pressure the negative glow moves from the cathode and a second dark space, the Crookes's dark space, develops [Fig. 503 (*b*)]. This is about as far as the process can be carried with even a good air-pump, but tubes can be obtained in which the pressure has been reduced still further by special means. Such further lowering of the pressure results first in the shortening of the positive column and a lengthening of the Crookes's dark space. At still lower pressure the positive column breaks up into a number of discs and, finally, the Crookes's dark space fills the whole tube. At this stage the glass, particularly towards the anode end, exhibits a greenish fluorescence.

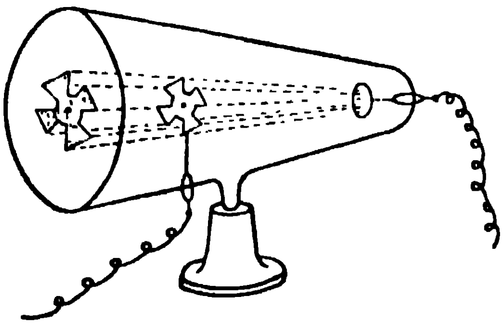


FIG. 504.

This fluorescence can be shown to be due to a radiation referred to as Cathode Rays, travelling in straight lines from the cathode. For instance, if a tube is fitted with an anode in the form of a Maltese cross as illustrated in Fig. 504, a shadow of it will be formed on the end of the tube. The cathode rays exert force on obstacles placed in their path. This has been shown by mounting a small wheel fitted with light vanes with its axle resting on two rails inside a discharge tube. The anode and cathode are so placed that the rays will fall on the upper vanes and not the lower ones, and when the discharge takes place the wheel is driven away from the cathode. This suggests that the rays consist of streams of particles with definite masses. The rays are also deflected by magnetic and electrical fields and the direction of deflection is that which would be given to a current of negative electricity flowing from the cathode. This deflection can be shown by the movement of the fluorescence on the walls of the tube when the pole of a magnet is brought near.

By experiments whose descriptions are beyond the scope of this book, the actual mass and charge of particles forming the cathode rays have been determined, and it has been established that they are electrons moving with very high velocities. They may penetrate thin layers of light substances such as aluminium but are usually absorbed when they fall on solids, and the fluorescence they excite in glass and a number of other substances is due to the transformation of the kinetic energy they lose into energy of wave motion giving rise to waves in the range of wave-lengths corresponding to the visible part of the spectrum. When they fall on solids with very heavy atoms, for example, platinum, they are stopped more quickly and waves in the X-ray range are emitted.

X-ray Tubes

Early X-ray tubes (Fig. 505) had concave cathodes. Since the cathode rays leave the cathode normally this enabled a beam of them to be concentrated on a target of platinum or tungsten which was embedded in a block of copper to conduct away the heat into which some of the kinetic energy of the electrons is converted. The anode was placed in a side tube. Its position does not affect the paths of the cathode rays. The target and anode were usually connected. This arrangement produced a steadier discharge than if the target alone was made the anode. When a discharge was passed through the tube a beam of X-rays was thrown out from the target. The type of X-ray produced depended on the gas pressure in the tube. The potential difference needed to produce the discharge is a minimum when the pressure is about 1 or 2 mm. At this pressure only a weak beam of X-rays of comparatively long wave-length and little penetrating power, so-called "soft rays," is produced. With lower pressures, requiring a higher voltage for discharge, a powerful beam of "hard rays" with shorter wave-length and more penetrating power is given out.

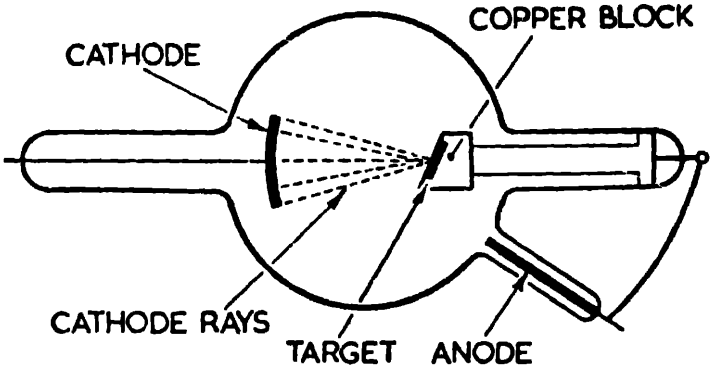


FIG. 505.

Modern X-ray tubes are usually modifications of the Coolidge tube, the principle of which is indicated in Fig. 506. The tube is almost completely evacuated and the source of electrons is the coil X which is heated by an electric current. (See "Thermionic Emission".) X is surrounded by a molybdenum cylinder C which concentrates the electrons into a narrow beam. A potential difference is applied between X and the target T which is at the higher potential so that the electrons are driven towards it. Increasing the current through X will raise its temperature and thus increase the rate of emission of electrons and consequently the intensity of the beam of X-rays produced. Increasing the potential difference between X and T increases the velocity of the electron stream and so produces X-rays of greater hardness. Thus the hardness and intensity of the X-ray beam can be separately controlled.

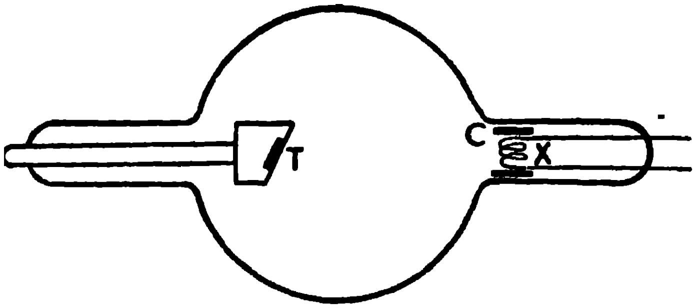


Fig. 506.

Cathode Ray Tubes

The main features of a cathode ray tube are indicated in Fig. 507. Electrons are emitted from a heated coil X in a highly-evacuated tube, as in the Coolidge tube. The anode A, maintained at a higher potential than X, is perforated so that a narrow pencil of electrons driven from X passes through it. The screen S is coated with a fluorescent substance so that a spot of light is produced at the point where the electron stream falls on it. At P the pencil passes between two horizontal plates. If a potential difference is applied to them the electric field between them will deflect the pencil and the spot on the screen will move. If the potential difference is alternating the spot will move up and down on the screen giving the appearance of a vertical line of light. At Q the pencil passes between a pair of horizontal plates so that a potential difference applied to them will deflect the spot horizontally. These plates are connected to what is known as a time-base circuit, which

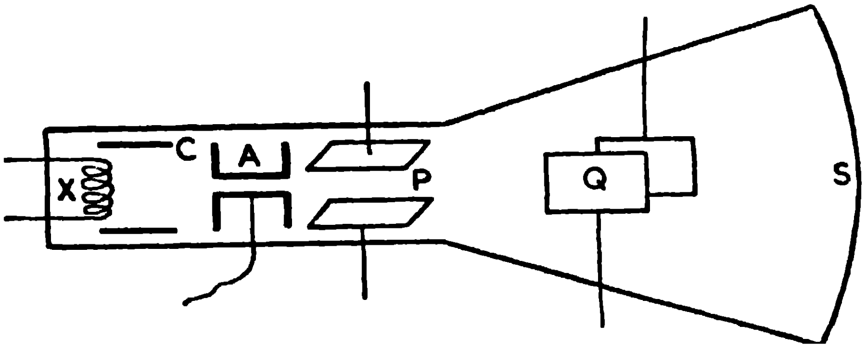


Fig. 507.

varies the potential difference in such a way that the spot will travel steadily from one side to the other of the screen and then flash back to its starting point. If an alternating potential difference is applied to P and the time-base circuit to Q, the spot will trace out a wavy line on the screen which will indicate the way in which the potential difference applied to P varies.

The cathode ray tube can be used for investigating alternating currents and for demonstrating various properties of wave motion, and it is used for the reception of television and in radar.

Thermionic Emission Valves

In previous chapters it has been indicated that metals act as conductors because they contain a large number of electrons very loosely attached to their atoms. These electrons can move within the metal almost as freely as the molecules in a liquid and have a tendency to make random movements similar to those of liquid molecules. The velocity of these movements increases with rise of temperature and at a certain temperature some of them acquire sufficient kinetic energy to escape through the surface of the metal much as molecules of liquids escape during evaporation. This escape of electrons from heated metals is known as thermionic emission. The rate of emission increases very rapidly with rise of temperature and, because it can be heated to a high temperature without melting, tungsten is the metal generally used for the production of electrons in this way. It has been found, however, that if a tungsten filament is coated with a thin layer of barium oxide or of thorium it will emit electrons much more readily and give off a good stream of them at dull-red heat, constituting what is known as a dull-emitter.

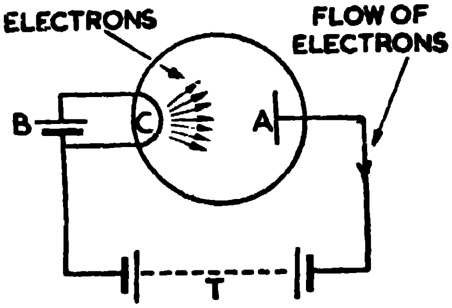


FIG. 508.

The **thermionic valve,** invented by Fleming in 1904, and now so widely used in connexion with radio, is based on thermionic emission. The **diode valve,** shown diagrammatically in Fig. 508, is so-called because it has two electrodes. The cathode C is a filament which will emit electrons when heated by a current from the source B and is enclosed in a vacuum bulb. The concentration of electrons produced in the bulb gives rise to what is known as a *negative space charge* which repels further electrons which might escape from the filament and tends to prevent continued emission. The anode A consists of a nickel plate and a big potential difference can be maintained between it and C by means of a high tension source T. If A is maintained at a higher potential than C the emitted electrons will be driven from C to A and then flow through T in the direction shown. If the connexions with T are reversed no current will flow, as A will be at lower potential than C, and electrons will not be driven towards it. If an alternating potential difference is introduced at T, A will be alternately at higher and lower potential than C and it is only during the periods when it is at higher potential that current will flow through T. It is because the arrangement will only allow current to flow one way that it is known as a valve and is frequently used as a **rectifier**, that is a means of obtaining current or potential difference in one direction only from an alternating source. If the circuit of Fig. 509 is set up, S being an alternating source, as explained above, current will flow through any system of conductors connected to the two terminals T_1 and T_2 only during those half-periods of S when A is at higher potential than C and the flow of electrons will always be from T_1 to T_2, that is, the conventional current will flow from T_2 to T_1. This **anode or plate current** will be pulsating, the variations in it being as indicated graphically in Fig. 510, but it may be used for such purposes as charging accumulators. If the circuit of Fig. 511 is used, R being a resistance coil, the potential difference between the ends of R will always be the product of the anode current and the resistance of R, so a pulsating potential difference always in the same direction will be applied to any set of conductors connected to X and Y.

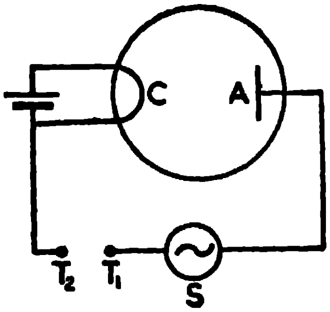


FIG. 509.

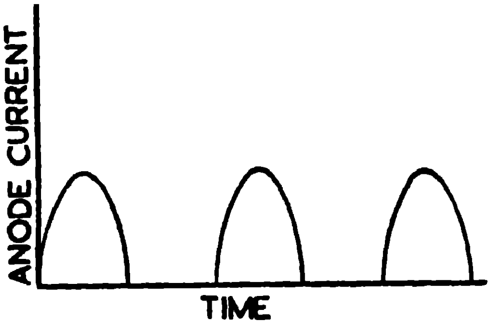


FIG. 510.

The strength of the anode current will depend on the temperature of C, which determines the rate at which electrons are liberated from it, and the potential difference between C and A (anode potential) which determines the speed with which the electrons travel from C to A. The relation between anode current and anode potential for a fixed filament temperature can be investigated by using the circuit of Fig. 508 with an ammeter inserted between T and A. The battery B should have a fixed E.M.F. of, say, 2 volts, and T should be a high tension battery of dry cells tapped so that various E.M.F.s can be applied by it. The results will be as illustrated in Fig. 512. At first raising the anode potential increases the current but at a certain stage the current remains constant as the potential increases. The reason for this is that at a certain voltage the electrons are attracted to the anode at the same rate as they are produced by the cathode so further voltage increase cannot increase the current.

The **triode valve**, as indicated in Fig. 513, has a third electrode, G, known as the grid because it is perforated, inserted between the cathode and anode. In practice the anode is an open-ended cylinder; the cathode is a straight filament passing along the axis of the anode and the grid is an open coil of wire surrounding the cathode filament. If a constant potential difference is maintained between C and A the resulting anode current can be regulated by giving various potentials to G. Thus if G has a positive potential electrons emitted from C will be attracted towards it and most of them will shoot through the spaces in it and then be attracted to A. If the potential of G is raised it will give a greater velocity to the electrons it attracts so that more of them reach A in a given time and the anode current is increased. On the other hand, a small negative potential at G will tend to repel the electrons so that few pass through the grid and the anode current is small. If G is given a sufficiently great negative potential it will repel the electrons so strongly that none will get through and there will be no anode current.

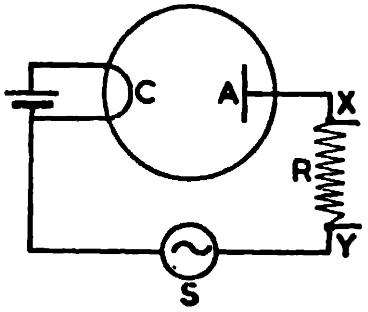


FIG. 511.

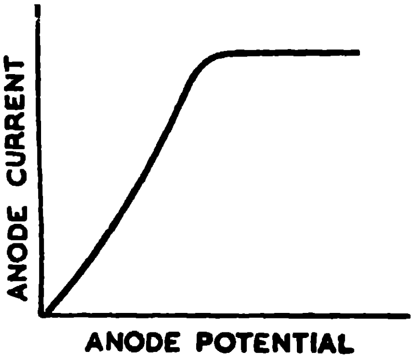


FIG. 512.

The relation between grid potential and anode current can be investigated by means of the circuit shown in Fig. 513. The steady potential difference between C and A is maintained by the high tension battery T, while different potentials can be applied to the grid by means of the tapped battery T_1. The anode current is measured by the milliammeter *m*A. The results plotted into a graph give a *characteristic curve* of the valve which is of the form shown in Fig. 514. The negative grid voltage at A will suppress the anode current altogether. Between B and C the graph is very steep showing that a small change of grid potential causes a relatively big change in the anode current.

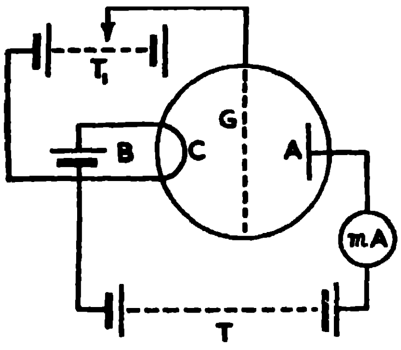


FIG. 513.

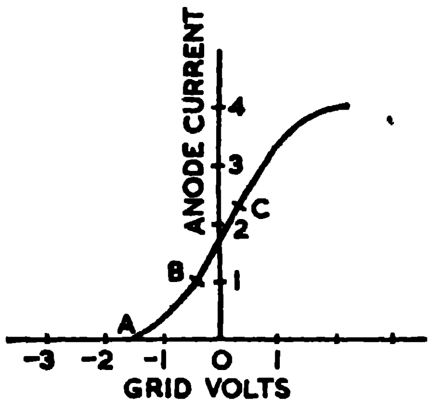


FIG. 514.

One of the main uses of the triode valve is to act as an **amplifier**. If an alternating source whose maximum and minimum E.M.F.s have the values of the grid volts corresponding to B and C is substituted for T_1 there will be big charges in the anode current and corresponding big charges in the potential difference between the ends of a conductor through which the anode current flows. Thus an oscillating potential difference of the same frequency as that applied to the grid but having a much greater amplitude can be obtained. It should be noted, however, that the amplified potential difference will always act in the same direction and not alternate.

ANSWERS

Chapter III. (*Page* 21)

1. 7·33 grm./c.cm., 43·2 lb./cub. ft., 1·1 grm./c.cm., 85·3 lb./cub. ft.
2. 975 lb., 1305·6 grm., 9 lb., 806 grm. **3.** 73·53 c.cm., 271·3 cub. in., 875 c.cm.
5. Zinc. **7.** 1483 cm. **8.** 0·871 grm./c.cm. **9.** 0·93 grm./c.cm.
10. 362 tons. **11.** (*a*) 52·51 c.cm., (*b*) 1·84, (*c*) 184 lb. **12.** 39 tons.
13. 2·5. **14.** 154·5 lb. **16.** 1950 grm. **17.** 0·8722, 2·18.

Chapter IV. (*Page* 39)

2. 7200 yd., 6 sec., 1666·7 cm./sec., 30 ml. per hr., 4166·7 cm.
3. $\frac{1}{6}$ ft./sec.^{2}, 7500 ft.; 150 cm./sec., 1125 cm.; 40 cm./sec., 4 cm./sec.^{2}; 26 ft./sec. 105 ft.; 16$\frac{2}{3}$ sec., 794·4 ft.; 40 cm./sec., 2 cm./sec.^{2}.
4. 5·59 sec., 178·9 ft./sec.; 176·4 m., 5880 cm./sec.; 14·3 sec., 140 m./sec.; 2304 ft., 12 sec.; 57,600 ft., 1920 ft./sec. **5.** 2 ft./sec.
6. (*a*) 900 ft./sec., (*b*) – 2·4 ft./sec.^{2}, (*c*) 8 ml. 3410 ft. **8.** 979 cm./sec.^{2}.
9. 252·98 ft./sec., 7·906 sec. **10.** (*a*) 3600 ft., (*b*) 480 ft./sec.
13. 12,000 dynes, 112 pndls., 2 lb., 4900 grm., 6·4 ft./sec.^{2}, 100 cm./sec.^{2}.
14. 18$\frac{1}{3}$ tons-wt., 1320 ft. **15.** (*a*) 1·92 ft./sec$'^2$, (*b*) 118·72 lb.-wt.
17. 2665 lb.-wt. **18.** 2$\frac{1}{3}$ ml. per hr. **19.** 120 ft./sec., 75 tons-wt. **22.** 4·33 sec.

Chapter V. (*Page* 52)

3. (*a*) 20 lb.-wt., 36° 52′: (*b*) 150 grm.-wt., 36° 52′; (*c*) 20·25 pndl., 32° 57′.
4. 21° 48′ E. of N., 16·16 ml. per hr.
6. 141·4 lb.-wt., 141·4 lb.-wt.; 75 grm.-wt., 129·9 grm.-wt.; 1·732 ton-wt., 1 ton-wt.
7. (*a*) 86·6 lb.-wt., (*b*) 50 lb.-wt., (*c*) 150 lb.-wt. **8.** 250 grm.-wt.
9. 3·214 units, 41° 17′ W. of N. **10.** 774·6 lb-wt.
11. (*a*) 2·598 grm.-wt., 1·299 grm.-wt.

Chapter VI. (*Page* 68)

4. 26 cm., 120 in., 712·5 grm.-wt., 150 grm.-wt. **7.** 50 grm.
8. 0·818 in. from centre of square. **9.** 0·1414 in. from centre of square.
11. 6° 51′. **16.** 7·765 lb., 4·235 lb., 20 lb. **17.** 6 tons-wt., 3·5 tons-wt.

Chapter VII. (*Page* 76)

3. (*a*) 504,000 ft.-lb., (*b*) 15·3 h.p. **4.** 4,242,857 ft.-lb., 2·14 h.p.
5. 3,960,000 ft.-lb., 89·52 kW. **6.** 162×10^{10} ergs.
7. 924,000 ft.-lb., 0·156 h.p. **11.** 20·17 ft.

Chapter VIII. (*Page* 97)

2. 4, $3\frac{1}{3}$, $\frac{5}{6}$. **4.** 5, $74\frac{2}{3}$ lb.-wt., 36 st. **5.** 9·6.
6. 373·3 lb.-wt., 14,933 ft.-lb., 11,200 ft.-lb. **7.** 0·8 in., 9·5 in.
9. 0·200. **10.** 24·02 lb.-wt. **13.** (*a*) 896 ft.-lb., (*b*) 995·6 ft.-lb.
14. 4, 3·33, 83·3%. **16.** 3·3 lb.-wt., 419 ft.-lb.

Chapter IX. (*Page* 105)

4. (*a*) 286,364 grm./cm.^{2}, (*b*) 0·0001786, (*c*) $1{\cdot}6 \times 10^9$ grm./cm.^{2}.
5. 106·9 kgrm. **6.** 4·716 lb.

Chapter X. (*Page* 115)

4. 32 grm., 0·8, 7·84 cm. **6.** 2·21 cm., 37·5 cm. **9.** 7 cm.

Chapter XI. (*Page* 132)

4. 999,600 dynes/cm.^{2}, 784·6 cm. **5.** 1·095 grm./c.cm.
9. 237·5 c.cm., 6 lb./sq. in., 94·55 cm., 9 lb./sq. in., 690·8 c.cm.
10. 1120 cub. in. **11.** 1440 lb./sq. in. **12.** 22·37 c.cm. **17.** 14·97 lb./sq. in.
18. 0·833. **19.** 27·2 cm. **20.** 1020 grm./sq. cm., 1020 cm.
21. 6 cub. m. **22.** 78 cm. **25.** 10000/14641 or 0·683 atmospheres.
26. 225 cm. **27.** 120 atmospheres; 96 atmospheres; 432 grm.
28. 76 cm. **29.** 75 cm.; 42·9 cm. **30.** 114 cm.; 5·02 m.
31. 48·2 in.; 8033 cub. ft.

Chapter XII. (*Page* 143)

3. 22 c.cm., 7·05, 137·4 grm. **4.** 0·343.
8. 0·5 grm./c.cm., 1·47 grm./c.cm. **9.** 112·5 grm. **11.** 10,667 kgrm.
12. 168 lb. **14.** 0·75. **16.** 4·6875 tons. **17.** 0·4468 lb.-wt.

Chapter XIII. (*Page* 159)

11. 93·8 sec.

Chapter XIV. (*Page* 173)

7. (*a*) − 58° F., − 25° C., − 5° C., 50° F., 15° C., 113° F., 167° F., 95° C.;
(*b*) − 40° C. or F. **9.** 23·9° C., 6·7° C.

Chapter XV. (*Page* 184)

1. 100·1425 cm., 100·09 cm., 100·06375 cm., 100·21 cm. **2.** 2 ft. 1·34 in.
3. 0·000017, 0·000012, 0·000028. **4.** 5280·6336 ft., 0·0118%.
5. (*a*) 0·00189 in., (*b*) 0·001134 in., (*c*) 0·04536 sq. in.

6. 5·57 ft., 2·57 ft. **7.** 1000·255 c.cm.
8. (*a*) 8·00832 cm., (*b*) 3·1494 sq. cm., (*c*) 25·2213 c.cm. **9.** 0·000010.
11. 75 cm., 50 cm. **12.** 50° C. **13.** 1·0008 : 1.

Chapter XVI. (*Page* 193)

2. 0·0008745. **3.** 0·000874, 0·0000087. **4.** 0·00018.
6. 997·6 grm. **7.** 0·0349 sq. mm.

Chapter XVII. (*Page* 203)

4. 237·5 c.cm., 67·6 cub. in., 752 c.cm., 75° C., — 140·3° C., 484·5° C.
5. 91·25 cm., 18·75 lb./sq. in., 56·5 cm., 68·25° C., — 39·875° C.
6. 5·024 litres, 244·3 cub. in., 79·5 cm., — 133·875° C.
7. 149 c.cm. **8.** 25·26 lb./sq. in. **9.** 10 atmos., 115° C.
13. (*a*) 186·5 c.cm., (*b*) 131·5 c.cm.

Chapter XIX. (*Page* 227)

5. $252\frac{3}{7}$ cals.
6. (i) 6300 cals., (ii) 5472 B.Th.U., (iii) 5200 B.Th.U., (iv) 325,000 cals., (v) 121·73 cals., (vi) 280·24 B.Th.U.
7. (i) $61\frac{2}{3}$° C., (ii) 168° F., (iii) 20·4° C., (iv) 48·7° C., (v) 105·8° F.
8. 217° C., 291·5° F., 824° C. **9.** (i) 0·0988, (ii) 0·0307, (iii) 0·0299.
10. 1673·3 cals./min., 0·5544. **11.** Coal. **13.** 4·53 grm.
14. 76 grm. **15.** 0·541. **16.** 102 cub. ft., 9·792 pence.
17. (*a*) 5·59 grm., (*b*) 0·264. **23.** 7·22 grm. **24.** 2·28 cu. ft.; 0·228 pence.
26. 0·12. **27.** 8·8 cals. per degree; 0·557.

Chapter XX. (*Page* 245)

3. (i) 1300 cals., (ii) 10,800 cals., (iii) 1575 cals.
4. (i) 29·1° C., (ii) 34·4° C., (iii) 35·2° C., (iv) 64° C.
5. 144 B.Th.U./lb., 972 B.Th.U./lb. **8.** 28,800 cals. **11.** 141·9 B.Th.U./lb.
13. 48·2° C. **14.** 200 grm., 1200 c.cm.

Chapter XXII. (*Page* 270)

3. 1·7° F. **4.** 0·438° C.

Chapter XXIII. (*Page* 277)

5. (*a*) 9 in., 15 in.; (*b*) 12 ft.

Chapter XXV. (*Page* 304)

2. $+\frac{6}{11}$ in.

6. (*a*) $+ 4\frac{2}{7}$ cm., $\frac{6}{7}$ cm.; (*b*) + 12 cm., 2 cm.; (*c*) − 3 in., $1\frac{1}{2}$ in.; (*d*) $-\frac{6}{7}$ in., $\frac{3}{7}$ in.; (*e*) − 2·4 cm., 0·3 cm.

8. 30 cm., + 10 cm.

13. Concave, 6·25 cm., $+ 8\frac{1}{3}$ cm.

Chapter XXVI. (*Page* 322)

5. 1·44, 0·76 cm.

Chapter XXVII. (*Page* 333)

6. (*a*) $+ 16\frac{2}{3}$ cm., $3\frac{1}{3}$ cm.; (*b*) + 30 in., 3 in.; (*c*) − 30 cm., 12 cm.; (*d*) + 20 in. 3 in.; (*e*) − 18·75 cm., 3 cm.; (*f*) $- 16\frac{2}{3}$ in., 2 in.; (*g*) − 12 cm., 2 cm.

10. (*a*) + 10 in., (*b*) + 20 in., (*c*) − 10 in., (*d*) $+ 6\frac{2}{3}$ in.

11. + 5 cm.

12. (*a*) $- 9\frac{1}{3}$ in., $4\frac{2}{3}$ in.; (*b*) $+ 23\frac{1}{2}$ in., $4\frac{2}{3}$ in.

Chapter XXVIII. (*Page* 348)

4. Concave, 720 in., 6·05 in.

5. 10 dioptres.

9. + 2 cm.

Chapter XXX. (*Page* 369)

3. 0·01 ft.-candle, 0·4 ft.-candle, 0·174 ft.-candle, 0·25 ft.-candle.

4. 14·14 ft., 7·07 ft., 4·47 ft., 3·16 ft.

5. 27 sec.

8. 2·828 ft.

9. 25 cm., still equal.

Chapter XXXI. (*Page* 381)

5. 4000.

Chapter XXXII. (*Page* 395)

9. 1100 ft./sec., 2750 yd.

13. 1116 ft./sec., 1138 ft./sec., 1147 ft./sec., 1156 ft./sec

Chapter XXXIII. (*Page* 402)

2.

Frequency	320	360	400	$426\frac{2}{3}$	480	$533\frac{1}{3}$	600	640
Wave-length	3·44	3·06	2·75	2·58	2·29	2·06	1·83	1·72 fe

8. 1100 ft./sec.

Chapter XXXIV. (*Page* 410)

3. 261 or 251.

Chapter XXXV. (*Page* 425)

4. 80 cm.

5. Reduce by 3·75 cm., Increase by 1·224 kgrm.

Chapter XXXVIII. (*Page* 456)

4. (i) 53° E. of N., (ii) 9° E. of S., (iii) 68° W. of S., (iv) 60° W. of N.

Chapter XXXIX. (*Page* 465)

1. $13\frac{1}{3}$ dynes, 300 webers, 40 cm.
2. 6 oersteds, 0·56 oersted, 0·104 oersted, 2·5 oersteds. **3.** 140 dynes.
4. 0·768 oersted, 0·444 oersted, 0·280 oersted, 0·188 oersted, 0·132 oersted; 34·1 cm.
5. 0·25 oersted, 0·15 oersted.
7. $M_1 : M_2 = 0·5774 : 1$; 57·74 webers; 133·3 webers.
8. $H_1 : H_2 = 1 : 0·7265$; 0·1453 oersted. **9.** 2700 c.g.s. units; 23·8 cm.
11. 0·2 oersted **12.** 1 : 3·375. **13.** 0·2 oersted; 12 cm.
14. 100 c.g.s. units; 12 cm.

Chapter XL. (*Page* 481)

6. 1·5 dynes, 8·17 cm. **7.** 0·833 e.s.u., 7·05 cm. from larger.

Chapter XLI. (*Page* 500)

6. 3 e.s.u., — 7·5 e.s.u.; — 3 e.s.u.; — 45 e.s.u., — 60 e.s.u.

Chapter XLIII. (*Page* 519)

6. 0·926 amp. **7.** 70 min. 38 sec.

Chapter XLVI. (*Page* 570)

2. $\frac{1}{2}$ ohm. **3.** 13·125 ohm. **6.** 0·00435 amp., 0,00066 amp.
8. (*a*) $\frac{5}{95}$ ohm., (*b*) 95 ohm. **10.** $5\frac{5}{99}$ ohm.

Chapter XLVII. (*Page* 581)

3. 79·3%. **4.** (*a*) 20 watts, 30 watts; (*b*) 125 watts, 83·3 watts.
5. 10 amps., $\frac{1}{2}$ amp., 3·3 amp.; 24·9 pence, 15 amp.
7. (*a*) 32 ohms, (*b*) 150 watts.
8. 80 ohms, 2·5 amp.; 20 ohms, 5 amp.; equal rates. **9.** 2500 watts.
11. (*a*) 2 amp.; (*b*) 10 amp.; (*c*) 15 amp.
12. (*a*) 52·9 ohms; (*b*) 174 min.; (*c*) 8·7 pence.

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